Empirical Analysis of a Ring Pendulum

INTRODUCTION

If you suspend an object in the shape of a ring on a knife edge and give it a slight push, it will swing back and forth, pivoting about the equilibrium position. The time required for the ring to complete one complete swing back and forth is called the **period of oscillation**. If you were to use a different sized ring, say one with a larger diameter, and measure its period of oscillation, you'd find that the larger ring has a longer period. To a physicist, several interesting questions come to mind: What properties of the ring determine its period of oscillation? What mathematical relationship among the relevant properties of a ring can be found that will enable me to predict the period of oscillation of any ring?

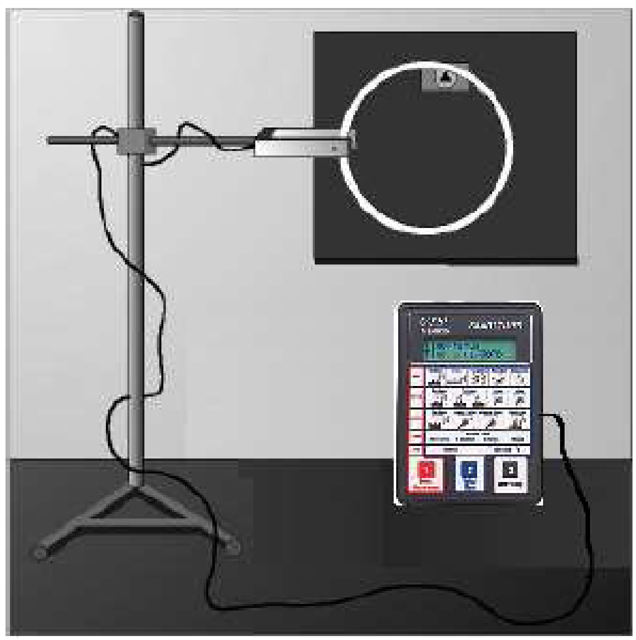

Figure 1

An alternative approach to deriving the mathematical equations that describe the swinging rings' motion is the **empirical method**, the performing of experiments and studying relationships between different parameters of the system to obtain a quantitative description. (A **parameter** is a property of a physical system that is relevant to the system's behavior—which may affect, or be affected by, other factors.) The objective is to obtain an equation, or equations, that *quantitatively* describe the dependence of one parameter on the others.

In the example of the swinging rings, the period of oscillation is one of the system parameters.Other parameters are the ring's diameter, mass, density, etc. Generally a scientist tries to isolate parameters, varying one parameter while holding as many as possible of the others constant and measuring one of the remaining parameters. In this way the scientist gets a feeling for how the measured parameter is affected by changes in the varied parameter. The varied parameter is referred to as the **independent** or **manipulated** parameter, and the measured parameter is referred the **dependent** or as **responding** parameter. This process is repeated for various dependent and independent parameters until the scientist understands how the system works. An empirical description of the system may then be constructed.

In this experiment you will construct an empirical description of how various properties of a ring affect its period of oscillation. You have undoubtedly observed many objects in oscillatory motion in the course of everyday life and have unconsciously made a large number of observations about such systems. You know, for example, that such factors as the time of day, the phase of the moon, the barometric pressure, and the Dow-Jones average will have little or no effect on the oscillating ring system. (Any of these factors might, of course, be of great importance for some other experiment).

Examination of the set of rings will reveal that they are all constructed in a similar way. They are all thin, and therefore the difference in the inner and outer diameters is small compared to either diameter. It is reasonable, then, to consider the mean diameter (the average of the inner diameter and the outer diameter) as one of the important parameters in the system.

The width (not to be confused with the ring's diameter or thickness) of a ring is another parameter which might at first be considered important, but this factor can be rejected after a little thought: suppose two identical rings were mounted on a knife edge and started swinging at the same time and with the same amplitude. Reason suggests that the rings would continue to swing at identical rates. This situation would be unchanged even if the rings were touching or in fact joined together. A pair of identical rings joined together is just a ring of twice the width, so it seems clear that the width of a ring will not matter in this system.)

The mass of the ring might be an important parameter in the experiment. You know by experience that an object's mass can affect its motion, so we must allow for that possibility with the swinging rings. Note that two rings can have identical dimensions but different masses, if they are made of materials of different densities.

The effect of the amplitude of oscillation on the period of the rings is an important issue; one you could easily spend the entire lab period studying. You will investigate this parameter first, but mostly from a qualitative perspective to avoid spending too much time.

PROCEDURE

Part 1: Effect of Oscillation Amplitude on Period

Select a large ring and mount it on the knife edge support. Position the photogate just beyond the outer edge of the ring as shown in the equipment setup figure. Gently nudge the ring and observe the red LED on the photogate. The LED should turn on only once per oscillation. If this does not occur, adjust the position of the photogate until the LED flashes at the same frequency as the ring is oscillating. The selecting the time and one gate mode on the smart-timer measures the elapsed time between two successive interruptions of the photogate beam, so it should now read the period of oscillation.

Record at least five measurements of the period for this "small" amplitude oscillation. Estimate the angular displacement (in degrees) for this “small” amplitude trial. This measurement can be made with a protractor held upside down at the pivot point and subtracting readings where the edge of the ring intersects the protractor. Record your findings (with uncertainty estimates).

Repeat the above procedure for “medium” and “large” amplitude oscillations, and record your data in a table. Note that larger amplitude oscillations can become chaotic, so do your best to gather accurate data from this upper limit, but do not spend more than about ten minutes on this part of the experiment.

Examine your data and discuss your findings with other students. What trends do you observe? How does the amplitude affect the period of oscillation? Is there a range of amplitudes for which the period is mostly constant?

Part 2: Effect of Mass on the Period

Consider now how the mass of the ring might affect the period. Since both the mass and the mean diameter of the rings may affect the period, we should hold the diameter of the rings constant while varying their mass.

Find a set of three rings all having the same physical dimensions but made of different materials and therefore different masses. Weigh each of the rings in this set and record the masses.

One by one, mount each ring on the knife-edge and measure its period, keeping the amplitude the same for each ring (i.e. $\sim$10 degrees). Record at least five values of the period for each ring using the photogate smart-timer as in Part 1.

Compute the average period for each of the rings. Estimate the uncertainty in the period for each ring, and report the average $\pm$ the uncertainty.

Study the results of this experiment. What can you conclude about the effect of a ring's mass on the period of oscillation? Is this what you expected? Why?

Part 3: Effect of Ring Diameter on the Period

Now you will determine the dependence of the period of oscillation due to the mean diameter of the rings. Notice that if we were to follow the same line of reasoning as in Part 2, it would be necessary to measure the period of a set of rings that are identical in every respect except the diameter; i.e., we would need to use rings of *varying size* but *constant mass*. This would be difficult to arrange: we would have to choose the sizes and materials just right. Fortunately, this is not necessary. We can, in fact, proceed using rings of varying diameters and ignore the variation in masses. Why is this valid?

In order to determine the dependence of the period of oscillation on the mean diameter of a ring, we first make a *guess* about the nature of this dependence. In particular, we will guess that the period is proportional to a power of the mean diameter. That is:

$$T = a \cdot D^n \tag{1}$$

where a and n are constants determined from your period vs. diameter data (which you will obtain below), T is the period of oscillation, and d is the mean diameter of the ring.

In fact, this is an educated guess: theory predicts a function of this form. But we could have arrived at an expression of this form without any theoretical knowledge; the expression is a fairly general one found in nature. However, D could appear in a more complicated way (for instance, there could be a second term of the form $b \cdot D^m$, where $m \neq n$). The simplest guess may not be the right one, but it is certainly a good place to start. If the guess is wrong, we'll know it, because the experimental results will not support the theory.

Let's take the natural logarithm of both sides of the power-law relation, and apply the properties of logarithms to obtain:

$$\ln(T) = n^* \ln(D) + \ln(a). \tag{2}$$

A quick review of logarithm properties:

$$\begin{aligned} \ln(ab) &= \ln(a) + \ln(b) \\ \ln\left(\tfrac{a}{b}\right) &= \ln(a) - \ln(b) \\ \ln\left(a^b\right) &= b \ln(a) \end{aligned}$$

Thus, if our guess is correct, a graph of $\ln(T)$ vs. $\ln(D)$ will be a straight line. From the graph we can obtain the values of a and n. If the power-law 'guess' is wrong, this graph will not give a straight line.

Select several rings with different diameters. Using Vernier calipers for the smaller rings and the metal metric rulers for the larger rings, measure and record the inner and outer diameters of these rings. Measure each diameter at several different places around the ring in order to average out any irregularities in the rings. Compute the average for the inner and outer diameter for each ring, then compute the average of those two numbers to obtain the average mean diameter. Record this value and its uncertainty for each ring.

Using the photogate smart-timer, measure the period of oscillation for each ring as in Part 2. Take several readings of the period for each ring, record the data, and compute an average period, along with an estimate of the uncertainty in the period. Report the average period ± uncertainty for each ring.

Before leaving the lab, initial your data sheets, have your instructor initial your data sheets, and hand in a copy of each data sheet to your instructor.

ANALYSIS

Part 1: Effect of Oscillation Amplitude on the Period

Compare the periods of oscillation you obtained for different amplitudes of the same ring. Is there a measurable difference in the average period for small versus large amplitudes? How significant is this difference compared to the change in angular displacement? Is your conclusion consistent with the findings of other students?

Part 2: Effect of Mass on the Period

What did you conclude about how the mass of a ring affects its period? Is your conclusion consistent with the findings of other students? Are any of your results (or those of other students) inconsistent with your conclusion? If so, what might account for the discrepancy?

Part 3: Effect of Ring Diameter on the Period

Make a table of your experimental results for Part 3 that contains the mean diameter, average period of oscillation, and the natural logarithm of each. Construct a graph of $\ln(T)$ vs. $\ln(D)$. See if points lie along a straight line. Note that a straight-line plot also indicates that our original 'guess' about the power-law dependence of T on D was correct. Determine the slope and vertical intercept from the graph. The slope of your straight line is the power, n, in the assumed power law expression. Record the value of the slope. The vertical intercept on your graph is the natural log of the constant a. Find the anti-log (recall $A = e^{\ln A}$) of the intercept and record your experimental determination of a. Finally, summarize your experimental results by writing an empirical equation of the form of eq. 1 where a and n are the experimental values determined from your graph. Does your empirical value of n agree with your prediction? Try to interpret the meaning of the constant a by comparing your experimental results with the equation for a simple pendulum (found in your textbook).

DISCUSSION

Summarize your findings in the discussion section of your report. What did you learn from this investigation? Did any of your results surprise you? What did you learn about the functional dependence of various parameters? When is a difference between results considered significant? Are there times when results can be measurably different but still "insignificant" (i.e. not meaningful)? What implications does this investigation have for other experiments?