Lesson 2 - The Copernican Revolution

READING ASSIGNMENT

- Chapter 2.1: Ancient Astronomy
- Chapter 2.2: The Geocentric Universe
- Chapter 2.3: The Heliocentric Model of the Solar System
  - Discovery 2-1: The Foundations of the Copernican Revolution
- Chapter 2.4: The Birth of Modern Astronomy
- Chapter 1.6: The Measurement of Distance
  - More Precisely 1-2: Measuring Distance with Geometry
- Chapter 2.5: The Laws of Planetary Motion
  - More Precisely 2-1: Some Properties of Planetary Orbits
- Chapter 2.7: Newton's Laws
- Chapter 2.8: Newtonian Mechanics
  - More Precisely 2-2: Weighing the Sun

SUMMARY OF HISTORICAL FIGURES AND TEXTS

Aristotle (384 - 322 BC)

Aristarchus (310 - 230 BC)

Ptolemy (2^{nd} century AD)

Megale Syntaxis tes Astronomias (*Great Syntaxes of Astronomy*), also known as *Syntaxis*, but more commonly known as *Almagest* (*The Greatest*) (c. 141 AD)

Nicholas Copernicus (1473 - 1543)

De Revolutionibus Orbium Coelestium (*On the Revolutions of the Celestial Spheres*), also known as *De Revolutionibus* (1543)

Tycho Brahe (1546 - 1601)

Often pictured with a metal nose, because he lost his real one in a dual. Sadly, he wasn't defending the honor of a lady or anything noble like that. A fellow student claimed to be the better mathematician and that pissed him off. So it was a nerd fight. History does not record what happened to the other student.

Galileo Galilei (1564 - 1642)

Sidereus Nuncius (The Starry Messenger) (1610)

Dialogue Concerning the Two Chief World Systems—Ptolemaic and Copernican, also known as *Dialogue* (1632)

Johannes Kepler (1571 - 1630)

Always desperate to get his hands on his boss Tycho's planetary measurements, history raises an eyebrow when a modern analysis of Tycho's exhumed fingernails showed that he had died from an extreme case of...mercury poisoning (not from drinking so much that his bladder burst, as his detractors rumored). Shortly after Tycho's death, his planetary data were found in Kepler's possession. Hmmm. Tycho's family sued for their return, but Kepler only partially complied. The result? Kepler's three laws of planetary motion. And one dead Tycho.

Isaac Newton (1642 - 1727)

Philosophie Naturalis Principia Mathematica (The Mathematical Principals of Natural Philosophy), also know as *Principia* (1687)

KEPLER'S LAWS OF PLANETARY MOTION

Read Chapter 2.5 and More Precisely 2-1.

Kepler's First Law

The orbital paths of the planets are elliptical (*not* circular), with the sun at one focus.

Kepler's Second Law

An imaginary line connecting the sun to any planet sweeps out equal areas of the ellipse in equal intervals of time.

Kepler's Third Law

The square of a planet's orbital period, P, is proportional to the cube of its semi-major axis, a: P^2 is proportional to a^3.

NEWTON'S LAWS OF MOTION

Read Chapter 2.7.

Newton's First Law

Every body continues in a state of rest or in a state of uniform motion in a straight line, unless it is compelled to change that state by a force acting on it.

Newton's Second Law

When a force, F, acts on a body of mass, m, it produces in it an acceleration, a, equal to the force divided by the mass: $a = F \ / \ m$ or $F = ma$.

Newton's Third Law

To every action, there is an equal and opposite reaction.

NEWTON'S LAW OF UNIVERSAL GRAVITATION

Read Chapter 2.7.

Newton's Law of Universal Gravitation

Every particle of matter in the universe attracts every other particle with a force that is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between their centers.

NEWTON'S FORMS OF KEPLER'S LAWS OF PLANETARY MOTION

Read Chapter 2.7 and Chapter 2.8.

Kepler's First Law

The orbital paths of the planets are elliptical (*not* circular), with the center of mass of the sun-planet system at one focus.

Kepler's Third Law

The square of a planet's orbital period, P, is proportional to the cube of its semi-major axis, a, and inversely proportional to the total mass, M_{total}, of the sun-planet system: P^2 is proportional to a^3/M_{total}.

MATH NOTES

Parallax

Read Chapter 1.7 and More Precisely 1-2.

- baseline = distance between different observing points
- angular shift = apparent shift in angular position of object when viewed from different observing points.
- distance = distance to object

- If you know the baseline and the angular shift (in degrees), you can calculate the distance using the following equation.

$$\text{distance} = \left(\frac{360^\circ}{2\pi}\right) \times \left(\frac{\text{baseline}}{\text{angular shift}}\right) \tag{1}$$

- If you know the baseline and the distance, you can calculate the angular shift using the following equation.

$$\text{angular shift} = \left(\frac{360^\circ}{2\pi}\right) \times \left(\frac{\text{baseline}}{\text{distance}}\right) \tag{2}$$

- Standard astronomical baselines
  - diameter of Earth = 12,756 km
  - diameter of Earth's orbit = 2 astronomical units (or AU)
  - 1 AU is the average distance between Earth and the sun.
- By the mid-19$^{\text{th}}$ century, telescopes were being built that were powerful enough to detect and measure stellar parallaxes. This confirmed the final major prediction of the heliocentric model. In the geocentric model, Earth does not move and consequently stellar parallaxes are not expected.

Kepler's Third Law

Read Chapter 2.5, Chapter 2.7, and Chapter 2.8

- P = orbital period
- a = orbital semi-major axis
- M_{total} = total mass of two orbiting objects
- G = Newton's gravitational constant
- $P^2 = \left(\frac{4\pi^2}{G}\right) \times \left(\frac{a^3}{M_{\text{total}}}\right)$
- We will now simplify this equation to make it easier to use, for two special cases. **In this course, you will never need to use Newton's gravitational constant to solve a problem.**

Case 1. For objects that orbit the sun

- Since the masses of all objects that orbit the sun are significantly less than the mass of the sun, $M_{\text{total}} \approx M_{\text{sun}}$.
- Hence, $P^2 \approx$ constant $\times\ a^3$, where constant $= 4\pi^2/GM_{\text{sun}}$.

- Consider the case of Earth orbiting the sun. For Earth, $P = 1$ yr and $a = 1$ AU. Hence, for Earth
  - (1 yr)$^2 \approx$ constant $\times$ (1 AU)^3.
- Dividing this equation into the previous equation yields
  - P^2 / (1 yr)$^2 \approx$ [constant $\times$ a^3] / [constant $\times$ (1 AU)^3].
- Simplification yields
  - (P / 1 yr)$^2 \approx$(a / 1 AU)^3.
- Solving for P and a yields the following.

$$P \approx \left(\frac{a}{1 \text{ AU}}\right)^{3/2} \text{ yr} \tag{3}$$

$$a \approx \left(\frac{P}{1 \text{ yr}}\right)^{2/3} \text{ AU} \tag{4}$$

- Hence, as long as P is measured in years, a is measured in AU, and the object orbits the sun, Kepler's Third Law should be very easy to use.

Case 2. For objects that orbit Earth

- Since the masses of all objects that orbit Earth are significantly less than the mass of Earth, $M_{\text{total}} \approx M_{\text{earth}}$.
- Hence, $P^2 \approx$ constant $\times$ a^3, where constant $= 4\pi^2/GM_{\text{earth}}$.
- Consider the case of the moon orbiting Earth. For the moon, $P = 1$ lunar month and $a = 1$ Earth-Moon distance. Hence, for the moon
  - (1 lunar month)$^2 \approx$ constant $\times$ (1 Earth-Moon distance)^3.
- Dividing this equation into the previous equation yields
  - P^2 / (1 lunar month)$^2 \approx$ [constant $\times$ a^3] / [constant $\times$ (1 Earth-Moon distance)^3.
- Simplification yields
  - (P / 1 lunar month)$^2 \approx$ (a / 1 Earth-moon distance)^3.
- Solving for P and a yields the following two equations.

$$P \approx \left(\frac{a}{1 \text{ Earth-Moon distance}}\right)^{3/2} \text{ lunar months} \tag{5}$$

$$a \approx \left(\frac{P}{1 \text{ lunar month}}\right)^{2/3} \text{ Earth-Moon distance} \tag{6}$$

- Hence, as long as P is measured in lunar months, a is measured in Earth-Moon distances, and the object orbits Earth, Kepler's Third Law should again be very easy to use.

Newton's Second Law

Read Chapter 2.7.

- F = force acting on object
- m = mass of object
- a = acceleration of object produced by force

$$F = ma \tag{7}$$

Newton's Universal Law of Gravitation

Read Chapter 2.7.

- F = force of gravity between two objects
- M = mass of first object
- m = mass of second object
- r = distance between first and second objects
- G = Newton's gravitational constant

$$F = \frac{GMm}{r^2} \tag{8}$$

Derivation of Kepler's Laws from Newton's Laws

Read Chapter 2.5, Chapter 2.7, Chapter 2.8, and More Precisely 2-2.

- All three of Kepler's laws can be derived from Newton's laws using calculus.

Special Case

For the **special case** of an object of mass m that travels at speed v in a **circular** orbit of radius r around an object of mass M, where $\boldsymbol{m}$ **is much less than** $\boldsymbol{M}$, Kepler's Third Law can be derived using only algebra.

- From More Precisely 2-2, the acceleration a of any object that travels at speed v in a circular orbit of radius r is given by $a = v^2/r$.

- By Newton's Second Law, the force on this object is then $F = ma = mv^2 \ / \ r$.
- By Newton's Universal Law of Gravitation, $F = GMm \ / \ r^2$.
- Hence, $mv^2 \ / \ r = GMm \ / \ r^2$. Solving for v yields the following equation. This is the speed at which an object in a circular orbit of radius r around an object of mass M travels.

$$v = \left(\frac{GM}{r}\right)^{1/2} \tag{9}$$

- [The speed that is necessary to escape an object of mass M from a distance r is only 41% greater.]

$$v_{esc} = \left(\frac{2GM}{r}\right)^{1/2} = 1.41\left(\frac{GM}{r}\right)^{1/2} \tag{10}$$

- Since v is simply distance / time, and the object of mass m travels the circumference $2\pi r$ of the orbit every orbital period P, $v = 2\pi r/P$.
- Hence, $2\pi r/P = (GM/r)^{1/2}$.
- Simplifications yields: $P^2 = (4\pi 2 \ / \ GM) \times r^3$, where $4\pi^2 \ / \ GM$ is a constant for all such objects that orbit the object of mass M, and where r is also the semi-major axis since the orbit is circular.

EXERCISE 4

Hold your index finger vertically in front of your nose and focus on some distant object, such as a wall. Close one eye and then open it while closing the other. Notice how much your finger appears to shift with respect to the far-off object. Now, hold your finger at arm's length and repeat this exercise. Do you notice a difference?

HOMEWORK 2

Download Homework 2 from WebAssign. Feel free to work on these questions together. Then submit your answers to WebAssign individually. Please do not wait until the last minute to submit your answers and please confirm that WebAssign actually received all of your answers before logging off.