CHAPTER 21  THERMODYNAMICS


†21-1. The change in internal energy for this ideal monatomic gas is the difference between the heat absorbed \( Q \) by the gas and the work that it does on the piston. The heat input into the system is

\[
Q = nC_p \Delta T = (1.0 \text{ mole})(20.8 \text{ J/(mole K)})(90 \text{ K}) = 1.9 \times 10^3 \text{ J}
\]

where \( C_p \) is given in Table 20.5. The change in the internal energy is then,

\[
\Delta E = Q - W = nC_p \Delta T - W
\]

\[
= (1.0 \text{ mole})(20.8 \text{ J/(mole K)})(90 \text{ K}) - 800 \text{ J} = 1.1 \times 10^3 \text{ J}
\]

21-2. Because there is no temperature change and no pressure change, the process results in no change in the internal energy of the gas. Therefore, the amount of heat transferred to the system must be equal to the work done by the gas as it expands against the piston.

\[
Q - W = 0
\]

\[
Q = W = p \Delta V = (1.01 \times 10^5 \text{ N/m}^2)(1.50 \times 10^{-5} \text{ m}^3) = 1.52 \text{ J}
\]

†21-3. (a) In step A, the gas isovolumetrically (\( \Delta V = 0 \)) expands, so the work done by the gas \( W = p \Delta V = 0 \). The heat absorbed by the ideal gas is \( Q = nC_v \Delta T \). From the ideal gas law, we have

\[
pV_1 = nRT_1 \quad \text{or} \quad V_1 = \frac{nRT_1}{p_1} = \frac{nRT_2}{p_2}
\]

since the volume is constant. Solving for \( \Delta T = T_2 - T_1 \), gives

\[
\Delta T = T_1 \left( \frac{p_2}{p_1} - 1 \right)
\]

The system is initially at standard temperature and pressure (STP), \( T_1 = 273 \text{ K} \) and \( P_1 = 1.0 \text{ atm} \). One mole of an ideal gas at STP has a volume of 22.4 liters. The heat absorbed in step A is

\[
Q = nC_vT_1 \left( \frac{p_2}{p_1} - 1 \right) = \text{4.0 liters} \left( \frac{\text{1 mole}}{\text{22.4 liter}} \right)(12.5 \text{ J/(mole K)}) (273 \text{ K}) \left( \frac{2p_2}{p_1} - 1 \right) = 610 \text{ J}
\]

The internal energy in this case is equal to the heat absorbed by the system, \( \Delta E = Q - W = Q = 610 \text{ J} \).

(b) In step B, the gas isobarically (\( \Delta P = 0 \)) expands by \( \Delta V = 4.0 \text{ liters} \) \( (10^{-3} \text{ m}^3/\text{liter}) = 4.0 \times 10^{-3} \text{ m}^3 \), so the work done by the gas is

\[
W = p \Delta V = (2.02 \times 10^5 \text{ N/m}^2)(4.0 \times 10^{-3} \text{ m}^3) = 808 \text{ J} = 810 \text{ J}
\]

The amount of heat absorbed by the system in this process is again \( Q = nC_v \Delta T \). From the ideal gas law, we have
\[ p_2 V_1 = nRT_1 \quad \text{and} \quad p_2 V_2 = nRT_3 \]

\[ T_2 = \frac{p_2 V_1}{nR} \quad \text{and} \quad T_3 = \frac{p_2 V_2}{nR} \]

\[ \Delta T = T_3 - T_2 = \frac{p_2 V_2 - p_2 V_1}{nR} = \frac{p_2 \Delta V}{nR} \]

\[ = \frac{2.02 \times 10^7 \text{ N/m}^2}{4.0 \text{ liters} \left( \frac{1 \text{ mole}}{22.4 \text{ liters}} \right) \left( 8.31 \text{ J/(mole} \cdot \text{K}) \right) (4.0 \times 10^{-3} \text{ m}^3)} = 544.5 \text{ K} \]

Then,
\[ Q = nC_p \Delta T = 4.0 \text{ liters} \left( \frac{1 \text{ mole}}{22.4 \text{ liters}} \right) (20.8 \text{ J/(mole} \cdot \text{K}) (544.5 \text{ K}) = 2022 \text{ J} = 2.0 \times 10^3 \text{ J} \]

The internal energy in this case is
\[ \Delta E = Q - W = Q - 810 \text{ J} = 1.2 \times 10^3 \text{ J} \]

21-4. The free expansion of the air is considered an adiabatic process \((Q = 0)\), so the change in the internal energy is equal to the work done by the gas against the atmosphere, \(\Delta E = Q - W = -1800 \text{ J} \). The change in internal energy is also \(\Delta E = nC_v \Delta T\) (see section 20.6). Therefore,
\[ nC_v \Delta T = -1800 \text{ J/mole} \]

\[ \Delta T = \frac{1800 \text{ J/mole}}{nC_v} = \frac{-1800 \text{ J/mole}}{1.0 \text{ mole}(20.8 \text{ J/(mole} \cdot \text{K})} = -86.5 \text{ K} \]

\[ T_2 = T_1 + \Delta T = 293 \text{ K} - 86.5 \text{ K} = 206.5 \text{ K} \text{ or } -66.5 \text{°C} \]

†21-5. In an adiabatic process, heat is neither absorbed nor emitted from a system; \(Q = 0\). Since work is being done on the gas instead of by the gas, the internal energy of the system increases
\[ \Delta E = Q - W = -W \]

Since work is done on the gas, the work done has a negative value and the internal energy is increasing in this situation. According to section 20.6, the change in internal energy is also equal to \(nC_v \Delta T\). Therefore,
\[ \Delta E = nC_v \Delta T = (0.0300 \text{ mole})(20.8 \text{ J/(mole} \cdot \text{K})((790 - 40) \text{ K} = 470 \text{ J} \]

The work done is \(W = -470 \text{ J}\).

21-6. Since all of the heat generated in the collision with the floor stays in the lead ball, \(Q = 0 \text{ J}\). Work is done on the ball by the floor causing a change in its kinetic energy and causing an increase in the internal energy of the ball. In the instant before impact, the ball has a kinetic energy \(K_1\) equal to its initial energy, which was its gravitational potential energy. Since its final speed is zero, its final kinetic energy is zero.
\[ W = \Delta K = K_2 - K_1 = 0 - mgh = -(0.25 \text{ kg})(9.81 \text{ m/s}^2)(0.80 \text{ m}) = -1.96 \text{ J} \]

The increase in the internal energy of the ball is \(\Delta E = -W = 1.96 \text{ J}\).
The increase in the internal energy will increase the temperature of the ball by
\[ \Delta T = \frac{\Delta E}{mc} = \frac{1.96 \text{ J}}{(0.25 \text{ kg})(130 \text{ J/(kg} \cdot \text{°C})} = 0.060 \text{°C} \]

†21-7. (a) Assuming the gas obeys the ideal gas law, \(pV = nRT\),
\[ n = \frac{pV}{RT} = \frac{(1.01 \times 10^5 \text{ N/m}^2)(0.100 \text{ m}^3)}{(8.31 \text{ J/mole} \cdot \text{K})(283 \text{ K})} = 4.29 \text{ moles} \]
(b) The work done by the gas as it expands is
\[ W = p\Delta V = \left(1.01 \times 10^5 \text{ N/m}^2\right) \left(0.110 - 0.100\right)\text{m}^3 = 1010 \text{ J} \]

(c) The change in internal energy is \( \Delta E = Q - W = 3500 \text{ J} - 1010 \text{ J} = 2490 \text{ J} \)

(d) Consider section 20.5 and equations 20.30 to 20.33, in particular. The internal energy of the gas is \( \Delta E = fnR\Delta T \), where \( f \) is a fraction related to the nature of the gas (monatomic or diatomic). For the situation given,
\[ f = \frac{\Delta E}{nRT} = \frac{2490 \text{ J}}{(4.29 \text{ moles})(8.31 \text{ J/(mole} \cdot \text{K})(28 \text{ K})} = \frac{5}{2} \]

Therefore, \( C_V = \frac{5R}{2} \) and the gas is diatomic.

21-8. (a) The initial volume of the water is 1.00 liter = 1.00 \times 10^{-3} \text{ m}^3. Assuming the water vapor obeys the ideal gas law, the volume of the steam vapor at 1 atm is given by
\[ V = \frac{nRT}{p} = \frac{1.00 \text{ kg}\left(\frac{1 \text{ mole}}{0.018 \text{ kg}}\right)(8.31 \text{ J/(mole} \cdot \text{K})(373 \text{ K})}{1.01 \times 10^5 \text{ N/m}^2} = 1.705 \text{ m}^3 \]

In the expansion of the water vapor, work is done by the gas against the atmosphere equal to
\[ W = p\Delta V = \left(1.01 \times 10^5 \text{ N/m}^2\right) \left(1.72 \times 10^{-5} \text{ m}^3\right) = 1.72 \times 10^0 \text{ J} \]

The change in the internal energy during the expansion is
\[ \Delta E = Q - W = mL_V - W = (1.00 \text{ kg})(2.26 \times 10^6 \text{ J/kg}) - (1.72 \times 10^5 \text{ J}) = 2.09 \times 10^6 \text{ J} \]

During the process the internal energy of the system increases due to the heat added to the system, but that energy is reduced by the work done by the vapor. About 8% of the heat input to the system goes into doing work.

(b) Because liquid water is incompressible, there is no volume change as the pressure is increased from 1 atm to 2 atm. No work is done, therefore, on the liquid water. Because there is also no temperature change, there is no change in the internal energy of the water. As for water vapor at 1 atm, the heat of vaporization is \( L_{V,1\text{ atm}} = 2.26 \times 10^6 \text{ J} \). Because the temperature is constant at 373 K, \( \Delta E = 0 \). To compress water vapor from 1 atm to 2 atm, the work done, applying the Ideal-Gas Law, \( p = nRT/V \) and \( p_1V_1 = p_2V_2 \), is
\[ W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} nRT \frac{dV}{V} = nRT \ln \frac{V_2}{V_1} \]
\[ = \left(1.00 \text{ kg} \frac{1 \text{ mole}}{0.018 \text{ kg/mole}}\right)(8.31 \text{ J/(mole} \cdot \text{K})(373 \text{ K}) \ln (0.50) = -1.19 \times 10^5 \text{ J} \]

The heat of vaporization at 2 atm is
\[ L_{V,2\text{ atm}} = L_{V,1\text{ atm}} + W = 2.26 \times 10^6 \text{ J} - 1.19 \times 10^5 \text{ J} = 2.14 \times 10^6 \text{ J} \]

21-9. (a) \[ W = p\Delta V = (1.01 \times 10^5 \text{ N/m}^2)\left(1.000 \times 10^{-3} \text{ m}^3 - 1.091 \times 10^{-3} \text{ m}^3\right) = -9.19 \text{ J} \]

\[ \Delta E = Q - W = mL_V - W = (1.00 \text{ kg})(3.34 \times 10^5 \text{ J/kg}) - (-9.19 \text{ J}) = 3.34 \times 10^5 \text{ J} \]

The work done by the atmosphere on the ice is negligible, so the change in internal energy is that of the heat added to the ice to melt it.

(b) Since the densities, and hence the volumes, of the water and ice are assumed to remain the same at 2.00 atm as at 1.00 atm, the work done by the atmosphere will be
\[ W = p\Delta V = (2.02 \times 10^5 \text{ N/m}^2)\left(1.000 \times 10^{-3} \text{ m}^3 - 1.091 \times 10^{-3} \text{ m}^3\right) = -18.4 \text{ J} \]
CHAPTER 21

Given the assumptions that the volumes of water and ice do not change between 1.00 and 2.00 atm and the negligible amount of work done by the atmosphere, one concludes that the heat of vaporization remains unchanged.

21-10. (a) \[
\int_0^W dW = \int_V nRT \frac{dV}{V}
\]

\[
W = nRT \left[ \ln V' - \ln V \right] = nRT (\ln V' - \ln V) = nRT \ln \left( \frac{V'}{V} \right)
\]

(b) In an isothermal expansion, the change in internal energy \( \Delta E = Q - W = 0 \) J. Therefore, \( W = Q \). Using the result from part (a),

\[
W = nRT \ln \left( \frac{V'}{V} \right)
\]

\[
e^{\frac{W}{nRT}} = e^{\ln \left( \frac{V'}{V} \right)}
\]

\[
e^{\frac{W}{nRT}} = \frac{V'}{V}
\]

\[
V' = V e^{\frac{W}{nRT}} = \left( 0.0020 \text{ m}^3 \right) e^{500 \text{ J}/\text{(0.10 mole)(8.31 J/mole K)(700 K)}} = 0.0047 \text{ m}^3
\]

†21-11. The work done during this process is

\[
W = p \Delta V = 0.25 \text{ atm} (1.01 \times 10^5 \text{ N/m}^2/1.0 \text{ atm})(0.5 \times 10^{-3} \text{ m}^3 - 2.0 \times 10^{-3} \text{ m}^3) = -37.9 \text{ J}
\]

During the process 75 J flows out of the system, therefore \( Q = -75 \text{ J} \). The change in internal energy is \( \Delta E = Q - W = -75 \text{ J} - (-37.9 \text{ J}) = -37.1 \text{ J} \)

21-12. The change in the volume of the cylinder in which the gas is contained decreases by

\[
\Delta V = Ah = \left( 90 \text{ cm}^2 \right) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right)^2 (0.12 \text{ m}) = 1.08 \times 10^{-3} \text{ m}^3
\]

where \( A \) is the area of the cylinder and \( h \) is the displacement of the cylinder. The work done during this process is

\[
W = p \Delta V = 15 \text{ atm} (1.01 \times 10^5 \text{ N/m}^2/1.0 \text{ atm})(-1.08 \times 10^{-3} \text{ m}^3) = -1636 \text{ J}
\]

Because work is done on the gas to compress it and the internal energy decreases, heat must be removed from the system. The quantity of this heat is found by applying the First Law of Thermodynamics,

\[
\Delta E = Q - W
\]

\[
Q = \Delta E + W = -15 \text{ J} + 15 \text{ atm} \left( \frac{1.01 \times 10^5 \text{ N/m}^2}{1.0 \text{ atm}} \right)(-1.08 \times 10^{-3} \text{ m}^3) = -1651 \text{ J}
\]

†21-13. In the temperature range of 5°C to 30°C, ethanol is a liquid. The amount of heat added to the ethanol is

\[
Q = mc \Delta T
\]

According to Table 20.1, the specific heat of ethanol at 20°C is 2430 J/(kg • °C). To determine the mass of the liquid, we need to know its density, which is given for 20°C. Therefore, we need to know the volume of the ethanol at 20°C. At 5.0°C, the volume is 1.00 liter.

As the alcohol is heated, it undergoes volumetric thermal expansion

\[
\Delta V = \beta V \Delta T = (1.01 \times 10^{-3}/°C)(1.00 \text{ liter}) \left( \frac{1.00 \times 10^{-3} \text{ m}^3}{1.00 \text{ liter}} \right)(20°C - 5°C) = 1.51 \times 10^{-5} \text{ m}^3
\]

where \( \beta \) is the volumetric thermal expansion coefficient listed in Table 20.2. The volume at 20°C is

\[
V + \Delta V = (1.00 \times 10^{-3} \text{ m}^3) + (1.51 \times 10^{-5} \text{ m}^3) = (1.015 \times 10^{-3} \text{ m}^3)
\]
The mass of the ethanol is therefore,
\[ m = \rho V = (789 \text{ kg/m}^3)(1.015 \times 10^{-3} \text{ m}^3) = 0.801 \text{ kg} \]
Then,
\[ Q = mc \Delta T = (0.801 \text{ kg})(2430 \text{ J/(kg} \cdot \text{°C}))(30^\circ \text{C} - 5^\circ \text{C}) = 4.87 \times 10^4 \text{ J} \]
The volume expansion between 5°C and 30°C is
\[ \Delta V = \beta V \Delta T = (1.01 \times 10^{-5} / \circ \text{C})(1.00 \text{ liter})(\frac{1.00 \times 10^{-3} \text{ m}^3}{1.00 \text{ liter}})(30^\circ \text{C} - 5^\circ \text{C}) = 2.53 \times 10^{-5} \text{ m}^3 \]
The work done by the alcohol in its expansion on the surrounding air is
\[ W = p \Delta V = 1.00 \text{ atm}(1.01 \times 10^5 \text{ N/m}^2/1.00 \text{ atm})(2.53 \times 10^{-5} \text{ m}^3) = 2.56 \text{ J} \]
21-14. In an adiabatic compression, \( Q = 0 \) J and \( \Delta E = W \). Assuming air behaves as an ideal gas,
\[ \Delta E = W = p \Delta V = (1.01 \times 10^5 \text{ N/m}^2)(20 \text{ cm}^3 - 80 \text{ cm}^3)(\frac{1 \text{ m}}{100 \text{ cm}})^3 = -6.06 \text{ J} \]
To determine the new pressure when the gas is compressed, we use the fact that \( pV^\gamma \) is constant for an ideal gas, where \( \gamma = 1.40 \) for a diatomic gas.
\[ p_1V_1^{\gamma = 1.40} = p_2V_2^{\gamma = 1.40} \]
\[ p_2 = p_1 \left( \frac{V_1}{V_2} \right)^{\gamma = 1.40} = (1.01 \times 10^5 \text{ N/m}^2) \left( \frac{80 \text{ cm}^3}{20 \text{ cm}^3} \right)^{1.4} = 7.03 \times 10^5 \text{ N/m}^2 \]
Using the Ideal-Gas Law, the number of moles of air present and the temperature of the compressed gas may be determined.
\[ p_1V_1 = nRT \]
\[ n = \frac{p_1V_1}{RT_1} = \frac{(1.01 \times 10^5 \text{ N/m}^2)(80 \text{ cm}^3)(\frac{1 \text{ m}}{100 \text{ cm}})^3}{(8.31 \text{ J/(mole} \cdot \text{K})}(300 \text{ K}) = 0.0032 \text{ moles} \]
\[ T_2 = \frac{p_2V_2}{nR} = \frac{(1.01 \times 10^5 \text{ N/m}^2)(20 \text{ cm}^3)(\frac{1 \text{ m}}{100 \text{ cm}})^3}{(8.31 \text{ J/(mole} \cdot \text{K})}(0.0032 \text{ moles}) = 75 \text{ K} \]
The gas then loses heat to its surroundings until the temperature reaches 300 K. This occurs at constant volume, or isovolumetrically. The work done is equal to 0 J, since there is no volume change. The mass of 1 mole of air is 29.0 g. Therefore, the change in internal energy is
\[ \Delta E = Q = mc \Delta T = 0.0032 \text{ moles}(\frac{0.0029 \text{ kg}}{1 \text{ mole}})(20.8 \text{ J/(mole} \cdot \text{K})}(300 \text{ K} - 75 \text{ K}) = 0.043 \text{ J} \]
The initial internal energy of the air is found by integrating equation (20.21)
\[ dE = nC_v dT \]
\[ \int dE = nC_v \int dT \]
\[ E_i = nC_v T_i = (0.0032 \text{ moles})(20.8 \text{ J/(mole} \cdot \text{K})}(300 \text{ K}) = 19.97 \text{ J} \]
As a result of the two steps taken to compress and cool the air, the final internal energy is
\[ E_f = E_i + \Delta E_i + \Delta E_2 = 19.97 \text{ J} - 6.06 \text{ J} + 0.043 \text{ J} = 13.95 \text{ J} \]
†21-15. This is an example of an isothermal process, \( \Delta T = 0 \) K. Since the temperature does not change, there is no change in the internal energy of the ideal gas; and \( W = Q \). The work done by the gas in its expansion, as determined in problem 21-10(a), is
\[ W = nRT \ln \frac{V_2}{V_1} = (10 \text{ moles})(8.31 \text{ J/(mole} \cdot \text{K})}(323 \text{ K}) \ln \left( \frac{10 \text{ liters}}{2 \text{ liters}} \right) = 4.3 \times 10^4 \text{ J} \]
21-16. This is an example of an isothermal process, $\Delta T = 0\, \text{K}$. Since the temperature does not change, there is no change in the internal energy of the ideal gas; and $W = Q$.

$$W = Q = nRT \ln \frac{V_2}{V_1} = (2.0\, \text{moles})(8.31\, \text{J/(mole}\cdot\text{K}))(373\, \text{K}) \ln(10) = 1.4 \times 10^4\, \text{J}$$

21-17. The temperature may be determined from the Ideal-Gas Law,

$$pV = nRT$$

$$T = \frac{p_2V_2}{nR} = \frac{(1.01 \times 10^5\, \text{N/m}^2)(0.10\, \text{m}^3)}{(8.31\, \text{J/(mole}\cdot\text{K})}(5.00\, \text{moles})} = 243\, \text{K}$$

Because this is an isothermal process, $\Delta T = 0\, \text{K}$; and there is no change in the internal energy of the ideal gas; and the work done as the gas expands is

$$W = Q = nRT \ln \frac{V_2}{V_1}$$

from which the initial volume may be determined since the quantity of work done by the gas is given.

$$\frac{W}{nRT} = \ln \frac{V_2}{V_1}$$

$$\frac{W}{nRT} = \frac{V_2}{V_1}$$

$$V_1 = V_2 e^{-\frac{W}{nRT}} = (0.10\, \text{m}^3) e^{-(2.0 \times 10^4\, \text{J})[(5.00\, \text{moles})(8.31\, \text{J/(mole}\cdot\text{K})][243\, \text{K}]} = 0.014\, \text{m}^3$$

The initial pressure was

$$p_i = \frac{nRT}{V_1} = \frac{(5.00\, \text{moles})(8.31\, \text{J/(mole}\cdot\text{K})}(243\, \text{K})}{0.014\, \text{m}^3} = 7.2 \times 10^5\, \text{N/m}^2$$

21-18. At 10 m below the surface of the water, the hydrostatic pressure is (see example 6, chapter 18),

$$p_i = p_0 - \rho g y = 1.01 \times 10^5\, \text{N/m}^2 - (1000\, \text{kg/m}^3)(9.81\, \text{m/s}^2)(-10\, \text{m}) = 1.99 \times 10^5\, \text{N/m}^2$$

The bubble forms from the diver’s airline from an initial volume of zero m$^3$. So, the work done in forming the bubble is done against the surrounding water is

$$W = p \Delta V = (1.99 \times 10^5\, \text{N/m}^2)[(4.0\, \text{cm}^3)(\frac{1\, \text{m}}{100\, \text{cm}})^3 - 0\, \text{m}^3] = 0.80\, \text{J}$$

As this is an isothermal process, $\Delta T = 0\, \text{K}$; and there is no change in the internal energy of the ideal gas; and the heat absorbed by the bubble as the gas expands is equal to the work,

$$W = Q = 0.80\, \text{J}$$

21-19. The diagram shows the indirect process described in the problem. The net work done against the atmosphere is the same in the indirect process as in the direct process since the changes in the volume are the same. The net changes in the internal energies is also the same. Because the work done and the internal energy are the same, then the heat must also be the same. For 1.0 kg of water, the latent heat per kilogram is

![Diagram showing indirect process](image-url)
\[ L_{V(20\degree C)} = c\Delta T + L_{V(100\degree C)} + c_p\Delta T \]
\[ = (4187 \text{ J/(kg} \cdot \text{C})(80\degree C) + 2.26 \times 10^6 \text{ J/kg} - (2010 \text{ J/(kg} \cdot \text{C})(80\degree C) \]
\[ = 2.43 \times 10^6 \text{ J/kg} \]

21-20. \[ L_{V(140\degree C)} = c\Delta T + L_{V(100\degree C)} + c_p\Delta T \]
\[ = (4187 \text{ J/(kg} \cdot \text{C})(-40\degree C) + 2.26 \times 10^6 \text{ J/kg} + (2010 \text{ J/(kg} \cdot \text{C})(40\degree C) \]
\[ = 2.17 \times 10^6 \text{ J/kg} \]

21-21. \[ e = \frac{W}{Q} = \frac{(W/t)}{(Q/t)} = \frac{300 \text{ MW}}{850 \text{ MW}} = 0.35 \text{ or 35}\% \]

21-22. \[ e = \frac{120 \text{ hp}(\frac{746 \text{ W}}{1 \text{ hp}})}{4.40 \times 10^5 \text{ W}} = 0.203 \text{ or 20.3}\% \]

†21-23. The work done by the runner is \( W = \Delta U = mgh \). So, the rate that work is being done is \( \frac{\Delta W}{\Delta t} = mg \frac{\Delta h}{\Delta t} = mgv \)

The efficiency is determined by the ratio of this work to the total energy generated by the body that includes the work done plus the waste heat generated.

\[ e = \frac{(\Delta W/\Delta t)}{(\Delta W/\Delta t) + (\Delta Q/\Delta t)} = \frac{mgv}{mgv + (\Delta Q/\Delta t)} \]
\[ = \frac{(70 \text{ kg})(9.81 \text{ m/s}^2)(0.30 \text{ m/s})}{(70 \text{ kg})(9.81 \text{ m/s}^2)(0.30 \text{ m/s}) + (1300 \text{ W})} = 0.14 \text{ or 14}\% \]

21-24. \[ e = \frac{W}{Q_l} = \frac{2.0 \times 10^4 \text{ J}}{3.0 \times 10^4 \text{ J}} = 0.67 \text{ or 67}\% \]

The waste heat produced is \( Q_2 = Q_1 - W = 1.0 \times 10^4 \text{ J} \)

†21-25. According to equation 21.5, \( e = \frac{W}{Q_l} = 1 - \frac{Q_2}{Q_l} \), the efficiency may be determined in more than one way. In this case, we are given \( Q_1 \) and \( Q_2 \) and asked to find the efficiency and the work done by the engine.

\[ e = 1 - \frac{Q_2}{Q_l} = 1 - \frac{8.0 \times 10^6 \text{ J}}{2.0 \times 10^7 \text{ J}} = 0.60 \text{ or 60}\% \]

The work done is \( W = Q_1 - Q_2 = 1.2 \times 10^7 \text{ J} \)

21-26. \[ \frac{W}{t} = e \frac{Q_l}{t} = 0.95(3.0 \times 10^3 \text{ W}) = 2.85 \times 10^3 \text{ W or 2.85 kW} \]
\[ (Q_2/t) = (Q_1/t) - (W/t) = 0.15 \text{ kW} \]

21-27. The overall efficiency is the product of the individual efficiencies.

\[ e = (0.90)(0.50)(0.99) = 0.445 \text{ or 44.5}\% \]

21-28. Summing the individual expenditures of mechanical power gives \( 2.77 \text{ hp} \). The efficiency of the runner is

\[ e = \frac{2.77 \text{ hp}}{13 \text{ hp}} = 0.21 \text{ or 21}\% \]
21-29. The maximum efficiency of the body as a heat engine would be
\[ e = 1 - \frac{T_e}{T_i} = 1 - \frac{293}{310} = 0.055 \text{ or } 5.5\% \]

The problem asks the student to calculate the work that he or she does to climb to a height of 3.0 m, the work done by the body would be \( W = \Delta U = mgh \). The student would insert his/her mass in this calculation. Therefore, the general result for the work done is
\[ W = m_{\text{you}} \left( 9.81 \text{ m/s}^2 \right) (3.0 \text{ m}) \]

With the assumption that the body is an inefficient heat engine, the amount of kcal one would have to consume to climb 3.0 m is
\[ Q = \frac{W}{e} = \frac{m_{\text{you}} \left( 9.81 \text{ m/s}^2 \right) (3.0 \text{ m})}{0.055} \left( \frac{1 \text{ kcal}}{4187 \text{ J}} \right) \]

21-30. The rate at which chemical energy is consumed is
\[ \frac{Q}{t} = \frac{1}{e} \frac{W}{t} = \frac{1}{0.37} (50 \text{ W}) = 135 \text{ W} \]

Because, \( e = 1 - \frac{Q_2}{Q_1} \), the rate at which waste heat is produced is
\[ \frac{Q_2}{Q_1} = (1 - e) = (135 \text{ W})(1 - 0.37) = 85 \text{ W} \]

The rate at which glucose is consumed is
\[ \frac{Q_1}{\text{glucose yield}} = \frac{135 \text{ J/s}}{3.7 \times 10^3 \text{ kcal/kg} \left( \frac{4187 \text{ J}}{1 \text{ kcal}} \right)} = 8.7 \times 10^{-6} \text{ kg/s} = 8.7 \text{ mg/s} \]

21-31. The engines each do 1100 hp of work per second and the efficiency is 20%, so the heat input to the system to deliver this work is somewhat greater. To find that rate of heat input, remember that efficiency is defined as \( e = W/Q \). Dividing both factors on the right side of the equation by \( t \) gives the ratio of the rate of work to the rate of heat input.
\[ e = \frac{W/t}{Q/t} \]

\[ Q_1/t = \frac{W/t}{e} = \frac{2200 \text{ hp} \left( \frac{745.7 \text{ W}}{1 \text{ hp}} \right)}{0.20} = 8.2 \times 10^6 \text{ W} = 8.2 \times 10^6 \text{ J/s} \]

The rate of fuel input needed to supply this heat input is
\[ \frac{m}{t} \frac{E}{m} = \frac{8.2 \times 10^6 \text{ J/s}}{4.4 \times 10^7 \text{ J/kg}} = 0.19 \text{ kg/s} \]

21-32. (a) 
\[ Q_1, T_1 \quad \text{High pressure stage} \quad \text{Steam} \quad \text{W} \quad Q_2, T_2 \quad \text{Low pressure stage} \quad \text{Steam} \quad \text{W} \quad Q_3, T_3 \quad \text{Water} \]

(b) Because some heat is put back into the system in the low-pressure stage, there is a gain in efficiency because additional work is the result. Therefore, the overall efficiency \( e \) is calculated as follows. Given that,
\[ e_1 = 1 - \frac{T_2}{T_1} \quad \text{and} \quad e_2 = 1 - \frac{T_1}{T_2} \]
\[ e_1 T_1 = T_2 - T_1 \quad \text{and} \quad e_2 T_2 = T_2 - T_3 \]

The overall efficiency is
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\[ e = 1 - \frac{T_2}{T_1} \]

\[ eT_1 = T_1 - T_3 = (T_1 - T_2) + (T_2 - T_3) = e_1T_1 + e_2T_2 \]

\[ e = e_1 + e_2 \left( \frac{T_2}{T_1} \right) = e_1 + e_2 - e_1e_2 = 0.4 + 0.2 - (0.4)(0.2) = 0.52 \text{ or } 52\% \]

†21-33. The efficiency is determined by the temperatures of the high temperature \((T_1)\) and low temperature \((T_2)\) reservoirs.

\[ e = 1 - \frac{T_2}{T_1} = 1 - \frac{300 \text{ K}}{1 \times 10^{11} \text{ K}} = 1 - (3 \times 10^9) = 0.999999997 \]

This is very close to 100% efficiency. However, working with such high temperatures can be a difficult engineering problem in practice.

21-34. If the efficiency is 33% and the amount of useful power produced is \(1 \times 10^9 \text{ W}\), then the rate of waste heat produced is \(\Delta Q/\Delta t = 2 \times 10^9 \text{ W}\). This the heat that raises the temperature of the water,

\[ \frac{\Delta Q}{\Delta t} = \frac{\Delta m}{c\Delta T} \]

\[ \Delta m = \frac{\Delta Q / \Delta t}{c\Delta T} = \frac{2 \times 10^9 \text{ J/s}}{(4187 \text{ J/(kg*K)})(8 \text{ K})} = 6 \times 10^4 \text{ kg/s} \]

\[ = 60 \text{ m}^3/\text{s} \]

†21-35. The efficiency is determined by the temperatures of the high temperature \((T_1)\) and low temperature \((T_2)\) reservoirs. The efficiency \(e\) is the ratio of the work done \(W\) by the engine to the amount of heat absorbed \(Q_1\). Therefore,

\[ e = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1} \]

\[ Q_1 = \frac{W}{1 - \frac{T_2}{T_1}} = \frac{5.0 \times 10^4}{1 - \left( \frac{273 \text{ K}}{373 \text{ K}} \right)} = 1.9 \times 10^4 \text{ J} \]

The amount of waste heat produced is the difference between the heat absorbed and the work done,

\[ Q_2 = Q_1 - W = (1.9 \times 10^4 \text{ J}) - (5.0 \times 10^4 \text{ J}) = 1.4 \times 10^5 \text{ J} \]

21-36. \[ e = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1} = 1 - \frac{300 \text{ K}}{400 \text{ K}} = 0.25 \text{ or } 25\% \]

Increasing the efficiency by 10%, gives

\[ 1 - \frac{T_2}{T_1} = 0.35 \]

\[ T_2 = 0.65T_1 \]

Therefore, if \(T_2 = 300 \text{ K}\), then \(T_1 = 462 \text{ K}\). Alternatively, if \(T_1 = 400 \text{ K}\), then \(T_2 = 260 \text{ K}\).

†21-37. According to equation 21.5, \[ e = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1}, \] the efficiency may be determined in more than one way. In this case, we are given \(Q_1\) and \(Q_2\) and asked to find the efficiency and the mechanical power output (the work done per second) the engine. Therefore,

\[ e = 1 - \frac{Q_2}{Q_1} = 1 - \frac{Q_2/t}{Q_1/t} = 1 - \frac{1400 \text{ J/s}}{2500 \text{ J/s}} = 0.44 \text{ or } 44\% \]

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The power output is
\[ e = \frac{W}{Q} = \frac{W/t}{Q_1/t} \]
\[ P = \frac{W}{t} = e(Q_1/t) = (0.44)(2500 \text{ J/s}) = 1100 \text{ J/s} = 1100 \text{ W} \]

21-38. The efficiency may be calculated in several ways. Of relevance here is
\[ e = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1} \]
\[ \frac{Q_2}{Q_1} = 1 - e \]
\[ Q_1 = \frac{Q_2}{1 - e} = \frac{5.0 \times 10^3 \text{ J}}{1 - 0.15} = 5880 \text{ J} = 5.9 \times 10^3 \text{ J} \]

Given the heat absorbed for one cycle, we can determine the amount of work done during the cycle.
\[ W = eQ_1 = (0.15)(5880 \text{ J}) = 882 \text{ J} = 8.8 \times 10^2 \text{ J} \]

Since the rate of work done is \( P = 12 \text{ kW} = 1.2 \times 10^3 \text{ J/s} \), the time for one cycle is
\[ \frac{882 \text{ J}}{1.2 \times 10^3 \text{ J/s}} = 0.74 \text{ s} \]

†21-39. The flowchart for the operation of a heat pump is shown in Figure 21.16. Heat is taken from the inside of the house at \( T_2 \) and sent to the high temperature reservoir at \( T_1 \) outside through the work done by the heat pump unit. The efficiency of the heat pump is
\[ e = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \]

To apply the thermodynamic relationships, temperatures must be converted into degrees Kelvin. Each \( ^\circ \text{F} = (5/9)^\circ \text{C} \). Since the freezing point of water is at \( 32^\circ \text{F} \), the temperatures given are:
\[ T_2 = (75^\circ \text{F} - 32^\circ \text{F}) \left( \frac{5^\circ \text{C}}{9^\circ \text{F}} \right) + 273^\circ \text{C} = 297 \text{ K} \]

and
\[ T_1 = (104^\circ \text{F} - 32^\circ \text{F}) \left( \frac{5^\circ \text{C}}{9^\circ \text{F}} \right) + 273^\circ \text{C} = 313 \text{ K} \]

Therefore, the heat transferred to the hot reservoir by the heat pump operating in cooling mode each hour will be
\[ \frac{Q_2}{Q_1} = \frac{T_2}{T_1} \]
\[ Q_1 = Q_2 \left( \frac{T_1}{T_2} \right) = \left( 5.0 \times 10^6 \text{ J} \right) \left( \frac{313 \text{ K}}{297 \text{ K}} \right) = 5.27 \times 10^6 \text{ J} \]

Therefore, the work input to the heat pump is
\[ W = Q_1 - Q_2 = 5.27 \times 10^6 \text{ J} - 5.0 \times 10^6 \text{ J} = 2.7 \times 10^5 \text{ J} \]

The rate of work done is
\[ \frac{W}{t} = \frac{2.7 \times 10^5 \text{ J}}{1 \text{ h}} \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) = 75 \text{ W} \]
21-40. From the efficiency relationship, we may find the temperature of the cold reservoir.

\[ e = 1 - \frac{T_2}{T_1} \]
\[ T_2 = T_1(1 - e) = (350 + 273 \text{ K})(1 - 0.15) = 530 \text{ K} \]

Then, to increase the efficiency while maintaining the temperature of the cold reservoir, the temperature of the hot reservoir should be increased to

\[ T_1 = \frac{T_2}{1 - e} = \frac{530 \text{ K}}{1 - 0.25} = 707 \text{ K} \text{ or } 434 \degree \text{C} \]

†21-41. When a heat pump is used to provide heat to a home, the coefficient of performance (CP) is defined as

\[ \text{CP} = \frac{\text{energy transferred to the high temperature reservoir}}{\text{work done by the heat pump}} = \frac{Q_1}{W} \]

The efficiency of a Carnot engine is

\[ e = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} = \frac{T_1 - T_2}{T_1} \]

Therefore,

\[ \text{CP} = \frac{Q_1}{W} = \frac{T_1}{T_1 - T_2} = \frac{293 \text{ K}}{293 - 278 \text{ K}} = 19.5 \]

This value represents the maximum CP for a system operating between these two temperatures. In practice, the CP values for real heat pumps is somewhat smaller than this value.

21-42.

\[ e = 1 - \frac{T_2}{T_1} \]
\[ T_1 = \frac{T_2}{1 - e} = \frac{273 \text{ K}}{1 - 0.322} = 403 \text{ K} \]

21-43.

\[ W = Q_1 - Q_2 \]
\[ Q_1 = W + Q_2 = 2.5 \times 10^3 \text{ J} + 6.0 \times 10^3 \text{ J} = 8.5 \times 10^3 \text{ J} \]
\[ \text{CP} = \frac{Q_1}{W} = \frac{8.5 \times 10^3 \text{ J}}{2.5 \times 10^3 \text{ J}} = 3.4 \]

21-44.

\[ e = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1} = \frac{T_1 - T_2}{T_1} = \frac{373 \text{ K}}{373 \text{ K} - 273 \text{ K}} = 0.268 \]
\[ W = eQ_1 = e(W - Q_2) \]
\[ W = Q_2 \left( \frac{e}{1 - e} \right) = Q_2 \left( \frac{0.268}{1 - 0.268} \right) = 0.366Q_2 \]

For every joule of heat removed from the low temperature reservoir, 0.366 J of work must be done.

†21-45. (a) The work done during each step for the ideal monatomic gas is \( W = p\Delta V \). Therefore, the work done during step 1 is zero J, since there is no change in volume. During step 2, the work done is

\[ W_2 = p_2(V_2' - V_1') = (1 \times 10^5 \text{ N/m}^2)(0.03 \text{ m}^3 - 0.01 \text{ m}^3) = 2 \times 10^3 \text{ J} \]

In step 3, both the pressure and the volume change, such processes may be either isothermal or adiabatic. To determine which type of process occurs in this system, consider the beginning and ending points of step 1, applying the Ideal-Gas Law
\( PV_i = nRT_i \)

\[
T_i = \frac{P_i V_i}{nR} = \frac{(3 \times 10^5 \text{ N/m}^2)(0.01 \text{ m}^3)}{(1.00 \text{ mole})(8.31 \text{ J/(mole K)})} = 361 \text{ K}
\]

\[
P_2 = \frac{P_1}{P_1} T_2 = \frac{T_1}{T_1} = \frac{1}{3} T_1 = \frac{1}{3}(361 \text{ K}) = 120 \text{ K}
\]

Then, in step 2, there is an isobaric expansion, the temperature at the beginning of step 3 is

\[
T_3 = \frac{V_2}{V_3} T_2 = 3T_2 = 3\left(\frac{1}{3}T_1\right) = T_1
\]

The temperature in step 3 does not change, so the process is isothermal. The work done in an isothermal process is

\[
W_3 = \frac{p_2 V_3}{V_3} \ln \frac{V_1}{V_2} = \frac{(1 \times 10^5 \text{ N/m}^2)(0.03 \text{ m}^3)}{0.03 \text{ m}^3} \ln \frac{0.01 \text{ m}^3}{0.03 \text{ m}^3} = -3.3 \times 10^3 \text{ J}
\]

The total work done during one cycle is

\[
W = W_1 + W_2 + W_3 = 0 + 2.0 \times 10^3 \text{ J} - 3.3 \times 10^3 \text{ J} = -1.3 \times 10^3 \text{ J}
\]

The negative value of the net work indicates that work is done on the gas during the cycle.

(b) During the isobaric step 2, the heat flow \( Q = mc_p \Delta T \). Since the change in temperature is positive, there is a net flow of heat into the system. Heat is absorbed during step 2. During isothermal step 3, the heat flow is equal to the amount of work, since the work is negative, the heat flow is again negative and there is a net heat flow out of the system. Heat is rejected in step 3.

(c) During isovolumetric step 1, heat is rejected by the system, since \( Q = mc_p \Delta T \) and \( T_1 \) is greater than \( T_2 \).

(d) Just as in single step processes that occur between high and low temperature reservoirs, the efficiency is still defined in a cyclical processes as the ratio of the work done by the system to the quantity of heat rejected to the high temperature reservoir. Heat is rejected to the high temperature reservoir in step 3.

\[
e = \frac{W}{Q_3} = \frac{W}{W_3} = \frac{-1.3 \times 10^3 \text{ J}}{-3.3 \times 10^3 \text{ J}} = 0.39
\]

21-46. (a) In steps 2 and 4, no work is done since these are isovolumetric steps. During steps 1 and 3, the work done depends on whether these are isothermal or adiabatic processes. At the beginning of step 1, the temperature is given by the Ideal-Gas Law,

\( PV_i = nRT_i \)

\[
T_1 = \frac{P_1 V_1}{nR} = \frac{(2 \times 10^5 \text{ N/m}^2)(0.01 \text{ m}^3)}{(1.00 \text{ mole})(8.31 \text{ J/(mole K)})} = 240 \text{ K}
\]

At the end of step 1, the temperature is

\[
T_2 = \frac{P_2 V_2}{nR} = \frac{(1 \times 10^5 \text{ N/m}^2)(0.03 \text{ m}^3)}{(1.00 \text{ mole})(8.31 \text{ J/(mole K)})} = 360 \text{ K}
\]

Step 1 must be an adiabatic process, therefore the work done during step 1 is
\[ W_i = \frac{p_1V_i - p_2V_2}{\gamma} = \frac{(2 \times 10^5 \text{ N/m}^2)(0.01 \text{ m}^3) - (1 \times 10^5 \text{ N/m}^2)(0.03 \text{ m}^3)}{1.66} = -600 \text{ J} \]

At the beginning of step 3, the temperature is
\[ T_3 = \frac{P_3V_3}{nR} = \frac{(3 \times 10^5 \text{ N/m}^2)(0.03 \text{ m}^3)}{(1.00 \text{ mole})(8.31 \text{ J/(mole} \cdot \text{K})} = 1100 \text{ K} \]

At the end of step 3, the temperature is
\[ T_4 = \frac{P_4V_4}{nR} = \frac{(4 \times 10^5 \text{ N/m}^2)(0.01 \text{ m}^3)}{(1.00 \text{ mole})(8.31 \text{ J/(mole} \cdot \text{K})} = 480 \text{ K} \]

The work done during step 3 is
\[ W_3 = \frac{p_3V_3 - p_4V_4}{\gamma} = \frac{(3 \times 10^5 \text{ N/m}^2)(0.03 \text{ m}^3) - (4 \times 10^5 \text{ N/m}^2)(0.01 \text{ m}^3)}{1.66} = 3000 \text{ J} \]

Therefore, the net work done during one cycle is \( W = -600 \text{ J} + 0 + 3000 \text{ J} + 0 = 2400 \text{ J} \)

(b) Steps 2 and 4 are isovolumetric, so the heat transferred during these steps is \( Q = nC_v\Delta T \).
\[ Q_2 = nC_v\Delta T = (1.00 \text{ mole})(12.5 \text{ J/(mole} \cdot \text{K}))(1100 - 360 \text{ K}) = 9250 \text{ J} \]
\[ Q_4 = nC_v\Delta T = (1.00 \text{ mole})(12.5 \text{ J/(mole} \cdot \text{K}))(240 - 480 \text{ K}) = -3000 \text{ J} \]

(c) Since steps 1 and 3 are adiabatic, \( Q_1 = Q_3 = 0 \text{ J} \) and \( \Delta E = -W \).

(d) \[ e = \frac{W}{Q_2} = \frac{2.4 \times 10^7 \text{ J}}{9.25 \times 10^7 \text{ J}} = 0.26 \]

21-47.
(a) \[ e = \frac{W}{Q_1} = 1 - \frac{T_2}{T_i} = 1 - \frac{T_1 - T_2}{T_i} = \frac{298 \text{ K} - 278 \text{ K}}{298 \text{ K}} = 0.067 \]

(b) \[ W = e Q_1 \]
\[ Q_1 = \frac{W}{e} = \frac{1}{0.067}(1.00 \times 10^6 \text{ W}) = 1.49 \times 10^7 \text{ W} \]
\[ P_{\text{waste}} = \frac{Q_1}{t} - \frac{W}{t} = 1.49 \times 10^7 \text{ W} - 1.00 \times 10^6 \text{ W} = 1.39 \times 10^7 \text{ W} \]
\[ \frac{Q_1}{t} = \frac{m}{t} c\Delta T \]
\[ \frac{m}{t} c\Delta T = \frac{1.49 \times 10^7 \text{ W}}{4187 \text{ J/(kg} \cdot \text{C})} = 3250 \text{ kg/s} \]

21-48.
(a) \[ e = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_i} = 1 - \frac{T_1 - T_2}{T_i} \]
\[ Q_2 = \frac{W}{e} - W = W\left(1 - \frac{1}{e}\right) \]
\[ \frac{Q_2}{t} = \frac{W}{t}\left(\frac{T_1}{T_i} - 1\right) = 5 \times 10^8 \text{ W}\left(\frac{520 + 273 \text{ K}}{520 - 30 \text{ K}} - 1\right) = 3 \times 10^8 \text{ W} \]
\[ Q_1 - W = \frac{W}{e} - W = W\left(1 - \frac{1}{e}\right) \]
\[ Q_2 = W\left(1 - \frac{1}{e}\right) = 5 \times 10^8 \text{ W}\left(\frac{1}{0.33} - 1\right) = 1 \times 10^9 \text{ W} \]
(a) For the Carnot heat pump operating in cooling mode, the heat rejected each hour to the high temperature reservoir is found from

\[
\frac{Q_2}{Q_1} = \frac{T_2}{T_1}
\]

\[
Q_1 = Q_2 \left( \frac{T_1}{T_2} \right) = \left( 8.4 \times 10^6 \text{ J} \right) \left( \frac{300 \text{ K}}{294 \text{ K}} \right) = 8.57 \times 10^6 \text{ J}
\]

Therefore, the work input to the heat pump each hour is

\[
W = Q_1 - Q_2 = 8.57 \times 10^6 \text{ J} - 8.4 \times 10^6 \text{ J} = 1.71 \times 10^5 \text{ J}
\]

The rate of work done is

\[
P = \frac{W}{t} = \frac{1.71 \times 10^5 \text{ J}}{1 \text{ h}} \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) = 48 \text{ W}
\]

(b) The air conditioner unit requires 950 W of work input, which is somewhat more than the heat pump from part (a).

\[
\text{factor} = \frac{P_{ac}}{P_{hp}} = \frac{950 \text{ W}}{48 \text{ W}} = 20
\]

21-50. The heat removed from ice of mass \( m \) in cooling it from 30°C to -5°C is

\[
Q_2 = mc_{\text{water}}\Delta T_{\text{water}} + mL_{\text{f}} + mc_{\text{ice}}\Delta T_{\text{ice}}
\]

\[
Q_2 = \frac{m}{t} \left( c_{\text{water}}\Delta T_{\text{water}} + L_{\text{f}} + c_{\text{ice}}\Delta T_{\text{ice}} \right)
\]

\[
= 1 \times 10^4 \left( \frac{1 \text{ kg}}{\text{day} \times 24 \text{ h}} \right) \left[ (4187 \frac{1}{\text{kg} \cdot \text{°C}}) \text{(30°C)} + (3.34 \times 10^5 \frac{1}{\text{kg}}) + (2230 \frac{1}{\text{kg} \cdot \text{°C}}) \text{(5°C)} \right]
\]

\[
= 5.45 \times 10^4 \text{ W}
\]

The maximum efficiency of this ice making process is

\[
e = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} = \frac{T_1 - T_2}{T_1}
\]

\[
e = 1 - \frac{T_2}{T_1} = 1 - \frac{268 \text{ K}}{303 \text{ K}} = 0.116
\]

Combining the efficiency relations and the fact that \( W = Q_1 - Q_2 \), then dividing by the time, we find the power required.

\[
W = \frac{Q_2}{t} / \frac{1}{e - 1} = \frac{5.45 \times 10^4 \text{ W}}{0.116 - 1} = 7.2 \times 10^5 \text{ W}
\]

†21-51. (a) \( e_{\text{turbine}} = 1 - \frac{T_1}{T_2} = 1 - \frac{260 + 273 \text{ K}}{540 + 273 \text{ K}} = 0.34 \)

\( e_{\text{engine}} = 1 - \frac{T_1}{T_2} = 1 - \frac{38 + 273 \text{ K}}{260 + 273 \text{ K}} = 0.42 \)

(b) The maximum net efficiency for the two components would occur if all of the heat that is rejected from the turbine goes into the stream engine. In practice, heat would be lost to the surroundings. Here, we’ll calculate the maximum efficiency,
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\[ e = \frac{W_{\text{net}}}{Q_1} = \frac{W_{\text{turbine}} + W_{\text{engine}}}{Q_1} = e_{\text{turbine}} + e_{\text{engine}} \frac{Q_2}{Q_1} = e_{\text{turbine}} + e_{\text{engine}} \left( \frac{T_2}{T_1} \right) = 0.34 + 0.42 \left( \frac{260 + 273 \text{ K}}{540 + 273 \text{ K}} \right) = 0.62 \]

If a single engine were utilized between the high and low temperature reservoirs,

\[ e = 1 - \frac{T_i}{T_f} = 1 - \frac{38 + 273 \text{ K}}{540 + 273 \text{ K}} = 0.62 \]

The two efficiencies are the same.

21-52. From the Carnot efficiency relationships, the minimum amount of work needed to cool the water from 30°C to 5°C can be determined.

\[ e = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} = \frac{T_1 - T_2}{T_1} \quad \text{and} \quad W = Q_1 - Q_2 \]

\[ \Delta W = Q_2 \left( \frac{T_1}{T_2} - 1 \right) = Q_2 \left( \frac{T_1}{T_2} - 1 \right) = -mc\Delta T \left( \frac{T_1}{T_2} - 1 \right) \]

This assumes that the heat rejected from the low temperature reservoir has been used to cool the water. Assuming that a sequence of Carnot engines is utilized, the above equation may be written in infinitesimal form. The mass of one liter of water is one kilogram.

\[ dW = -mc \left( \frac{T_1}{T_2} - 1 \right) dT \]

\[ \int dW = -mc \int_{T_1}^{T_2} \left( \frac{T_1}{T} - 1 \right) dT = mc \left( T_1 \ln T - T \right) \bigg|_{278 \text{ K}}^{303 \text{ K}} \]

\[ = (1 \text{ kg})(4187 \text{ J/(kg} \cdot ^\circ \text{C)}) \left[ (303 \text{ K}) \ln \left( \frac{303 \text{ K}}{278 \text{ K}} \right) - 303 \text{ K} + 278 \text{ K} \right] \]

\[ = 4570 \text{ J} \]

†21-53. The change in entropy as the water vaporizes is

\[ \Delta S_1 = \int \frac{dQ}{T} = \int \frac{\Delta Q}{T} = \frac{mL_v}{T} = \frac{(1.0 \text{ kg})(2.26 \times 10^6 \text{ J/kg})}{373 \text{ K}} = 6.06 \times 10^3 \text{ J/K} \]

In addition, the water vapor expands as the vaporization occurs and does work against the surrounding air. For the expanding gas, \( dQ = dW = pdV \), so the change in entropy due to the expansion is

\[ \Delta S_2 = \int \frac{dQ}{T} = \int \frac{pdV}{T} = \int nR \frac{dV}{V} = nR \ln \frac{V_2}{V_1} \]

The initial volume of the water, assuming thermal expansion is negligible, is

\[ V_1 = \frac{m}{\rho} = 1.0 \text{ kg}/(1000 \text{ kg/m}^3) = 0.0010 \text{ m}^3 \]

The volume of 1.0 kg of water vapor at 1.0 atm is

\[ V_2 = \frac{nRT}{p} = \frac{(1.0 \text{ kg})(1 \text{ mole/0.018 kg})(8.31 \text{ J/(mole} \cdot \text{K}))(373 \text{ K})}{1.01 \times 10^4 \text{ N/m}^2} = 1.705 \text{ m}^3 \]

Therefore,

\[ \Delta S_2 = nR \ln \frac{V_2}{V_1} = (1.0 \text{ kg})(1 \text{ mole/0.018 kg})(8.31 \text{ J/(mole} \cdot \text{K}) \ln \frac{1.705 \text{ m}^3}{0.001 \text{ m}^3} = 3.44 \times 10^3 \text{ J/K} \]

The net change in entropy is \( \Delta S = \Delta S_1 + \Delta S_2 = 6.06 \times 10^3 \text{ J/K} + 3.44 \times 10^3 \text{ J/K} = 9.5 \times 10^3 \text{ J/K} \)
21-54. The net change in entropy is
\[ \Delta S = \frac{\Delta Q_1}{T_1} + \frac{\Delta Q_2}{T_2} = \Delta Q_1 \left( \frac{1}{T_1} + \frac{1}{T_2} \right) \]

The rate of change of entropy is then,
\[ \frac{\Delta S}{\Delta t} = \frac{\Delta Q_1}{T_1} \left( -\frac{1}{T_1} + \frac{1}{T_2} \right) \]
\[ = \left( 2.5 \times 10^4 \text{ kcal h}^{-1} \left( \frac{4187 \text{ J}}{1 \text{ kcal}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \right) \left( -\frac{1}{21 + 273 \text{ K}} + \frac{1}{-5 + 273 \text{ K}} \right) = 9.6 \text{ W/K} \]

†21-55. From problem 49, the rate of heat removed from the inside is
\[ \frac{\Delta Q_2}{\Delta t} = 8.4 \times 10^6 \text{ J h}^{-1} \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) = 2.33 \times 10^4 \text{ W} \]

The total entropy change is the sum of the entropy change on the inside and outside. The change in heat on the inside is \( \Delta Q_2 \) where the temperature is \( T_2 \); and \( \Delta Q_1 \) on the outside where the temperature is \( T_1 \).
\[ \Delta S = \frac{\Delta Q_1}{T_1} - \frac{\Delta Q_2}{T_2} \]
\[ = \frac{\Delta Q_1}{\Delta t} \left( \frac{1}{T_2} - \frac{1}{T_1} \right) \]
\[ = \left( 2.0 \times 10^3 \text{ kcal h}^{-1} \left( \frac{4187 \text{ J}}{1 \text{ kcal}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \right) \left( \frac{1}{293 \text{ K}} - \frac{1}{310 \text{ K}} \right) = 0.44 \text{ W/K} \]

21-56. Typical body temperature is \( 37^\circ \text{C} = 310 \text{K} \); and typical room temperature is around \( 20^\circ \text{C} = 293 \text{K} \).
\[ \Delta S = \frac{\Delta Q_1}{T_1} + \frac{\Delta Q_2}{T_2} = \Delta Q_1 \left( -\frac{1}{T_1} + \frac{1}{T_2} \right) \]
\[ \frac{\Delta S}{\Delta t} = \frac{\Delta Q_1}{\Delta t} \left( \frac{1}{T_2} - \frac{1}{T_1} \right) \]
\[ = \left( 2.0 \times 10^3 \text{ kcal h}^{-1} \left( \frac{4187 \text{ J}}{1 \text{ kcal}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \right) \left( \frac{1}{293 \text{ K}} - \frac{1}{310 \text{ K}} \right) = 0.44 \text{ W/K} \]

21-57. Using the Carnot efficiency relationships
\[ e = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} = \frac{T_1 - T_2}{T_1} \quad \text{and} \quad W = Q_1 - Q_2 \]
we can find the rate of change of entropy for this system. The absolute temperatures are \( T_1 = 480 + 273 \text{ K} = 753 \text{ K} \) and \( T_2 = 27 + 273 \text{ K} = 300 \text{ K} \)
\[ \Delta S = \frac{\Delta Q_1}{T_1} + \frac{\Delta Q_2}{T_2} \]

The rate of change of the entropy for this system is
\[ \frac{\Delta S}{\Delta t} = \frac{\Delta Q_1}{\Delta t} \left( \frac{1}{T_2} - \frac{1}{T_1} \right) + \frac{\Delta Q_2}{\Delta t} \]
\[ = \left( \left( 2000 \text{ hp} \right) \left( \frac{745.7 \text{ W}}{1 \text{ hp}} \right) \left( \frac{1}{753 \text{ K}} \right) \right) + \left( \left( 2000 \text{ hp} \right) \left( \frac{745.7 \text{ W}}{1 \text{ hp}} \right) \left( \frac{1}{300 \text{ K}} \right) \right) = 12400 \text{ W/K} \]
21-58. $\Delta S = \frac{\Delta Q}{T} = \frac{mL_v}{T} = \frac{(1.0 \text{ kg})(3.34 \times 10^5 \text{ J/kg})}{273 \text{ K}} = 1.22 \times 10^3 \text{ J/K}$

21-59. $\Delta S = \frac{\Delta Q}{T} = \frac{mL_v}{T} = \frac{(1.0 \text{ kg})(5.8 \times 10^5 \text{ J/kg})}{-79 + 273 \text{ K}} = 3.0 \times 10^3 \text{ J/K}$

21-60. $\Delta S_{\text{melt}} = \frac{\Delta Q}{T_{\text{melt}}} = \frac{mL_v}{T_{\text{melt}}} = \frac{(1.00 \text{ kg})(2.05 \times 10^6 \text{ J/kg})}{1083 + 273 \text{ K}} = 150 \text{ J/K}$

$\Delta S_{\text{vapor}} = \frac{\Delta Q}{T_{\text{vapor}}} = \frac{mL_v}{T_{\text{vapor}}} = \frac{(1.00 \text{ kg})(5.2 \times 10^6 \text{ J/kg})}{2567 + 273 \text{ K}} = 1.8 \times 10^3 \text{ J/K}$

†21-61. The melting temperatures and heats of fusion are given in Table 20.4.

$\Delta S_{\text{Al}} = \frac{\Delta Q}{T_{\text{melt}}} = \frac{mL_v}{T_{\text{melt}}} = \frac{(1.00 \text{ kg})(3.99 \times 10^4 \text{ J/kg})}{660 + 273 \text{ K}} = 430 \text{ J/K}$

$\Delta S_{\text{Fe}} = \frac{mL_v}{T_{\text{melt}}} = \frac{(1.00 \text{ kg})(2.7 \times 10^5 \text{ J/kg})}{1535 + 273 \text{ K}} = 150 \text{ J/K}$

$\Delta S_{\text{Ag}} = \frac{mL_v}{T_{\text{melt}}} = \frac{(1.00 \text{ kg})(9.9 \times 10^4 \text{ J/kg})}{962 + 273 \text{ K}} = 80 \text{ J/K}$

$\Delta S_{\text{Hg}} = \frac{mL_v}{T_{\text{melt}}} = \frac{(1.00 \text{ kg})(1.1 \times 10^4 \text{ J/kg})}{-39 + 273 \text{ K}} = 47 \text{ J/K}$

The change in entropy seems to decrease with increasing atomic number. The largest value occurs for aluminum ($Z = 13$) and the smallest value occurs for mercury ($Z = 80$).

21-62. As the parachute descends, it does work against air friction and absorbs heat.

$W = f s = \Delta Q$

Because the parachutist is failing at a constant rate, the upward force due to air friction $f$ is equal in magnitude to the downward force due to gravity, $mg$. The rate of work done is thus,

$\frac{W}{t} = mg \frac{\Delta v}{\Delta t} = mg v = \frac{\Delta Q}{\Delta t}$

The rate of change of entropy is then

$\frac{\Delta S}{\Delta t} = \frac{\Delta Q}{T} = \frac{mg v}{T} = \frac{(80 \text{ kg})(9.81 \text{ m/s}^2)(5.0 \text{ m/s})}{20 + 273 \text{ K}} = 13 \text{ W/K}$

†21-63. As the automobile moves, it does work against the various sources of friction and absorbs heat.

$W = f s = \Delta Q$

The rate of work done is thus,

$\frac{W}{t} = f \frac{\Delta x}{\Delta t} = \frac{\Delta Q}{\Delta t}$

The rate of change of entropy is then

$\frac{\Delta S}{\Delta t} = \frac{\Delta Q}{T} = \frac{12 \times 10^3 \text{ W}}{20 + 273 \text{ K}} = 41 \text{ W/K}$

21-64. (a) The work done by the brakes is equal to the change in kinetic energy. This is also equal to the heat absorbed by the brakes.
\[ W = \Delta K = \frac{1}{2} mv_i = \Delta Q \]

\[
\int dS = \int \frac{dQ}{T} = \int mc dT = mc \ln \frac{T_2}{T_1} = \frac{\Delta Q}{\Delta T} \ln \frac{T_2}{T_1}
\]

\[
= \frac{1}{2}mv_i^2 \Delta T \ln \frac{T_2}{T_1}
\]

\[
= \frac{1}{2}(2100 \text{ kg}) \left[ \left( \frac{80 \text{ km}}{h} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) \right]^2 \ln \frac{60 + 273 \text{ K}}{20 + 273 \text{ K}} = 1.66 \times 10^3 \text{ J/K}
\]

(b) \[ \Delta S = \Delta S_{\text{air}} + \Delta S_{\text{brake}} = \frac{\Delta Q}{T_1} + \frac{\Delta Q}{\Delta T} \ln \frac{T_2}{T_1} \]

So, the additional entropy is

\[
\Delta S_{\text{air}} = \frac{1}{2}(2100 \text{ kg}) \left[ \left( \frac{80 \text{ km}}{h} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) \right]^2 \ln \frac{20 + 273 \text{ K}}{20 + 273 \text{ K}} = 1.8 \times 10^3 \text{ J/K}
\]

†21-65. The potential energy of the water is dissipated as heat at a rate

\[
\frac{\Delta m}{\Delta t} \frac{g \gamma}{v} = \frac{\Delta Q}{\Delta t}
\]

The rate of change of entropy is then

\[
\frac{\Delta S}{\Delta t} = \frac{\Delta Q}{T} \frac{T}{\Delta T} = \frac{\left( \frac{5700 \text{ m}^3/\text{s}}{} \right) \left( \frac{1000 \text{ kg}}{1 \text{ m}^3} \right) \left( 9.81 \text{ m/s}^2 \right) (50 \text{ m})}{20 + 273 \text{ K}} = 9.5 \times 10^6 \text{ W/K}
\]

21-66. Entropy of the water increases as it does work on the diver to change his kinetic energy. That energy is then dissipated as heat in the water.

\[
\frac{\Delta S}{\Delta t} = \frac{\Delta Q}{T} \frac{T}{\Delta T} = \frac{m g \gamma}{v} = \frac{(70 \text{ kg})(9.81 \text{ m/s}^2)(36 \text{ m})}{295 \text{ K}} = 84 \text{ J/K}
\]

21-67. \[ \Delta S = \frac{\Delta Q}{T_1} + \frac{\Delta Q}{T_2} \]

\[
\frac{\Delta S}{\Delta t} = \frac{-\Delta Q}{T_1} + \frac{\Delta Q}{T_2} = \frac{\Delta Q}{T} \left( -\frac{1}{T_1} + \frac{1}{T_2} \right)
\]

\[
= 1.8 \times 10^3 \text{ W} \left( -\frac{1}{21 + 273 \text{ K}} + \frac{1}{-18 + 273 \text{ K}} \right) = 0.94 \text{ W/K}
\]

21-68. The melting represents an increase in entropy. The change in entropy is

\[
\frac{\Delta S}{\Delta t} = \frac{\Delta Q}{T} \frac{T}{\Delta T} = \frac{m L_f}{T} = \frac{(20 \text{ kg})(2.9 \times 10^4 \text{ J/kg})}{328 + 273 \text{ K}} = 965 \text{ J/K}
\]

†21-69. In the free expansion of the gas, we remember from the First Law of Thermodynamics that

\[ \Delta E = \Delta Q - W = \Delta Q - p \Delta V \]

Since the process is an isothermal one, \( \Delta E = 0 \text{ J} \) and \( \Delta Q = p \Delta V \)

If we consider the system by adding and removing infinitesimally small amounts of heat, the change in entropy is

\[
dS = \frac{dQ}{T} = \frac{pdV}{T} = nR \frac{dV}{V}
\]

\[
\Delta S = \int dS = \int \frac{nR}{V} \frac{dV}{V} = nR \ln \frac{V'}{V} = (1.00 \text{ mole}) \left( 8.31 \text{ J/(mole \cdot K)} \right) \ln \frac{2.0 \text{ liters}}{1.0 \text{ liter}} = 5.8 \text{ J/K}
\]
21-70. At its initial temperature, the silver is solid since it is below its melting temperature. Heat will be transferred from the silver to the water until the equilibrium temperature is reached.

\[
m_w c_w \Delta T_w = m_{Ag} c_{Ag} \Delta T_{Ag}
\]

heat gained by the water

heat lost by the silver

\[
m_w c_w (T - T_w) = m_{Ag} c_{Ag} (T_{Ag} - T)
\]

\[
m_w c_w T - m_w c_w T_w = m_{Ag} c_{Ag} T_{Ag} - m_{Ag} c_{Ag} T
\]

\[
T = \frac{m_w c_w T_w + m_{Ag} c_{Ag} T_{Ag}}{m_w c_w + m_{Ag} c_{Ag}}
\]

\[
= \frac{5.0 \text{ kg} \left(4187 \text{ J/(kg} \cdot \text{°C)}\right)(20\text{°C}) + (0.50 \text{ kg})(240 \text{ J/(kg} \cdot \text{°C)})(950\text{°C})}{5.0 \text{ kg}(4187 \text{ J/(kg} \cdot \text{°C)}) + (0.50 \text{ kg})(240 \text{ J/(kg} \cdot \text{°C)})}
\]

\[= 25.3\text{°C} = 298 \text{ K}
\]

The heat lost by the silver has been added to the water, so there is a change in entropy of the water.

\[\Delta S = \frac{\Delta Q}{T} = \frac{m_{Ag} c_{Ag} \Delta T_{Ag}}{T} = \frac{5.0 \text{ kg}(4187 \text{ J/(kg} \cdot \text{°C)})(25.3 - 20\text{°C})}{298 \text{ K}} = 370 \text{ J/K}
\]

21-71. (a) In the limit of adding or removing infinitesimally small quantities of heat from a system, we have

\[
\int dS = \int \frac{dQ}{T} = \int mc dT = mc \ln \frac{T_2}{T_1}
\]

\[S_2 - S_1 = \Delta S = mc \ln \frac{T_2}{T_1}
\]

(b) \[\Delta S = mc \ln \frac{T_2}{T_1} = (1.0 \text{ kg})(4187 \text{ J/(kg} \cdot \text{°C}) \ln \frac{80 + 273 \text{ K}}{20 + 273 \text{ K}} = 780 \text{ J/K}
\]

21-72. The total entropy change is \(\Delta S = \Delta S_1 + \Delta S_2\) where \(\Delta S_1\) is the change in entropy of the boiling water and \(\Delta S_2\) is the change in entropy of the ice cubes.

\[\Delta S_1 = mc \ln \frac{T_2}{T_1} = (1.0 \text{ kg})(4187 \text{ J/(kg} \cdot \text{°C}) \ln \frac{40 + 273 \text{ K}}{100 + 273 \text{ K}} = -734 \text{ J/K}
\]

\[\Delta S_2 = \frac{mL_v}{T_0} + mc \ln \frac{T_2}{T_0} = \frac{(0.5 \text{ kg})(3.34 \times 10^6 \text{ J/kg})}{273 \text{ K}} + (0.5 \text{ kg})(4187 \text{ J/(kg} \cdot \text{°C}) \ln \frac{313 \text{ K}}{273 \text{ K}}
\]

\[= 900 \text{ J/K}
\]

\[\Delta S = \Delta S_1 + \Delta S_2 = -734 \text{ J/K} + 900 \text{ J/K} = 164 \text{ J/K}
\]

†21-73. Because we are mixing equal amounts of warm and cool water, the final temperature will be 50\text{°C}, the mid-point between the two temperatures. The total entropy change is \(\Delta S = \Delta S_1 + \Delta S_2\) where \(\Delta S_1\) is the change in entropy of the initially cooler water and \(\Delta S_2\) is the change in entropy of the initially warmer water.

\[\Delta S_1 = mc \ln \frac{T_2}{T_1} = (1.0 \text{ liter})(\frac{1 \text{ kg}}{1 \text{ liter}})(4187 \text{ J/(kg} \cdot \text{°C}) \ln \frac{50 + 273 \text{ K}}{20 + 273 \text{ K}} = 408 \text{ J/K}
\]

\[\Delta S_2 = mc \ln \frac{T_2}{T_1} = (1.0 \text{ liter})(\frac{1 \text{ kg}}{1 \text{ liter}})(4187 \text{ J/(kg} \cdot \text{°C}) \ln \frac{50 + 273 \text{ K}}{80 + 273 \text{ K}} = -371 \text{ J/K}
\]

\[\Delta S = \Delta S_1 + \Delta S_2 = 408 \text{ J/K} - 371 \text{ J/K} = 37 \text{ J/K}
\]
(a) In an isobaric process, the heat transferred into or out of a system is given by \( \Delta Q = nC_p \Delta T \). If the heat is transferred in infinitesimally small amounts, the change in entropy is
\[
\Delta S = \int \frac{dQ}{T} = \int \frac{nC_p dT}{T} = nC_p \ln \frac{T_f}{T_i}
\]

(b) In an isovolumetric process, the heat transferred into or out of a system is given by \( \Delta Q = nC_v \Delta T \).
If the heat is transferred in infinitesimally small amounts, the change in entropy is
\[
\Delta S = \int dS = \int \frac{dQ}{T} = \int \frac{nC_v dT}{T} = nC_v \ln \frac{T_f}{T_i}
\]

21-75. In the free expansion of the gas, we remember from the First Law of Thermodynamics that
\( \Delta E = \Delta Q - W = \Delta Q - p \Delta V \)
Because the process is an isothermal one, \( \Delta E = 0 \) J and \( \Delta Q = p \Delta V \)
If we consider the system by adding and removing infinitesimally small amounts of heat, the change in entropy is
\[
\Delta S = \int dS = \int \frac{dQ}{T} = \int \frac{p dV}{T} = nR \ln \frac{V_f}{V_i}
\]

21-76. \( \Delta E = \Delta Q - W = 3000 \text{ J} - 2000 \text{ J} = 1000 \text{ J} \)
The work done is \( W = p \Delta V = nR \Delta T \), therefore
\[
\Delta T = \frac{p \Delta V}{nR} = \frac{W}{nR} = \frac{2000 \text{ J}}{(1 \text{ mole})(8.31 \text{ J/(mole K)})} = 240 \text{ K}
\]

21-77. For \( n \) moles of a diatomic gas at temperature \( T \), the internal energy is \( E = \frac{5}{2} nRT \)
The change in internal energy is \( \Delta E = \Delta Q - W = -W \)
Because work is done on the system, the sign for the work is negative, \( W = -2500 \text{ J} \)
Combining the above equations, gives
\[
\Delta E = \frac{5}{2} nR \Delta T = -W
\]
\[
\Delta T = \frac{-2W}{5nR} = \frac{-2(-2500 \text{ J})}{5(1 \text{ mole})(8.31 \text{ J/(mole K)})} = 120 \text{ K}
\]

21-78. (a) \( W = \frac{1}{2}(p_1 - p_2)(V_i' - V_1) + p_2(V_2' - V_i) = \frac{1}{2}(p_1 + p_2)(V_2 - V_i') \)
(b) \( \Delta E = C_v(T_2 - T_i) = C_v(p_2V_2 - p_1V_1) = \frac{5}{2}(p_2V_2 - p_1V_1) \)
(c) \( Q = \Delta E + W = \frac{5}{2}(p_2V_2 - p_1V_1) + \frac{1}{2}(p_1 + p_2)(V_2 - V_i) \)
\[
= \frac{5}{2}(p_2V_2 - p_1V_1) + \frac{1}{2}(p_1 + p_2)(V_2 - V_i)
\]

21-79. (a) \( p_1 = \frac{nRT}{V_1} = \frac{(0.20 \text{ mole})(8.31 \text{ J/(mole K)}) (300 \text{ K})}{0.0012 \text{ m}^3} = 4.16 \times 10^5 \text{ N/m}^2 \)
\[
p_2 = \frac{nRT}{V_2} = \frac{(0.20 \text{ mole})(8.31 \text{ J/(mole K)}) (300 \text{ K})}{(0.0012 + 0.0010) \text{ m}^3} = 2.27 \times 10^5 \text{ N/m}^2
\]
(b) In an isothermal compression, the work done on the gas is
\[ W = p_2 V_2 \ln \left( \frac{V_2}{V_1} \right) \]
\[ = \left( 2.27 \times 10^3 \text{ N/m}^2 \right) \left( 0.0022 \text{ m}^3 \right) \ln \left( \frac{0.0012 \text{ m}^3}{0.0022 \text{ m}^3} \right) = -3.0 \times 10^2 \text{ J} \]

Also, since \( \Delta E \) is equal to 0 J in such a process, \( \Delta Q = W = -3.0 \times 10^2 \text{ J} \)

21-80. (a) At 0°C, one mole of an ideal gas occupies 22.4 liters.
\[ p_2 V_2 = nRT_2 \]
\[ V_2 = \frac{nRT}{p} = \frac{4.0 \text{ liters} \left( \frac{1 \text{ mole}}{22.4 \text{ liters}} \right) (8.31 \text{ J/(mole \cdot K)})(60 + 273 \text{ K})}{1.01 \times 10^5 \text{ N/m}^2} = 0.0049 \text{ m}^3 = 4.9 \text{ liters} \]

(b) \[ W = p \Delta V = (1.01 \times 10^5 \text{ N/m}^2)(0.9 \times 10^{-3} \text{ m}^3) = 91 \text{ J} \]
\[ \Delta E = Q - W = nC_v \Delta T - W \]
\[ = (4.0 \text{ liters}) \left( \frac{1 \text{ mole}}{22.4 \text{ liters}} \right) (29.1 \text{ J/(mole \cdot K)})(60 \text{ K}) = 220 \text{ J} \]
\[ Q = (4.0 \text{ liters}) \left( \frac{1 \text{ mole}}{22.4 \text{ liters}} \right) (29.1 \text{ J/(mole \cdot K)})(60 \text{ K}) = 310 \text{ J} \]

21-81. \( e = \frac{W}{Q} = \frac{W/\Delta t}{Q/\Delta t} = \frac{20 \text{ hp} \left( \frac{745.7 \text{ W}}{1 \text{ hp}} \right)}{6.3 \times 10^4 \text{ W}} = 0.24 = 24\% \)
\[ e = 1 - \frac{Q_2}{Q_1} \cdot \frac{Q_2}{Q_1} \]
\[ (1 - e) \frac{Q_2}{\Delta t} = (1 - 0.24)(6.3 \times 10^4 \text{ W}) = 4.8 \times 10^4 \text{ W} \]

21-82. \( e = \frac{W}{Q} = \frac{W/\Delta t}{Q/\Delta t} = \frac{869 \text{ W}}{(2.1 \times 10^4 \text{ J/liter}) \left( \frac{10.8 \text{ liters}}{1 \text{ min}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right)} = 0.23 = 23\% \)

21-83. \( \frac{Q_2}{Q_1} = \frac{T_2}{T_1} \)
\[ \frac{\Delta Q_2/\Delta t}{\Delta Q_1/\Delta t} = \frac{T_2}{T_1} \]
\[ \frac{\Delta Q_2}{\Delta t} = \frac{\Delta Q_1}{\Delta t} \left( \frac{T_1}{T_2} \right) = \left( 3.8 \times 10^7 \text{ J} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \left( \frac{32 + 273 \text{ K}}{21 + 273 \text{ K}} \right) = 1.095 \times 10^4 \text{ W} \]
\[ W = Q_1 - Q_2 \]
\[ \frac{\Delta W}{\Delta t} = \frac{\Delta Q_1 - \Delta Q_2}{\Delta t} = 1.0950 \times 10^4 \text{ W} - \left( 3.8 \times 10^7 \text{ J} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) = 4.0 \times 10^2 \text{ W} \]

21-84. (a) \[ e = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} = 1 - \frac{30 + 273 \text{ K}}{500 + 273 \text{ K}} = 0.61 \]
(b) \[ e = \frac{W}{Q_1} \]
\[ Q_1 = \frac{W}{e} = 1.5 \times 10^3 \text{ J} \]
\[ 0.61 = 2460 \text{ J} \]

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†21-85. (a) Beginning with the point at the upper left, the gas undergoes an isobaric expansion in step 1 as the volume is increased, followed by a isovolumetric reduction of pressure in step 2 as the temperature is reduced. The gas is then compressed isobarically in step 3 by reducing the volume, before an isovolumetric increase in pressure in step 4 by increasing the temperature.

(b) Step 1: \[ W_1 = p \Delta V = (1.5 \times 10^5 \text{ N/m}^2)(0.021 - 0.007 \text{ m}^3) = 2100 \text{ J} \]
Step 2: \[ W_2 = 0 \text{ J} \] (no work is done in an isovolumetric process)
Step 3: \[ W_3 = p \Delta V = (0.5 \times 10^5 \text{ N/m}^2)(0.007 - 0.021 \text{ m}^3) = -700 \text{ J} \]
Step 4: \[ W_4 = 0 \text{ J} \] (no work is done in an isovolumetric process)

(c) Step 1: At the beginning of step 1, the temperature is
\[ T_1 = \frac{P_i V_i}{nR} = \frac{(1.5 \times 10^5 \text{ N/m}^2)(0.007 \text{ m}^3)}{(1.00 \text{ mole})(8.31 \text{ J/(mole \cdot K)})} = 126 \text{ K} \]
At the end of step 1, the temperature is
\[ T_2 = \frac{P_2 V_2}{nR} = \frac{(1.5 \times 10^5 \text{ N/m}^2)(0.021 \text{ m}^3)}{(1.00 \text{ mole})(8.31 \text{ J/(mole \cdot K)})} = 379 \text{ K} \]
\[ Q_1 = nC_p \Delta T_1 = (1 \text{ mole})(20.8 \text{ J/(mole \cdot K)})(379 \text{ K} - 126 \text{ K}) = 5260 \text{ J} \]
Step 2: At the end of step 2, the temperature is
\[ T_3 = \frac{P_3 V_3}{nR} = \frac{(0.5 \times 10^5 \text{ N/m}^2)(0.021 \text{ m}^3)}{(1.00 \text{ mole})(8.31 \text{ J/(mole \cdot K)})} = 126 \text{ K} \]
\[ Q_2 = nC_v \Delta T_2 = (1 \text{ mole})(12.5 \text{ J/(mole \cdot K)})(126 \text{ K} - 379 \text{ K}) = -3160 \text{ J} \]
Step 3: At the end of step 3, the temperature is
\[ T_4 = \frac{P_4 V_4}{nR} = \frac{(0.5 \times 10^5 \text{ N/m}^2)(0.007 \text{ m}^3)}{(1.00 \text{ mole})(8.31 \text{ J/(mole \cdot K)})} = 42 \text{ K} \]
\[ Q_3 = nC_p \Delta T_3 = (1 \text{ mole})(20.8 \text{ J/(mole \cdot K)})(42 \text{ K} - 126 \text{ K}) = -1750 \text{ J} \]
Step 4: \[ Q_4 = nC_v \Delta T_4 = (1 \text{ mole})(12.5 \text{ J/(mole \cdot K)})(126 \text{ K} - 42 \text{ K}) = 1050 \text{ J} \]
In part (b), the net work done is 2100 – 700 J = 1400 J. From part (c), the sum of the four quantities of heat also equals 1400 J.

(d) The efficiency of this four-step cycle is
\[ e = \frac{W}{Q_2} = \frac{1400 \text{ J}}{3160 \text{ J}} = 0.44 \text{ or } 44\% \]

21-86. \[ \frac{Q_2}{Q_1} = \frac{T_2}{T_1} \]
\[ Q_2 = Q_1 \frac{T_2}{T_1} = (1200 \text{ J}) \frac{200 \text{ K}}{600 \text{ K}} = 400 \text{ J} \]
\[ W = Q_1 - Q_2 = 1200 \text{ J} - 400 \text{ J} = 800 \text{ J} \]

21-87. \[ e = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} = 1 - \frac{25 + 273 \text{ K}}{300 + 273 \text{ K}} = 0.48 \text{ or } 48\% \]
\[ \frac{Q_1}{t} = \frac{W}{e} = \frac{1 \times 10^7 \text{ W}}{0.48} = 2 \times 10^7 \text{ W} \]
The rate of waste heat is then,
\[ \frac{Q_2}{t} = \frac{Q_1}{t} - \frac{W}{t} = 2 \times 10^7 \text{ W} - 1 \times 10^7 \text{ W} = 1 \times 10^7 \text{ W} \]
21-88. \( \frac{Q_2}{Q_1} = \frac{T_2}{T_1} \)

\( Q_2 = Q_1 \frac{T_2}{T_1} = (5.0 \times 10^3 \text{ J}) \frac{(4 + 273) \text{ K}}{(20 + 273) \text{ K}} = 4727 \text{ J} \)

\( W = Q_1 - Q_2 = 5.0 \times 10^3 \text{ J} - 4727 \text{ J} = 270 \text{ J} \)

21-89. The boiling point temperatures and heats of vaporization are given in Table 20.4.

\[ \Delta S_N = \frac{\Delta Q}{T_{\text{boil}}} = \frac{m_{LV}}{T_{\text{boil}}} = \frac{(1.00 \text{ kg})(2.00 \times 10^5 \text{ J/kg})}{-196 + 273 \text{ K}} = 2600 \text{ J/K} \]

\[ \Delta S_O = \frac{\Delta Q}{T_{\text{boil}}} = \frac{m_{LV}}{T_{\text{boil}}} = \frac{(1.00 \text{ kg})(2.1 \times 10^5 \text{ J/kg})}{-183 + 273 \text{ K}} = 2300 \text{ J/K} \]

\[ \Delta S_H = \frac{\Delta Q}{T_{\text{boil}}} = \frac{m_{LV}}{T_{\text{boil}}} = \frac{(1.00 \text{ kg})(4.5 \times 10^5 \text{ J/kg})}{-253 + 273 \text{ K}} = 2.25 \times 10^4 \text{ J/K} \]

Hydrogen is largest and oxygen the smallest.

21-90. (a) Because both the lead and the ice are at their respective melting temperatures, some of the ice will melt into water and the lead will solidify. Because the amount of heat removed from the lead in solidifying is much less than the heat required to melt all of the ice, the equilibrium temperature will be 273 K. As a check, let’s calculate how much of the ice melts.

\[
\frac{m_{\text{lead}}}{c_{\text{lead}}}(T_{\text{lead}} - T) + m_{\text{lead}}L_{F,\text{lead}} = m_{\text{ice}}L_F
\]

\[
m_{\text{ice}} = \frac{m_{\text{lead}}}{c_{\text{lead}}}(T_{\text{lead}} - T) + m_{\text{lead}}L_{F,\text{lead}}
\]

\[
= \frac{(1.0 \text{ kg})[(130 \text{ J/(kg} \cdot \text{°C})](328 \text{ °C}) + 2.32 \times 10^4 \text{ J/kg}]}{3.34 \times 10^4 \text{ J/kg}} = 0.20 \text{ kg}
\]

(b) The entropy of the lead decreases by

\[
\Delta S = mc \ln \frac{T_2}{T_1} = (1.0 \text{ kg})(130 \text{ J/(kg} \cdot \text{°C}) \ln \left( \frac{273 \text{ K}}{328 + 273 \text{ K}} \right) = -100 \text{ J/K}
\]

(c) \( \Delta S_{\text{water}} = \frac{\Delta Q}{T} = \frac{m_{LV}}{T} = \frac{(0.20 \text{ kg})(3.34 \times 10^4 \text{ J/kg})}{273 \text{ K}} = 245 \text{ J/K} \)

21-91. (a) \( S = \frac{Q_1}{T_1} = \frac{800 \text{ J}}{700 \text{ K}} = 1.1 \text{ J/K} \)

(b) \( \frac{Q_2}{Q_1} = \frac{T_2}{T_1} \)

\( Q_2 = Q_1 \frac{T_2}{T_1} = (800 \text{ J}) \frac{300 \text{ K}}{700 \text{ K}} = 340 \text{ J} \)

\( S = \frac{Q_2}{T_2} = \frac{340 \text{ J}}{300 \text{ K}} = 1.1 \text{ J/K} \)

(c) The entropy remains constant during adiabatic processes. \( \Delta S = 0 \text{ J/K} \)