Answers to Selected Exercises

Chapter 1

Section 1.1

1. Yes **3.** No **5.** Yes

7. a. $-4, -3, -2, 2, 3$ **b.** $0, 4.5$
c. $[-4, -3], [-2, 2], [3, 5]$
d. $[-5, -4], [-3, -2], [2, 3]$

9. a. Fourth; **b.** third; **c.** second;
d. first. The remaining graph is the last: Profits have steadily decreased.

11. $x \neq 1$ **13.** $x \geq 0$ **15.** $x > 1$

17. $x \neq 2, 3$ **19.** All real numbers

21. $(-2, 0) \cup (0, \infty)$ **23.** All real numbers

25. Domain: $[0, 2]$ **27.** Domain: $[0, 2]$

29. Domain: $[0, 6]$

31. (25) increasing on $(0, 1)$, decreasing on $(1, 2)$, concave down on $(0, 2)$; (26) increasing on $(-1, 1)$, concave up on $(-1, 0)$, concave down on $(0, 1)$; (28) increasing on $(-1, 1)$ and $(1, 2)$, concave up on $(1, 2)$, concave down on $(-1, 1)$

33. $\frac{1}{2}, \frac{2}{3}, -1, \frac{1}{\sqrt{2}}, \frac{1}{x+3}, \frac{x+1}{x+2}$

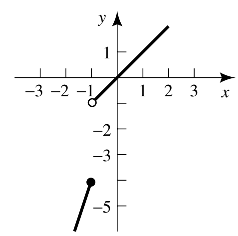
35. $3, 0, \sqrt{2x^2-2}, \sqrt{\frac{2}{x}-1}, \sqrt{-2x-1}, -\sqrt{2x-1},$

37. $1, -1, |x|, x^2$ **39.** $1, 2, 2, 2$

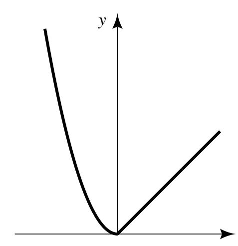
41. $0, 1, 0, 1$ **43.** Yes

45. No **47.** No **49.** Yes

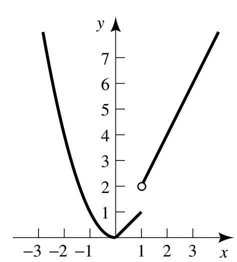
51. Continuous everywhere except $x = -1$.



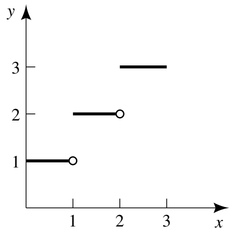
53. Continuous everywhere



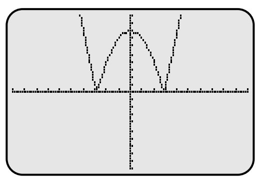
55. Continuous everywhere except $x = 1$



57. Continuous except at $x = 1$ and $x = 2$

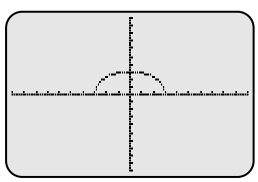


59. All real numbers



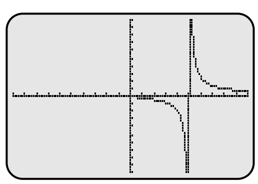
$[-10, 10] \times [-10, 10]$

61. $[-3, 3]$



$[-10, 10] \times [-10, 10]$

63. $[0, 5) \cup (5, \infty)$



$[-10, 10] \times [-10, 10]$

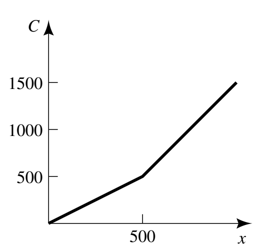
65. 0, 2, 3, 4

67. $V(x) = 3x^3$

69. $d(t) = 60t$

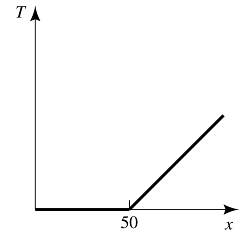
71. $R(x) = 60x$

73. $C(x) = \begin{cases} x & \text{if } 0 \le x \le 500 \\ 500 + 2(x - 500) & \text{if } x > 500 \end{cases}$



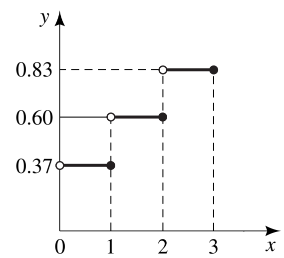
This function is continuous on $[0, \infty)$

75. $T(x) = \begin{cases} 0 & \text{if } 0 \le x \le 50 \\ 0.01(x - 50) & \text{if } x > 50 \end{cases}$



Continuous on $[0, \infty)$

77. $P(x) = \begin{cases} 0.37 & \text{if } 0 \le x \le 1 \\ 0.60 & \text{if } 1 < x \le 2 \\ 0.83 & \text{if } 2 < x \le 3 \end{cases}$

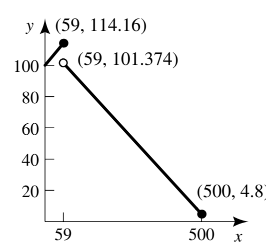


Continuous on $[0, 3]$ except at $x = 1$ and 2.

79. $A(10) = 0.80$. There are 0.80 fish on average in a cubic meter of water that is at 10°C. $A(13) = 1.15$. There are 1.15 fish on average in a cubic meter of water that is at 13°C. $A(19) = 0$. There are no fish on average in a cubic meter of water that's at 19°C.

81. $N(0) = 0$. There are no fish at the surface. $N(120) = 20$. Twenty fish were caught at a depth of 120 meters. $N(260) \approx 3$. There are about 3 fish at a depth of 260 meters.

83. 109.6, 70.5



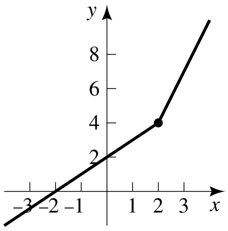
85.

$$T(x) = \begin{cases} 0.15x & \text{if } x \le 27,050 \\ 4057.50 + 0.275(x - 27,050) & \text{if } 27,050 < x \le 65,550 \\ 14,645 + 0.305(x - 65,550) & \text{if } 65,550 < x \le 136,750 \\ 36,361 + 0.355(x - 136,750) & \text{if } 136,750 < x \le 297,350 \\ 93,374 + 0.391(x - 297,350) & \text{if } x > 297,350 \end{cases}$$

87. $S(118) \cong 7$. This means that there will be about 7 plant species in 118 square meter area of woodland. S is reduced by a factor of $(0.5)^{0.248} \approx 0.82$

89. 3

91. $6x + 3h$

93. 

$$f(x) = \begin{cases} 3x - 2 & \text{if } x \geq 2 \\ x + 2 & \text{if } x < 2 \end{cases}$$

95. The two functions are the same.

97. $f(a + b) = m(a + b) = ma + mb = f(a) + f(b)$

99. $3f(x) = 3 \cdot 3^x = 3^{x+1} = f(x + 1)$, $f(a + b) = 3^{a+b} = 3^a \cdot 3^b = f(a) \cdot f(b)$

Section 1.2

1. $C = 3x + 10{,}000$

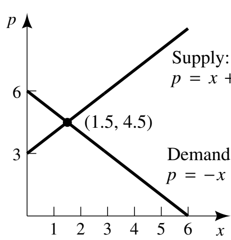
3. $R = 5x$

5. $P = 2x - 10{,}000$

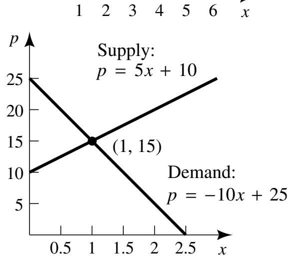
7. $x = 2$

9. $x = 20$

11. $(1.5, 4.5)$

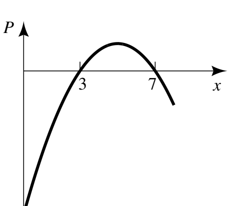


13. $(1, 15)$

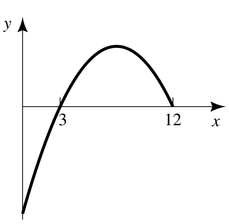


15. $x = 5$

17. $x = 1.5$

19. 

$P(x) = -2x^2 + 20x - 42$, maximum when $x = 5$, break-even quantities are $x = 3, 7$

21. 

$P(x) = -x^2 + 15x - 36$, maximum when $x = \frac{15}{2}$, break-even quantities are $x = 3, 12$

23. Break-even quantities are $x = 0.9$ and 2.2. Maximum at $x = 1.55$.

25. Break-even quantities are approximately $x = 1.936436$, 7.663564. Maximum at $x = 4.8$.

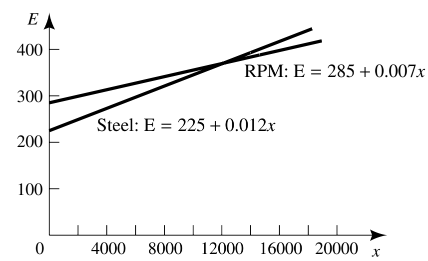
27. $V = -5000t + 50{,}000$, \$45,000, \$25,000

29. $p = -0.005x + 22.5$

31. Let x be the number of pairs of fenders manufactured. Then the costs functions in dollars are as follows:

Steel: $C(x) = 260{,}000 + 5.26x$
Aluminum: $C(x) = 385{,}000 + 12.67x$
RMP: $C(x) = 95{,}000 + 13.19x$
NPN: $C(x) = 95{,}000 + 9.53x$
PPT: $C(x) = 95{,}000 + 12.555x$

33. 12,000

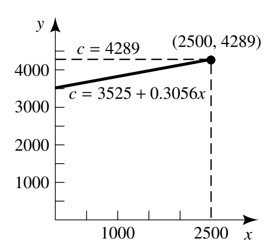


35. Let x be the number of cows, then the cost $C(x)$ in dollars is $C(x) = 13{,}386 + 393x$. The revenue function in dollars is $R(x) = 470x$. The profit function in dollars is $P(x) = 77x - 13{,}386$. The profit for an average of 97 cows is $P(97) = -5917$, that is, a loss of \$5917. For many such farms the property and buildings have already been paid off. Thus, the fixed costs for these farms are lower than stated in the table.

37. $C = 209.03x + 447{,}917$, $R = 266.67x$, $P = 57.64x - 447{,}917$

39. Outside

41. **a.** $f = 0.3056x$; **b.** $c = 3525 + 0.3056x$; **c.** \$4289



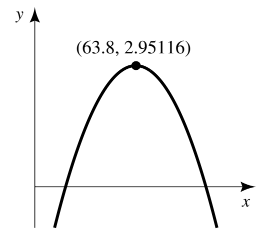
43. Let x be the number of copies. The cost in cents with no plastic card is $C(x) = 10x$. The cost with a plastic card is $C(x) = 50 + 7x$. 17.

45. Decreases by 0.108475 yen per ton. This number is the slope of the line.

47. 110

49. 5593 bales, 30,000 bales

51. 63.8, 2.95116



53. About 286 pounds, the relative yield of vegetables is maximized

55. \$11.667 billion, 58.46% **57.** 35

59. **a.** 30.74; **b.** 19.91; **c.** 41.57

61. $C = 5x + 1000$ **63.** $C = 3x + 2000$

65. $R = 11x$

67. **a.** Manual; **b.** automatic.

Volume	Total Costs Manual	Automatic
1000	\$2600	\$8020
10,000	\$17,000	\$8200

69.

Volume	Costs per Unit Manual	Automatic
1000	\$2.60	\$8.02
10,000	\$1.70	\$0.82
100,000	\$1.61	\$0.10

$\overline{C}_m = \dfrac{1000}{x} + 1.6$, $\overline{C}_a = \dfrac{8000}{x} + 0.02$. The cost per unit for a manual machine tends toward \$1.60 and becomes greater than the cost per unit for an automatic machine, which tends toward \$0.02.

71. $\alpha = 2$, $\beta = 8$

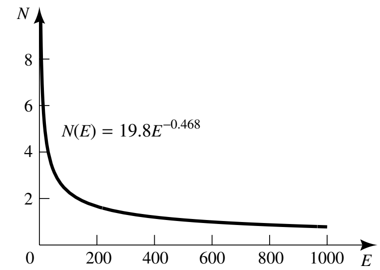
73. Since we expect profits to turn negative if too many products are made and sold (the price would have to be very low to unload so many products), a should be negative.

75. The graph of $p = mx + e$ should slope downward, requiring $m < 0$ and $e > 0$. Since $R = xp = mx^2 + ex$ and $R = ax^2 + bx$, $a = m < 0$ and $b = e > 0$.

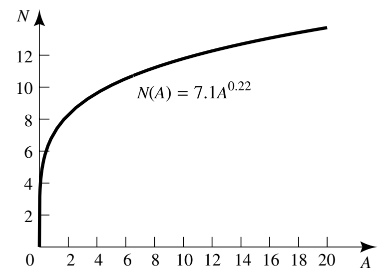
77. Approximately 2%, 4%.

79. The fewer the number of boat trips, the more the owner needs to charge to make a living. But at some point the price is so high that nobody will be willing to pay such a high price. The industry refers to this as the "choke" price.

81. About 20. There are about 0.72 times the original, or a 28% drop

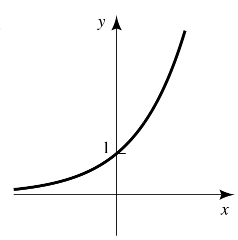


83. About 7. about 66%.

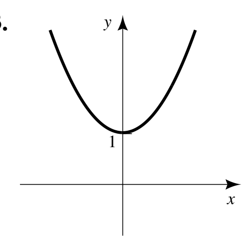


Section 1.3

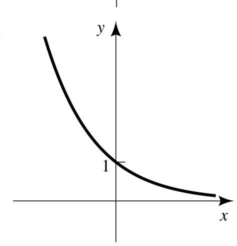
1.



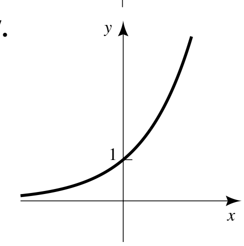
3.



5.



7.



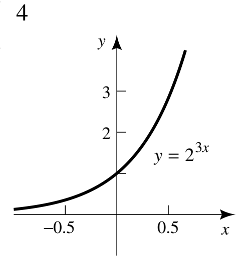
9. 2

11. $\dfrac{3}{4}$

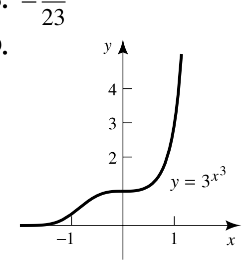
13. $\dfrac{3}{4}$

15. $-\dfrac{45}{23}$

17.



19.



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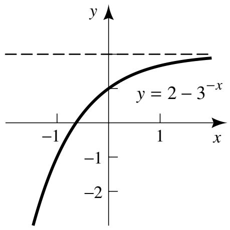
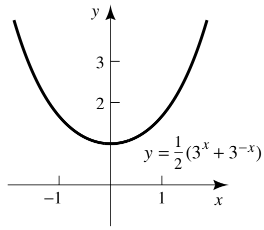


23.



25. -1 **27.** -3

29. $-\dfrac{1}{2}$ **31.** $-\dfrac{1}{2}$

33. 1 **35.** -1

37. 0, 1

39. **a.** \$1080; **b.** \$1082.43; **c.** \$1083.00; **d.** \$1083.22; **e.** \$1083.28; **f.** \$1083.29

41. **a.** \$3262.04; **b.** \$7039.99; **c.** \$14,974.46; **d.** \$31,409.42; **e.** \$93,050.97; **f.** \$267,863.55

43. **a.** 8.16%; **b.** 8.243%; **c.** 8.300%; **d.** 8.322%; **e.** 8.328% **f.** 8.329%

45. **a.** \$1784.31; **b.** \$1664.13; **c.** \$1655.56; **d.** \$1652.99

47. \$31,837.58 **49.** \$26,897.08

51. \$5.5 trillion **53.** \$1104.79, no

55. 6.66%, 6.82%, the second bank

57. \$67,297.13 **59.** 11.58 years

61. 8.2%

63. The second bank, the second effective interest rate is greater than the first one.

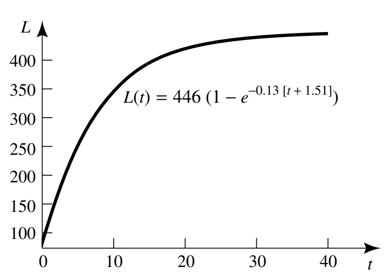
65. About 3.7 years **67.** \$568.95

69. 6.8 billion **71.** 5%

73. 16.82 **75.** $(0, \infty)$, $(-\infty, 0)$

77. $P = 32(0.5)^x$

79. Increasing. 346 mm. 736 grams.



81. 2.1

Section 1.4

1. **a.** 11 **b.** 3 **c.** 28 **d.** $\frac{7}{4}$

3. $x^2 + 2x + 4$, $-x^2 + 2x + 2$, $(2x + 3)(x^2 + 1)$, $\dfrac{2x + 3}{x^2 + 1}$. All domains are $(-\infty, \infty)$.

5. $3x + 4$, $x + 2$, $2x^2 + 5x + 3$, $\dfrac{2x + 3}{x + 1}$. The domain is $(-\infty, \infty)$ in the first three cases and $(-\infty, -1) \cup (-1, \infty)$ in the last case.

7. $\sqrt{x + 1} + x + 2$, $\sqrt{x + 1} - x - 2$, $\sqrt{x + 1}(x + 2)$, $\dfrac{\sqrt{x + 1}}{x + 2}$. Domains are all $[-1, \infty)$.

9. $2x + 1 + \dfrac{1}{x}$, $2x + 1 - \dfrac{1}{x}$, $\dfrac{2x + 1}{x}$, $x(2x + 1)$. Domains are all $(-\infty, 0) \cup (0, \infty)$.

11. $[-1, \infty)$ **13.** $\left[\dfrac{5}{2}, \infty\right)$

15. **a.** 125; **b.** -1

17. $6x - 3$, $6x + 1$. Domains are all $(-\infty, \infty)$.

19. x, x, Domains: $(-\infty, \infty)$, $(-\infty, \infty)$

21. **a.** 5; **b.** 54; **c.** 7; **d.** 16

23. $f(x) = x^5$, $g(x) = x + 5$

25. $f(x) = \sqrt[3]{x}$, $g(x) = x + 1$

27. $f(x) = |x|$, $g(x) = x^2 - 1$

29. $f(x) = \dfrac{1}{x}$, $g(x) = x^2 + 1$

31. **a.** 0; **b.** -1; **c.** 0; **d.** 1

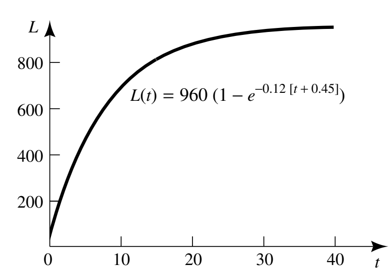
33. $9t + 10$. The firm has a daily start-up cost of \$10,000 and costs of \$9000 per hour.

35. $R(x) - C(x)$, which is the profit $P(x)$.

37. $V(t) = \dfrac{4\pi(30 - 2t)^3}{3}$

39. $g(x) = 40x$, $f(r) = 5.5r$, $y = f(g(x)) = (f \circ g)(x) = 40(5.5)x = 220x$. There are 220 yards in a furlong.

41. Increasing. 686 mm. 3178 grams. $W(L(t)) = 1.30 \times 10^{-6} \cdot (960(1 - e^{-0.12[t+0.45]}))^{3.31}$



43. $(-\infty, \infty)$ **45.** $[-5, \infty)$ **47.** $(-\infty, 5]$

Section 1.5

1. 100 **3.** $e^{-\pi}$

5. $3^{5/3}$ **7.** 4

9. -1

11. 2π

13. 8

15. 81

17. $2\log x + \frac{1}{2}\log y + \log z$

19. $\frac{1}{2}\log x + \frac{1}{2}\log y - \log z$

21. $\frac{1}{2}(\log x + \log y - \log z)$

23. $\log x^2 y$

25. $\log \dfrac{\sqrt{x}}{\sqrt[3]{y}}$

27. $\ln \dfrac{x^3 y}{\sqrt[3]{z}}$

29. $0.2\log(0.6)$

31. $-0.5\log 3$

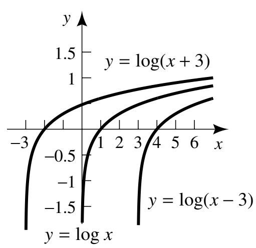
33. $\dfrac{1-\log 2}{3}$

35. $\pm\sqrt{\ln 4}$

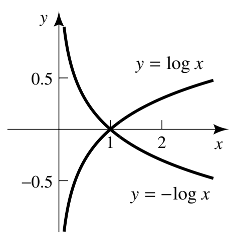
37. $10^{-1.5} - 7$

39. 2.5

41.



43.



45. $x \approx 0.01$ and $x \approx 1.4724$

47. $10^x = e^{(\ln 10)x} \approx e^{2.3026x}$

49. About 6.5 weeks

51. About 7.3 years

53. About 0.0084

55. 24.8 years

57.
$$
\begin{aligned}
y &= P_0 e^{-kt}\\
P_0 e^{-kT} &= \frac{1}{2}P_0\\
e^{-kT} &= \frac{1}{2}\\
-kT &= \ln\frac{1}{2} = -\ln 2\\
kT &= \ln 2
\end{aligned}
$$

59. $\ln 3 \approx 1.0986$ years

61. About 5.8 hours

63. 24.94%

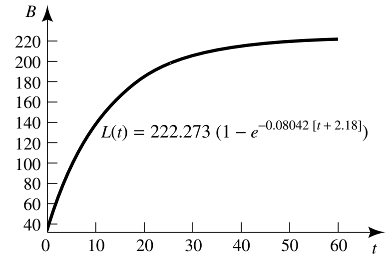
65. About \$40

67. $\log x = \dfrac{\ln x}{\ln 10}$

69. No. Since for $y > 0$, $\log_1 x = y$ if and only if $x = 1^y = 1$ has no solution if $x \neq 1$.

71. y_2

73. 5.25 years



Chapter 1 Review Exercises

1. $5, 11, 2x^2 + 4x + 5, 2x^4 + 3$

2. $0, -\frac{3}{5}, \dfrac{x+1}{x^2+4x+5}, \dfrac{x^3-1}{x^6+1}$

3. Yes

4. No

5. Yes

6. No

7. No

8. Yes

9. $\{x \mid x \neq 3\}$

10. $\{x \mid x > 5\}$

11. All x except for $x = \pm 1$

12. $|x| > 1$

13. $x \neq 0$

14. $x > 0$

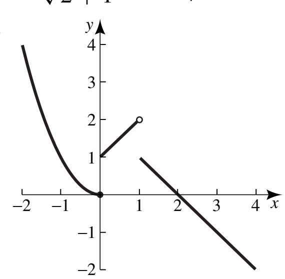
15. $x \leq 2$

16. All x

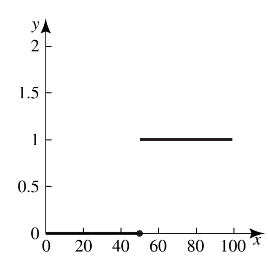
17. $1, \dfrac{1}{3}, \dfrac{1}{a^2 - 2a + 2}, \dfrac{1}{x^2 + 2x + 2}$

18. $1, \dfrac{1}{\sqrt{2}+1}, \dfrac{1}{a}, \dfrac{x+1}{x+2}$

19.



20. If your bill is \$50.00 you pay no tax. If your bill is \$50.01, you pay $.01(50.01) = 0.50$ dollars. A graph is shown.

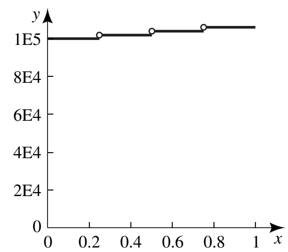


The break in the graph at 50.00 means this is a point of discontinuity.

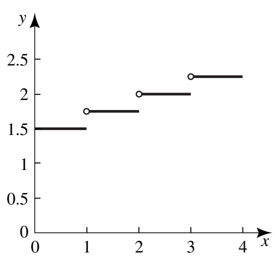
So a one cent increase in your bill, when your bill is \$50.00, results in an increase of \$0.50 in tax.

21. The account is at \$100,000 until the end of the first quartrer, when it abruptly becomes \$102,000. It stays at this amount until the end of the second quarter when it abruptly becomes $1.02(102,000) = 104,040$ dollars. It stays at this amount until the end of the third quarter when it abruptly becomes $1.02(104,000) = 106,120.80$ dollars. It stays at this az-mount until the end of the foursth quarter when it abruptly becomes $1.02(106,120.80) = 108,243.22$ dollars.

The graph is shown. Breaks occur in the graph at the quarter points, so these are points of discontinuity. Notice that if you withdrew your money 1 second before the end of the first quarter, you would obtain \$100,000. But if you withdrew your money 1 second into the second quarter, you would obtain \$102,000. Such a point of discontinuity would certainly affect when you withdraw your money.

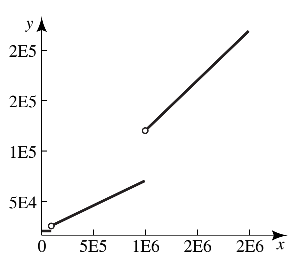


22. A graph is shown.



Discontinuities occur at the minute marks. For example, a 59.9 second call costs \$1.50, but a 60.1 second call costs \$1.75.

23. A graph is shown.



Discontinuities occur at 100,000 and 1,000,000. If the salesman sales \$100,000, he earns \$20,000. But if he sales \$100,000.01, he earns $20,000 + .05(100,000.01) = 25,000$ dollars. So a 1 cent increase in sales results in a \$5000 increase in salary.

If his sales are \$1,000,000, he earns $20,000 + 0.05(1,000,000 - 100,000) = 65,000$ dollars. But if sales are at \$1,000,000.01, he earns $20,000 + 0.10(1,000,000.01) = 120,000$ dollars. So a 1 cent increase in sales results in a $120,000 - 65,000 = 55,000$ dollar increase in earnings.

24. $x^3 + 2x + 3$. Domain: $(-\infty, \infty)$

25. $x^3 - 2x + 5$. Domain: $(-\infty, \infty)$

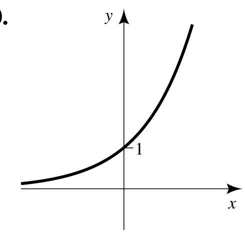
26. $(x^3 + 4)(2x - 1)$. Domain: $(-\infty, \infty)$

27. $\dfrac{x^3 + 4}{2x - 1}$. Domain: $(-\infty, 0.50) \cup (0.50, \infty)$

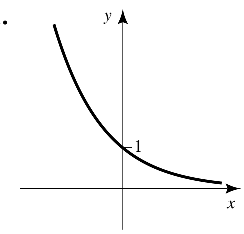
28. $9x^2 + 6x + 3$, $3x^2 + 7$. Domains: $(-\infty, \infty)$, $(-\infty, \infty)$

29. $\sqrt{x}$, $\sqrt{x-1} + 1$. Domains: $[0, \infty)$, $[1, \infty)$

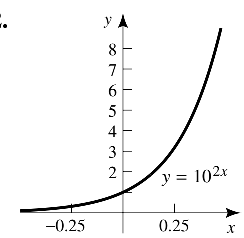
30.



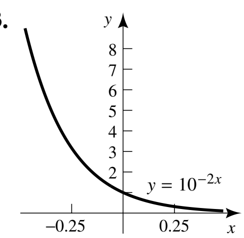
31.



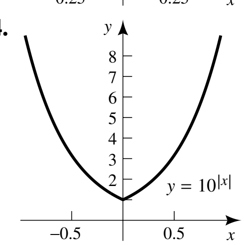
32.



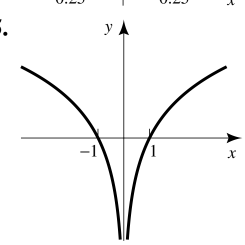
33.



34.



35.



36. $\dfrac{3}{2}$

37. 3

38. e

39. 0.50

40. 4

41. 5

42. Approximately day 227

43. $x \approx 1.5953$

44. $C = 6x + 2000$

45. $R = 10x$

46. $P = 4x - 2000$, 500

47. $x = 150$

48. $x = 1000$, $b = 2000$

49. 150 miles

50. 42 minutes

51. $p - 1 = -\dfrac{1}{200,000}(x - 100,000)$

52. \$29.37

53. 12,000

54. $4.5x + 5.5y = 15$. $m = -\frac{9}{11}$. For every additional 11 cups of kidney beans, she must decrease the cups of soybeans by 9.

55. If x is tons, then $C(x) = 447,917 + 209.03x$, $R(x) = 266.67x$, $P(x) = 57.64x - 447,917$

56. **a.** Boston **b.** Houston

57.

Quadruped	l	h	$l : h^{2/3}$
Ermine	12 cm	4 cm	4.762
Dachshund	35 cm	12 cm	6.677
Indian tiger	90 cm	45 cm	7.114
Llama	122 cm	73 cm	6.985
Indian elephant	153 cm	135 cm	5.814

58. 10, 20, 30

59. For each unit distance away from the pocket margin, the proportion of dead roots decrease by 0.023 unit.

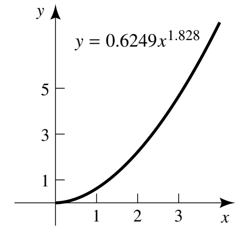
60. For 10,000 more units of shoot length, there will be 275 more buds.

61. $c = -6x + 12$

62. 6.84 years, 508.29 pounds

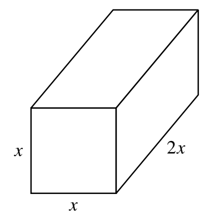
63. 103.41 pounds, 4294.39 pounds

64. 149.31, 313.32, 657.48

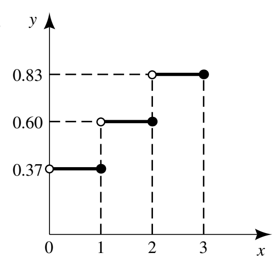


65. 11.18

66. $10x^2$



67.



68. $4\pi t^2$

69. 13.25 years

70. $k = 0.02$

71. In the year 2022

72. $\ln 10/k$

73. Day 253

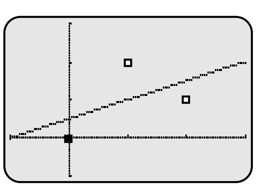
74. 0.47

75. 231 years

Chapter 2

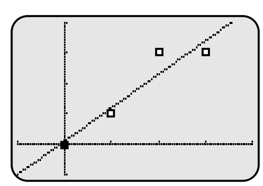
Section 2.1

1. $y = 0.50x + 0.50$, 0.5000



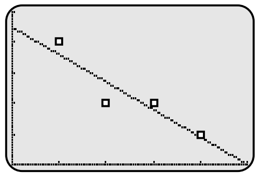
[−1, 3] × [−1, 3]

3. $= 1.1x + 0.1$, 0.9467



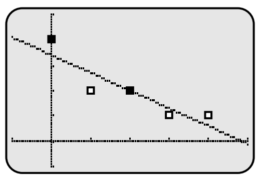
[−1, 4] × [−1, 4]

5. $y = -0.9x + 4.5, -0.9234$



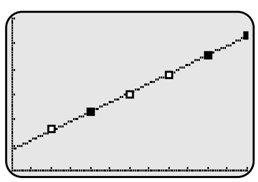
$[0, 5] \times [0, 5]$

7. $y = -0.7x + 3.4, -0.9037$



$[-1, 5] \times [-1, 5]$

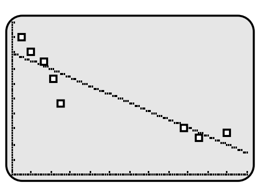
9. If x is thousands of bales and y is thousands of dollars, then $y = 37.32x + 82.448$, $r = 0.9994$



$[0, 12] \times [0, 600]$

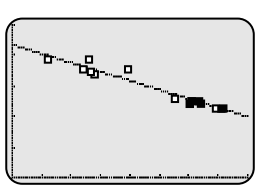
(b) 3180; (c) \$4659 loss; \$21,271 profit

11. **a.** $y = -0.0055x + 1.8244$, $r = -0.9262$.
b. Since the slope is negative, the larger the machine size, the fewer employee-hours are needed per ton.
c. 1.27
d. 149



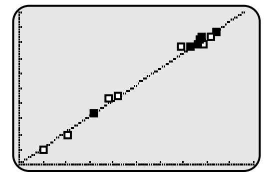
$[40, 280] \times [0, 2]$

13. $y = -0.0005892x + 0.07298$, $r = -0.9653$. The line slopes down. Thus, an increase in price leads to a decrease in demand.



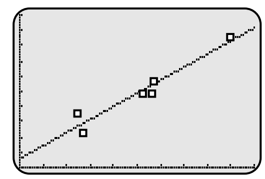
$[50, 90] \times [0, 0.05]$

15. **a.** $y = 2.0768x + 1.4086$, $r = 0.9961$.
b. \$209,000
c. 59,511 dozen



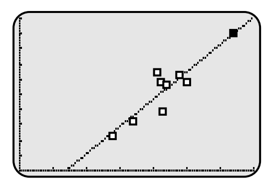
$[0, 150] \times [0, 300]$

17. **a.** $y = 0.8605x + 0.5638$, $r = 0.9703$.
b. 6.6%
c. 7.5%



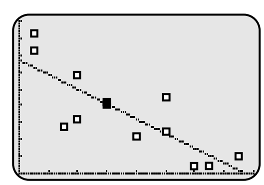
$[0, 10] \times [0, 10]$

19. **a.** $y = 0.3601x - 2.5162$, $r = 0.9070$.
b. 8.3%
c. 26.4%



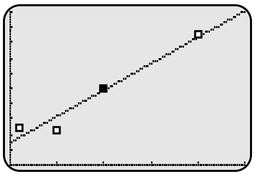
$[0, 35] \times [0, 10]$

21. $y = -87.16x + 1343$, $r = -0.8216$.



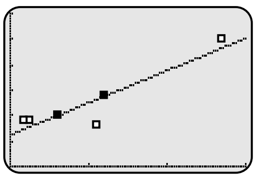
$[0, 16] \times [0, 1800]$

23. **a.** $y = 3.4505x + 14.4455$, $r = 0.9706$, $z = 2.0446x + 1.0990$, $r = 0.9413$;
b. For each additional aphid per plant the percentage of times the virus is transmitted to the fruit increases by about 3.5 for the brown aphid and about 2 for the melon aphid;
c. The melon aphid is more destructive, since the melon aphid transmits the virus more often than the brown aphid.



$[0, 25] \times [0, 100]$

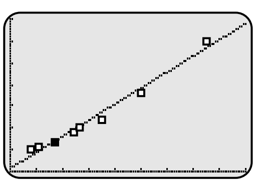
25. a. $y = 0.1264x - 19.0979$;
b. $r = 0.9141$;
c. for each additional trichome per 6.25 mm^2, there is an increase 0.1264% of damaged pods.



$[200, 350] \times [0, 30]$

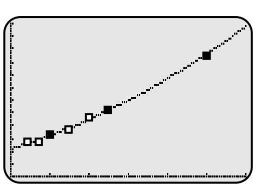
Section 2.2

1. a. $y = 0.1848x + 0.6758$, $r^2 = 0.9800$.
b. $y = 0.0006357x^2 + 0.0807x + 3.5705$, $r^2 = 0.9992$. Quadratic regression is better because of its higher correlation coefficient.



$[0, 180] \times [0, 35]$

3. a. $y = 0.0011x^2 + 0.0943x + 2.1498$, $r^2 = 0.9978$.
b. \$5932;
c. 16,544;
d. approximately 45.1



$[0, 60] \times [0, 12]$

5. a. $y = 0.000083x^2 - 0.02396x + 2.80925$; about 144

7. $y = 0.0031x^2 - 0.04x + 0.494$. About 6 years.

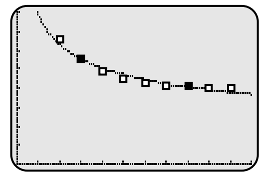
9. a. Linear gives $y = 28.545x + 155.237$, $r^2 = 0.9995$, quadratic gives $y = 0.1150x^2 + 26.0465x + 165.0653$, $r^2 = 0.99999997$. Quadratic is better.
b. 4499 bales. **c.** 20,000 bales

11. $y = -0.0044x^2 + 0.2851x - 3.0462$, about 32.45

13. $-0.00349x^2 + 0.39551x + 21.8891$. About 57.

Section 2.3

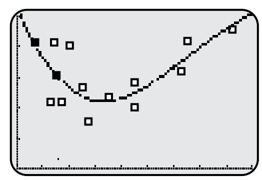
1. a. $y = 80.55x^{-0.3145}$, $r = -0.9758$.
b. \$20.72; **c.** about 23



$[0, 110] \times [0, 40]$

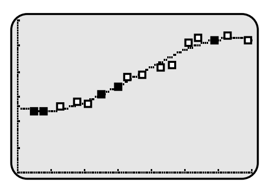
3. a. $y = 136.04x^{0.5263}$, $r^2 = 0.9678$, quadratic is best;
b. 5037 bales; **c.** 20,000 bales

5. $y = -0.00000217x^3 + 0.000425x^2 - 0.021x + 1.034$, $r^2 = 0.5356$.
Minimum at $x = 33.218567$



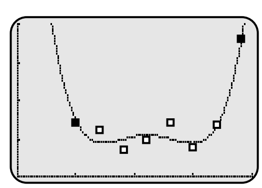
$[0, 90] \times [0, 1]$

7. Cubic regression is $y = -0.00031x^3 + 0.0523x^2 - 2.182x + 51.17$, $r^2 = 0.9838$.
Linear and quadratic regressions are $y = 0.548x + 7.958$, $r^2 = 0.9548$, $y = -0.00354x^2 + 0.588x + 6.968$, $r^2 = 0.9550$. Cubic is better.



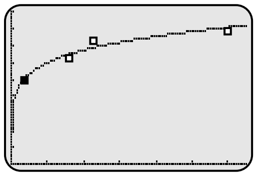
$[20, 90] \times [0, 60]$

9. $y = 0.0056x^4 - 0.581x^3 + 22.61x^2 - 388x + 2484$, $r^2 = 0.9393$.



$[15, 35] \times [0, 20]$

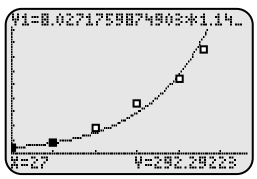
11. $y = 36.23x^{0.167}$, $r = 0.9738$



[0, 130] × [0, 90]

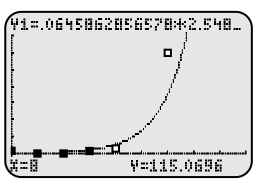
Section 2.4

1. $y = 8.027(1.1424)^x$, $r = 0.9932$.
Payments in 1997 are about \$292 billion.



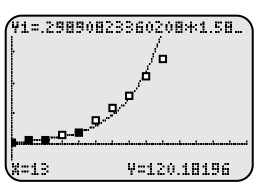
[0, 28] × [–20, 200]

3. $y = 0.0646(2.5489)^x$, $r = 0.9809$
The number in 1997 is about 115 million.



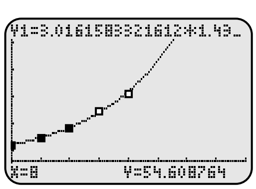
[0, 9] × [–5, 40]

5. a. $y = 0.2989(1.5861)^x$, $r = 0.9889$.
The number in 1997 is 120 million.
b. 1995



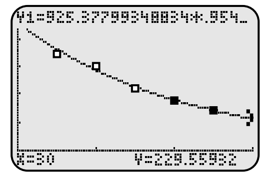
[0, 14] × [–5, 20]

7. a. $y = 3.016(1.4362)^x$, $r = 0.9985$
The number in 1997 is about 115 million.
b. According to the model, the number reached 20 million about one and a quarter years after 1993.



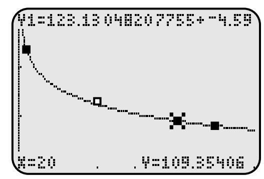
[0, 9] × [–3, 25]

9. a. $y = 925(0.9546)^x$, $r = -0.9975$.
The model predicts about 230 ordinations in 1997.
b. According to the model, the number reached 150 in 2006.



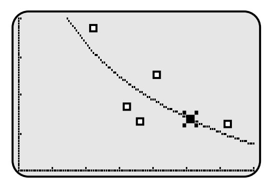
[0, 30] × [–100, 1000]

11. $y = 123 - 4.599 \ln x$, $r = -0.9986$.
The model predicts about 109 days with 20%.



[0, 30] × [100, 130]

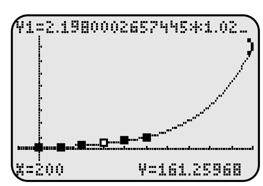
13. $y = 1457 - 212 \ln x$, $r = -0.7525$
According to the model, there would be a 142% increase with a 1988 population of 500.



[0, 700] × [0, 400]

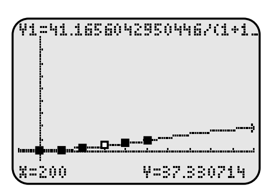
Section 2.5

1. a. $y = 2.198(1.0217)^x$, $r = 0.9961$. This model estimates the population in 1990 to be 161.3 million.



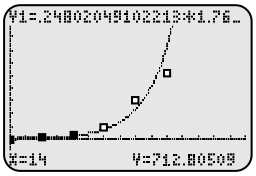
[–20, 200] × [–40, 200]

b. $y = 41.2/(1 + 18.33e^{-0.259})$. This model estimates the population in 1990 to be 37.3 million, a much better approximation.



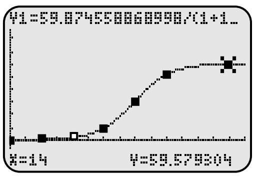
[–20, 200] × [–40, 200]

3. a. $y = 0.248(1.766)^x$, $r = 0.9938$. This model estimates the sales in 1992 to be 713 million.



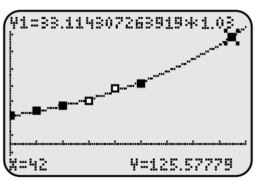
$[0, 15] \times [-20, 100]$

b. $y = 59.9/(1 + 1038e^{-0.8752})$. This model estimates the sales in 1992 to be 60 million, a much better approximation.



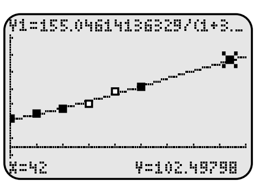
$[0, 15] \times [-20, 100]$

5. a. $y = 33.11(1.0322)^x$, $r = 0.9939$. This model estimates the sales in 1992 to be 125.6 quadrillion Btu.



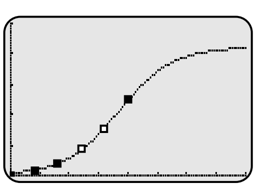
$[0, 45] \times [-30, 150]$

b. $y = 155/(1 + 3.800e^{-0.0477})$. This model estimates the sales in 1992 to be 102.5 quadrillion Btu, a much better approximation.



$[0, 45] \times [-30, 150]$

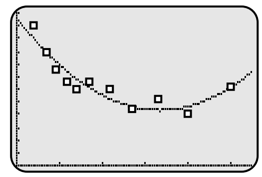
7. $y = 422.5/(1 + 55.03e^{-0.0218})$. The limiting value is 422.5 million.



$[0, 400] \times [0, 500]$

Section 2.6

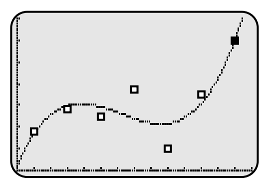
1. Exponential regression gives $y = 85.59(0.9859)^x$ with $r^2 = 0.5209$. This is not an outstanding fit. Power regression gives $y = 156.75x^{-0.3225}$ with $r^2 = 0.7570$. This is an improvement over exponential regression. Quadratic regression gives $y = 0.0645x^2 - 4.325x + 116.05$ with $r^2 = 0.8787$. This is a distinct improvement over power regression. The graph is shown below. Notice the good fit and notice that the graph of the quadratic turns up at the right. The graph looks very much like we would expect based on our business insights.



$[0, 55] \times [0, 120]$

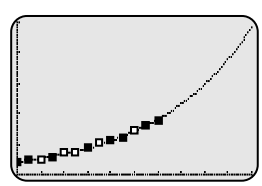
Naturally, cubic regression will lead to a better fit than quadratic regression. Doing cubic regression on a TI-83 gives $r^2 = 0.8946$. However, this is probably not a big enough improvement to justify using the more complicated cubic model. We conclude that the quadratic model is best. The quadratic model gives an excellent fit, is simple, and gives a graph that is reasonable based on basic business considerations.

3. Cubic is best with $r^2 = 0.7751$. (Quartic gives only a very slight improvement with $r^2 = 0.7856$.)



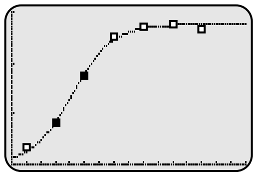
$[0, 14] \times [0, 7]$

5. Exponential ($r^2 = 0.9973$) is best, since quadratic ($r^2 = 0.9982$) turns up just to the left of the our prescribed interval.



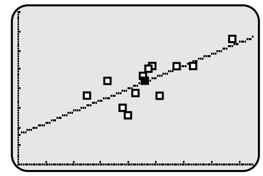
$[1, 21] \times [0, 2500]$

7. Graphs clearly show the logistic curve fits well. The cubic ($r^2 = 0.9824$) is rejected, since the graph turns down at the end of the interval. The quartic ($r^2 = 0.9981$) is also rejected, since the graph dips on the latter part of the interval, and also because the graph initially decreases.



$[0, 16] \times [0, 3]$

9. Linear ($r^2 = 0.5684$). Quadratic ($r^2 = 0.6048$) probably does not give a sufficient improvement for a more complicated model.



$[0, 170] \times [0, 40]$

Chapter 3

Section 3.1

1. 2, 2, 2

3. 2, 2, 2

5. 1, 2, does not exist

7.

x	0.9	0.99	0.999	$\to$	1	$\leftarrow$	1.001	1.01	1.1
$f(x)$	4.7	4.97	4.997	$\to$	5	$\leftarrow$	5.003	5.03	5.3

$$\lim_{x \to 1} f(x) = 5$$

9.

x	1.9	1.99	1.999	$\to$	2	$\leftarrow$	2.001	2.01	2.1
$f(x)$	2.859	3.881	3.988	$\to$	4	$\leftarrow$	4.012	4.121	5.261

$$\lim_{x \to 2} f(x) = 4$$

11.

x	1.9	1.99	1.999	$\to$	2	$\leftarrow$	2.001	2.01	2.1
$f(x)$	-10	-100	-1000	$\to$	undefined	$\leftarrow$	1000	100	10

$\lim_{x \to 2} f(x)$ does not exist

13.

x	3.9	3.99	3.999	$\to$	4	$\leftarrow$	4.001	4.01	4.1
$f(x)$	3.9748	3.9975	3.9998	$\to$	4	$\leftarrow$	4.0002	4.0025	4.0248

$$\lim_{x \to 4} f(x) = 4$$

15. 1, 0, does not exist

17. 1, 1, 1

19. 0, does not exist, does not exist

21. -1, 1, does not exist

23. None of the limits exist.

25. 4, does not exist, does not exist

27. 13

29. 13

31. -3

33. $-\dfrac{3}{4}$

35. 1

37. 4

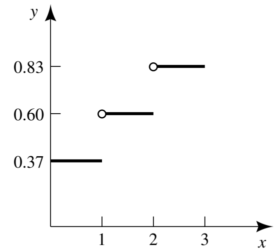
39. 2

41. -2

43. Does not exist

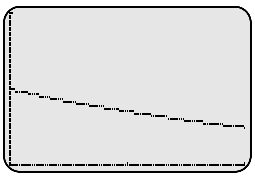
45. 4

47. **a.** 0.37, 0.60, does not exist;
b. 0.60, 0.60, 0.60



49. 1, 10, 100, 1000. $C(x)$ becomes large without bound

51. a.

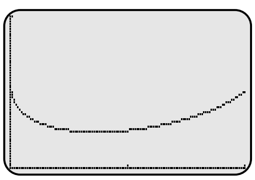


$[0, 1] \times [0, 2]$

x	-0.1	-0.01	-0.001	$\rightarrow$	0	$\leftarrow$	0.001	0.01	0.1
$f(x)$	0.9330	0.9931	0.9993	$\rightarrow$	1	$\leftarrow$	0.9993	0.9931	0.9330

$$\lim_{x\to 0} f(x) = 1$$

b.

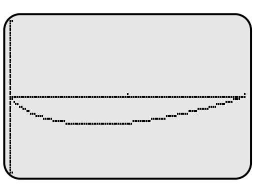


$[0, 1] \times [0, 2]$

x	0	$\leftarrow$	0.0001	0.001	0.01	0.1
$f(x)$	1	$\leftarrow$	0.9982	0.9863	0.9120	0.6310

$$\lim_{x\to 0^+} f(x) = 1$$

c.

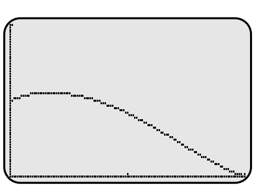


$[0, 1] \times [-1, 1]$

x	0	$\leftarrow$	0.0001	0.001	0.01	0.1
$f(x)$	0	$\leftarrow$	-0.0009	-0.0069	-0.0461	-0.2303

$$\lim_{x\to 0^+} f(x) = 0$$

d.

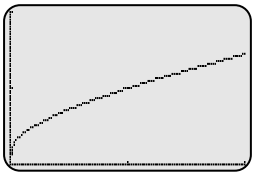


$[0, 1] \times [0, 2]$

x	0	$\leftarrow$	0.0001	0.001	0.01	0.1
$f(x)$	1	$\leftarrow$	1.0002	1.0019	1.0154	1.0870

$$\lim_{x\to 0^+} f(x) = 1$$

e.



[0, 2] × [0, 2]

x	0.9	0.99	.999	$\to$	1	$\leftarrow$	1.001	1.01	1.1
$f(x)$	0.9491	0.9950	0.9995	$\to$	1	$\leftarrow$	1.0005	1.0050	1.0492

$$\lim_{x\to 1} f(x) = 1$$

53. Since $\lim_{x\to 0^-} f(x) = -1 \neq 1 = \lim_{x\to 0^+} f(x)$, it is impossible to define $f(0)$ so that $\lim_{x\to 0} f(x)$ exists.

55.

$$\frac{\sqrt{x}-1}{x-1} = \frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1} = \frac{(x-1)}{(x-1)(\sqrt{x}+1)} = \frac{1}{\sqrt{x}+1},$$

As $x \to 1$, this goes to $1/2$.

57. No. For example, $f(x) = \frac{1}{x}$, $g(x) = 1 - \frac{1}{x}$. Both $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0}(f(x)+g(x)) = \lim_{x\to 0}\left(\frac{1}{x} + 1 - \frac{1}{x}\right) = 1$.

59. For very small farms of, say, less than an acre, the owner has the additional time to give additional nurture to the plants and to protect them from disease, insects, and animals.

61. Everywhere

63. Everywhere except $x = 0$.

65. Everywhere except $x = 1$.

67. Everywhere except $x = 0$.

69. Everywhere

71. Everywhere except $x = \pm 1$

73. Continuous everywhere.

75. Everywhere

77. Continuous on $[-8, \infty)$, where it is defined

79. $k = -1$

81. Every polynomial is continuous at every point. The indicated function in the graph is not continuous at the point where the graph "blows up" and therefore cannot be the graph of any polynomial.

Section 3.2

1. $-2, 2$ **3.** $-0.25, -0.05$ **5.** $-6, -2$

7. $-4, -9$ **9.** $2, -2$ **11.** 96

13. 96 **15.** 0 **17.** -64

19. $y - 48 = 32(t-1)$

21. 2 **23.** 2 **25.** -8

27. 0 **29.** -16 **31.** 3

33. BC, AB and DE, CD

35. Positive at A and D, negative at B, zero at C

37. 0.5 **39.** $y - 6 = 7(x-2)$

41. Peoria: -627. From 1980 to 1997, Peoria lost on average 627 people each year.
Springfield: 315. From 1980 to 1997, Springfield gained on average 315 people each year.

43. -1.2 percent inflation per percent unemployment. For each 1% increase in the percentage of unemployment, the inflation rate as measured by percent will decrease by 1.2.

45. 0.79 mm per day. On average they grow 0.79 mm each day.

47. Virginia's warbler: about -2.6 birds per centimeter of precipitation. For each centimeter increase in precipitation, there were on average about 2.6 fewer of these warblers.
Orange-crowned warbler: about 1.8 birds per centimeter of precipitation. For each centimeter increase in precipitation, there were on average about 1.8 more of these warblers.

49. About 0.25 kg of birth mass per yearly age. Each year the females birthed calves that were on average 0.25 kg heavier.

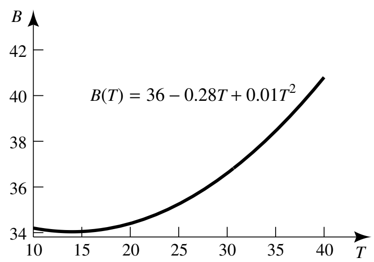
51. About \$0.40 per week worked. About \$6.00 per week worked. On average, hourly wages are rising much faster on the larger interval of weeks worked.

53. **a.** An increase of about 200 private-equity investments per quarter.
b. A drop of about 100 private-equity investments per quarter. Privite-equity investments increased during the first period and then decreased during the second period.

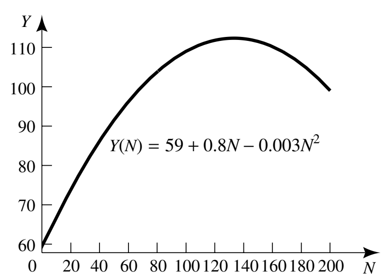
55. Increases about 0.4% per mile. Decreases about 1.3% per mile. The price of an average home first increases as you move away from the city and then sharply decreases.

57. **a.** Increases about \$300 per year.
b. Decreases about \$250 per year. Income increases when younger and then decreases as one gets older.

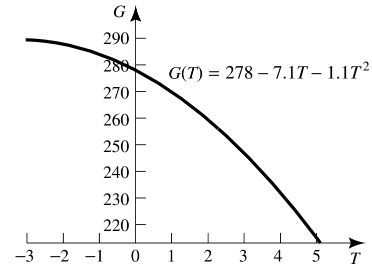
59. **a.** 0.12 degree body temperature per degree ambient temperature.
b. 0.32 degree body temperature per degree ambient temperature. The bird's body temperature increases at a more rapid rate for higher ambient temperatures.



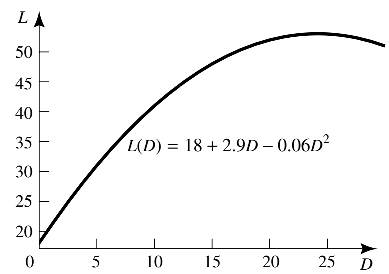
61. **a.** 0.2 bushel of soybeans per pound. At 100 pounds of fertilizer per acre the yield is increasing by about 0.2 bushel per pound of fertilizer.
b. −0.4 bushel of soybeans per pound. At 200 pounds of fertilizer per acre the yield is decreasing by about 0.4 bushel per pound of fertilizer. Initially, increasing the amount of fertilizer increases the yield, but at some point it begins to decrease the yield. Clearly, some fertilizer is helpful, but too much is actually counterproductive.



63. −0.5 day per degree. −7.1 days per degree. There is a more rapid increase in the early start in the growing season as the mean July temperature increases.



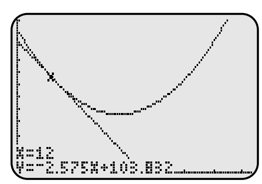
65. **a.** 1.7% per day.
b. −0.1% per day. On the given lactation interval, the percentage of lipid in the milk first increased and then decreased.



67. When 150 tons of steel is being produced, the cost is increasing at \$300 per ton. If cost were to increase at this constant rate, then the cost of the next ton would be \$300.

69. When the tax is set at 3% of taxable income, tax revenue is increasing at \$3 billion per percent increase in tax rate. If the tax revenue were to increase at this constant rate than increasing the tax rate by an additional 1% would increase tax revenue by \$3 billion.

71.

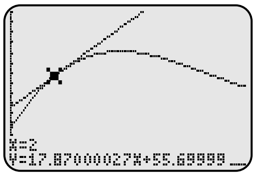


$[0, 75.2] \times [0, 120]$

x	12	20	32	40	52
Slope	−2.57	−1.58	−0.09	0.90	2.39

For example, when $x = 12$, the slope of the tangent line and the rate of change are both −2.57. When $x = 12$, the average costs were decreasing at the rate of \$2.57 per one thousand pairs of shoes increase in output. Average costs initially decrease but at some point begin to increase.

73.



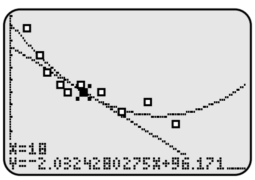
[0, 9.4] × [0, 150]

x	2	3	5	6
Slope	17.87	9.30	−2.98	−6.69

For example, when $x = 2$, the slope of the tangent line and the rate of change are both 17.87. When $x = 2$, the units of smoke in cities were increasing at the rate of 17.87 units per one thousand dollar increase in GDP per capita. The units of smoke in cities initially increase with increasing GDP per capita income but at some point begin to decrease.

77. a. $y = 0.057x^2 - 4.07x + 114.55$, $r^2 = 0.8702$

b.

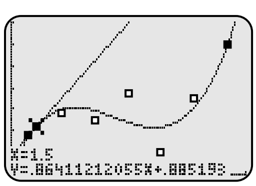


[0, 56.4] × [0, 120]

x	18	27	36	42	48
Slope	−2.03	−1.01	0.01	0.69	1.37

For example, when $x = 18$, the slope of the tangent line and the rate of change are both −2.03. When $x = 18$, the average costs were decreasing at the rate of \$2.03 per one thousand pairs of shoes increase in output. Average costs initially decrease but at some point begin to increase.

79. a. $y = 0.0174x^3 - 0.324x^2 + 1.72x + 0.274$, $r^2 = 0.7751$



[0, 14.1] × [0, 7]

x	1.5	2.7	3.9	6	9.6	12
Slope	0.86	0.35	−0.02	−0.29	0.30	1.45

For example, when $x = 1.5$, the slope of the tangent line and the rate of change are both 0.86. When $x = 1.5$, the units of coliform in waters were increasing at the rate of 0.86 units per one thousand dollar increase in GDP per capita. The units of coliform in waters initially increase with increasing GDP per capita income, then decrease, and then increase again.

Section 3.3

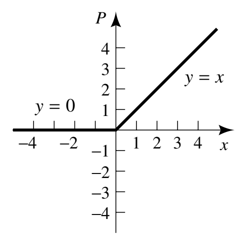
1. 5 **3.** $2x$ **5.** $6x + 3$

7. $-6x^2 + 1$ **9.** $\dfrac{-1}{(x+2)^2}$ **11.** $\dfrac{-2}{(2x-1)^2}$

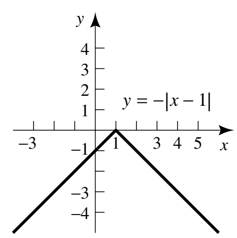
13. (1) $y - 2 = 5(x - 1)$; (3) $y - 5 = 2(x - 1)$; (5) $y - 5 = 9(x - 1)$

15. $y = x - 1$

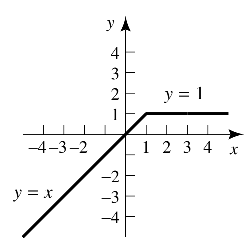
17. Everywhere except $x = 0$



19. Everywhere except $x = 1$



21. Everywhere except $x = 1$



23. 1, 2, 4

25. a. $(0, \infty)$

b. $(-\infty, 0)$

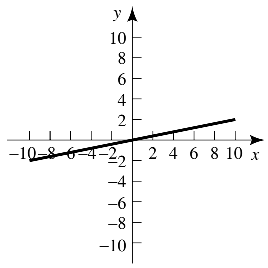
c. $f' \to 0$. f' becomes large. f' becomes a large negative number.

d. $(-\infty, -1)$. $(-1, 0)$ and $(0, \infty)$

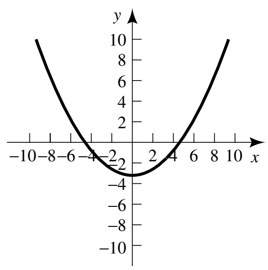
27. (b) **29.** (c) **31.** (e)

33. 3, 1, 0, −1, 0, 1 **35.** Does not exist

37.

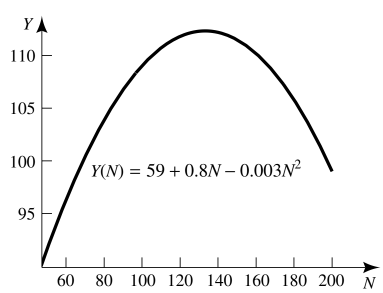


39.

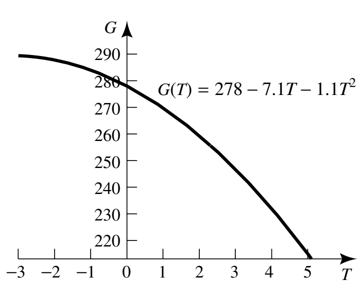


41. $B'(T) = -0.28 + 0.02T$ degree body temperature per degree ambient temperature. The bird's body temperature increases at a more rapid rate for higher ambient temperatures.

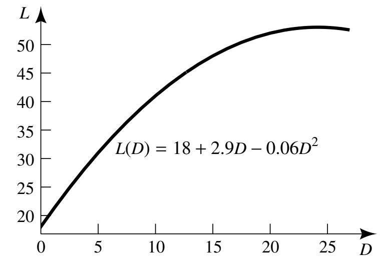
43. $Y'(N) = 0.8 - 0.006N$ bushels of soybeans per pound. Initially, increasing the amount of fertilizer increases the yield, but at some point it begins to decrease the yield. Clearly, some fertilizer is helpful, but too much is actually counterproductive.



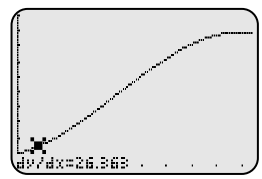
45. $G'(T) = -7.1 - 2.2T$ days per degree. There is a more rapid increase in the early start in the growing season as the mean July temperature increases.



47. $L'(D) = 2.9 - 0.12D$ % per day. On the given lactation interval, the percentage of lipid in the milk first increased, and then decreased.



49.

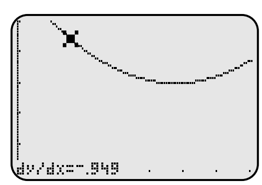


$[0, 47] \times [400, 2100]$

x	5	10	15	20
$f'(x)$	26.36	35.63	41.19	43.06
x	25	30	35	40
$f'(x)$	41.24	35.72	26.50	13.59

Since $f'(5) \approx 26.36$, the slope of the tangent line to the curve at $x = 5$ is approximately 26.36. Thus, when output was \$5000, yard labor costs were increasing at \$26.36 per \$1000 increase in output. Notice that the rate of increase of labor costs itself first increases, and then at larger values of output it decreases.

51.



$[0, 7.05] \times [5, 10]$

x	1.5	3	4.5	6	6.6
$f'(x)$	-0.95	-0.51	-0.073	0.365	0.54

Since $f'(1.5) \approx -0.95$, the slope of the tangent line to the curve at $x = 1.5$ is approximately -0.95. Thus, when assets were 1.5 million dollars, the margin ratio was decreasing at 0.95 units per 1 million dollar increase in assets. The rate of change of the margin ratio initially decreases then at higher values of assets begins to increase.

53. $\dfrac{1}{2\sqrt{t+1}}$ **55.** $\dfrac{1}{\sqrt{2t+5}}$

57. $f'(0) = \lim_{h \to 0} \dfrac{f(h) - f(0)}{h}$

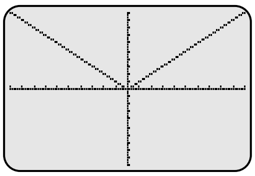
$= \lim_{h \to 0} \dfrac{\sqrt[3]{h^2} - 0}{h}$

$= \lim_{h \to 0} h^{2/3-1} = \lim_{h \to 0} h^{-1/3}$

$= \lim_{h \to 0} \dfrac{1}{\sqrt[3]{h}}$

does not exist

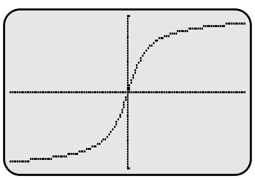
59. a.



$[-10, 10] \times [-10, 10]$

$f'(0)$ does not appear to exist since the graph appears to have a corner when $x = 0$,

b.

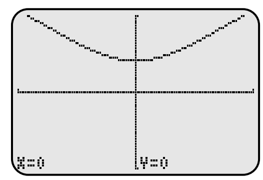


$[-0.01, 0.01] \times [-1, 1]$

From the graph it appears that as $h \to 0$, $\dfrac{\sqrt{h^2 + 0.000001} - 0.001}{h} \to 0$. This implies that $f'(0) = 0$.

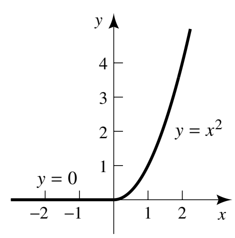
h	0.0001	0.00001
$\dfrac{\sqrt{h^2 + 0.000001} - 0.001}{h}$	0.0499	0.0050

c. The following screen was obtained after using the ZOOM about a half dozen times. Notice that the "corner" observed in part (a) now appears to have rounded out and the tangent to the curve at $x = 0$ now appears to be 0. This supports the conclusion found in part (b).



$[-0.00244, 0.00244] \times [-0.00244, 0.00244]$

61. From the figure it is difficult to tell whether the curve has a corner when $x = 0$.



h	-0.1	-0.01	-0.001	$\to$	0	$\leftarrow$	0.001	0.01	0.1
$\dfrac{f(0+h) - f(0)}{h}$	0	0	0	$\to$	0	$\leftarrow$	0.001	0.01	0.1

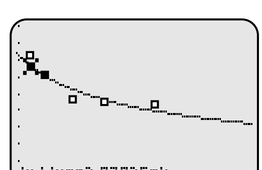
From the numerical work in the table it appears that $f'(0) = 0$. Since $\lim_{x \to 0^-} \dfrac{f(0+h) - f(0)}{h} = \lim_{x \to 0^-} \dfrac{0 - 0}{h} = 0$, and $\lim_{x \to 0^+} \dfrac{f(0+h) - f(0)}{h} = \lim_{x \to 0^+} \dfrac{h^2 - 0}{h} = \lim_{x \to 0^+} h = 0$, then $\lim_{x \to 0} \dfrac{f(0+h) - f(0)}{h} = 0$

63. $f'(0) = 1$ **65.** $f'(1.57) \approx -1$

67. The same, yes

69. a. $y = 29.434x^{-0.5335}$, $r = -0.9245$

b.



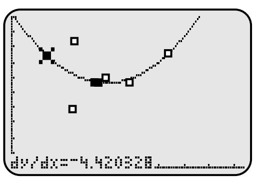
$[1000, 4760] \times [0, 0.9]$

x	1200	1600	2000
$f'(x)$	-0.000298	-0.000192	0.000136
x	2400	2800	3200
$f'(x)$	-0.000103	-0.000081	-0.000066

For example, since $f'(1200) \approx -0.000298$, the slope of the tangent line to the curve at $x = 1200$ is approximately -0.000298. Thus, when the total number of open accounts was 1200, salary cost per account was decreasing at \$0.000298 per account. Notice that the rate of decrease of salary cost per account lessens as the total number of open account increases. (Salary cost efficiencies lessen as the total number of open accounts increase.)

71. a. $y = 0.939x^2 - 7.237x + 30.785$, $r^2 = 0.2033$

b.



$[0, 9.4] \times [0, 30]$

x	1.5	3	4.5	6	6.6
$f'(x)$	-4.42	-1.60	1.21	4.03	5.16

For example, since $f'(1.5) \approx -4.42$, the slope of the tangent line to the curve at $x = 1.5$ is approximately -4.42. Thus, when the assets were \$1.5 million, the variable cost ratio was decreasing at 4.42 per \$1 million increase in assets. Notice that the rate of change of the variable cost ratio increased from negative to positive as assets increased.

Section 3.4

1. 4.03 **3.** 4.97 **5.** -0.05

7. $\ln 2 + 0.01 \approx 0.70$

9. 0.95 **11.** 0.51

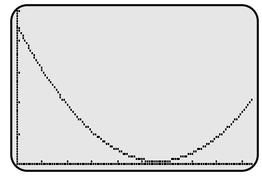
13. $f'(1) \approx \dfrac{1}{3}$ **15.** $f'(1) \approx 2.72$ **17.** $f'(1) \approx 1$

19. 1.0333 **21.** -0.272 **23.** -0.40

25. -1.6 **27.** 4 **29.** 2

31. -0.015 **33.** 288,000 **35.** 5.03

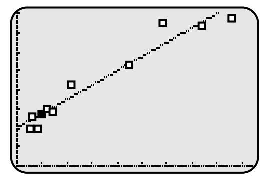
37. $C'(x) = 0.4490 - 0.03126x + 0.000555x^2$, $C'(10) \approx 0.19$, $C'(30) \approx 0.01$, $C'(40) \approx 0.09$. Marginal cost decreases and then increases.



$[0, 47] \times [0, 0.5]$

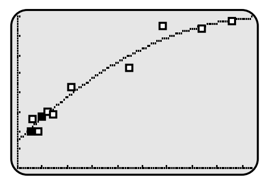
39. 0.02 **41.** Δ area $\approx 2\pi l \cdot \Delta r$

43. a. $y = 1.5x + 19.817$, $r = 0.9741$, $r^2 = 0.9488$



$[0, 47] \times [0, 80]$

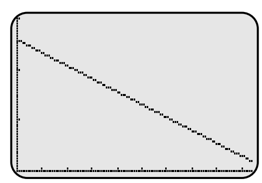
b. $y = -0.0254x^2 + 2.5787x + 14.292$, $r^2 = 0.9720$



$[0, 47] \times [0, 80]$

c. The quadratic with a larger square of the correlation coefficient.

d. marginal cost decreases



$[0, 47] \times [0, 3]$

Chapter 3 Review Exercises

1. $x = -1$: 1, 1, 1; $x = 0$: 2, 1, does not exist; $x = 1$: 2, does not exist, does not exist; $x = 2$: 1, 1, 1

2. $-1, 0, 1$ **3.** $-1, 0, 1, 2$ **4.** 12.7

5. π^3 **6.** 0 **7.** -3

8. -4 **9.** 1 **10.** -7

11. 3 **12.** 7 **13.** Does not exist

14. Does not exist **15.** 2 **16.** 3

17. 1.5 **18.** Does not exist **19.** -1

20. 0 **21.** 0.5 **22.** 2

23. 0 **24.** $4x$ **25.** $-2x$

26. $4x + 1$ **27.** $2x + 5$ **28.** $\dfrac{-2}{x^3}$

29. $\dfrac{-2}{(2x+5)^2}$ **30.** $\dfrac{1}{2\sqrt{x+7}}$ **31.** $\dfrac{1}{\sqrt{2x-1}}$

32. $y-9=12(x-2)$ **33.** $y-10=3(x-4)$

34. $y-6=11(x-1)$ **35.** $y-\dfrac{1}{3}=-\dfrac{1}{54}(x-9)$

36. $f'(1)\approx 0.20$ **37.** $f'(8)\approx 0.33$

38. $f'(2)\approx -0.19$ **39.** $f'(4)\approx 0.19$

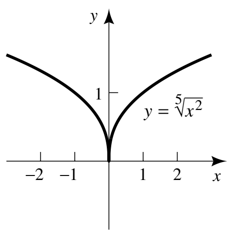
40. 1.01 **41.** 3.9

42. 0.11 **43.** 2.503

44. **a.** iii **b.** iv **c.** i **d.** ii

45. **a.** v **b.** i **c.** vi
d. iii **e.** iv **f.** ii

46.



From the graph, the tangent at $x=0$ appears vertical. Thus, $f'(0)$ does not exist.

h	-0.1	-0.01	-0.001	$\to$	0	$\leftarrow$	0.001	0.01	0.1
$\dfrac{f(0+h)-f(0)}{h}$	4.0	-15.9	-63.1	$\to$	No	$\leftarrow$	63.1	15.9	4.0

From the data in the table, $f'(0)$ does not appear to exist.

47. $$\lim_{x\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{x\to 0}\frac{\sqrt[5]{x^2}-0}{h}$$
$$=\lim_{x\to 0}h^{2/5-1}=\lim_{x\to 0}h^{-3/5}$$
does not exist, so $f'(0)$ does not exist.

48. Yes, since $\lim_{x\to 2}f(x)=f(2)$

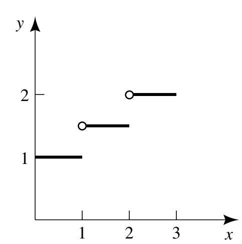
49. Since
$$\lim_{h\to 0^-}\frac{f(h)-f(0)}{h}=\lim_{h\to 0^-}\frac{0-0}{h}=0$$
and
$$\lim_{h\to 0^+}\frac{f(h)-f(0)}{h}=\lim_{h\to 0^+}\frac{h^3-0}{h}=\lim_{h\to 0^+}h^2=0,$$
then
$$\lim_{h\to 0^-}\frac{f(h)-f(0)}{h}=\lim_{h\to 0^+}\frac{f(h)-f(0)}{h}$$
$$=\lim_{h\to 0}\frac{f(h)-f(0)}{h}$$
$$=f'(0)=0$$

50. \$50

51. Does not exist. (i becomes large without bound.)

52. 0

53. Everywhere except the integer points of minute.



54. 200, 0, -200

55. -2, the distance s is decreasing at the rate of 2 ft/sec when $t=3$. 0, the distance s stops decreasing when $y=4$. 2, the distance s is increasing at the rate of 2 ft/sec when $t=5$.

56. $\Delta\approx 12.57$

57. About 22,580 per year. During this time there was on average an increase of about 22,580 Hispanic businesses each year.

58. About 0.05 pounds per year. During this time there was for each person on average an increase of about 0.05 pound of waste per year.

59. -80 farms per year. During this time there was a decrease of about 80 farms during each year.

60. \$0.0032 per hour worked during a year. Male workers who worked between 1000 and 3400 hours a year saw their average hourly wage increase on average by \$0.0032 for each additional hour they worked that year.

61. 0.7387% per hour. For each additional hour of light exposure, whole milk lost 0.7387% of its vitamin A.

62. −0.09% per day. For each additional day postpartum the percentage of protein in the milk dropped by 0.09.

63. 2.4722% per hour. For each additional hour of light exposure, nonfat milk lost 2.4722% of its vitamin A.

64. 6558 per mm. A female bass had 6558 more eggs for each millimeter increase in length.

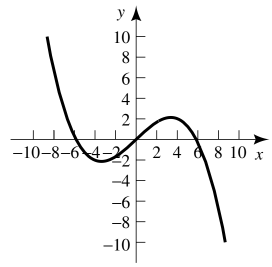
65. 0.042 per mm. For each mm increase in carapice length, the clutch size increased on average by 0.042.

66. 0.19 mm of length of prey for each millimeter length of bass. The length of the prey increased by 0.19 mm for each millimeter increase in the length of the bass.

67. About 0.00023 species per hectare. For each hectare increase in the size of a lake the number of species increased on average by 0.00023.

68. about 5 degrees per month, −4.5 degrees per month

69.



Chapter 4

Section 4.1

1. 0 **3.** 0 **5.** $23x^{22}$

7. $1.4x^{0.4}$ **9.** ex^{e-1} **11.** $\frac{4}{3}x^{1/3}$

13. $-5.1x^{-2.7}$ **15.** $0.12x^{-1/2}$ **17.** $2e^x$

19. $\pi + \frac{2}{x}$ **21.** $-2x - 1$ **23.** $\frac{2}{\pi}x + \pi$

25. $-3x^2 - 2x - 1 - \frac{10}{x}$ **27.** $5x^4 - 5e^x$

29. $0.7x - 0.3e^x$ **31.** $-\frac{1}{x} - 0.02x - 0.09x^2$

33. $\frac{1}{6}(e^x + 2x - 2)$ **35.** $9(x^2 - x^{-4})$

37. $2x + \frac{1}{x^2}$ **39.** $4x^{-1} + 9x^{-4} + 8x$

41. $-81t^2 - e^{t-3}$ **43.** $-\frac{1}{t^2} - \frac{2}{t^3} - \frac{3}{t^4}$

45. $\frac{1}{3}\left(\frac{1}{\sqrt[3]{t^2}} + \frac{1}{\sqrt[3]{t^4}}\right)$ **47.** $5u^{2/3} - 2u^{-1/3}$

49. $4u + 3$ **51.** $\frac{1}{2\sqrt{u}} - \frac{9}{2\sqrt{u^5}}$

53. $\frac{1}{\sqrt{x}} + \frac{1}{6\sqrt{x^3}}$ **55.** $\pi^2 x^{\pi-1} + \frac{2}{x}$

57. $\sqrt{2}(x^{\sqrt{2}-1} + x^{-\sqrt{2}-1})$

59. $-\frac{3}{2}x^{-5/2} - \frac{4}{5}x^{-9/5} + \frac{1}{2x}$

61. $\frac{2}{3}x^{-1/3}$ **63.** $-\frac{2}{5}x^{-7/5}$

65. $y = x + 2$ **67.** $x = \pm\sqrt{3}$

69. 10 **71.** 83

73. $B'(T) = -0.28 + 0.02T$ degree body temperature per degree ambient temperature. $B'(25) = 0.22$. At an ambient temperature of 25 degrees, the body temperature is increasing at 0.22 degree for each degree of ambient temperature.

75. $Y'(N) = 0.824 - 0.00638N$ bushels of soybeans per pound. $Y'(150) = -0.133$. When 150 pounds of fertilizer per acre is used, the yield of soybeans is decreasing at 0.133 bushel per acre for each additional pound of fertilizer per acre. Clearly, too much fertilizer is counterproductive.

77. $G'(T) = -7.1 - 2.2T$ days per degree. $G'(-1) = -4.9$. When the mean July temperature is -1^0C, the beginning of the growing season is 4.9 days earlier for each additional degree Celsius increase in the mean July temperature.

79. $0.053378x + 21.7397$, 63.25, $-0.053378x + 41.5103$

81. $g'(r) = -0.066r + 0.024r^2$ and is the rate of change of the growth rate with respect to the real interest rate; $g'(-2) \approx 0.228$ and means that when the real interest rate is −2%, growth is increasing at the rate 0.228% per each percentage increase in real interest rates; $g'(2) \approx -0.036$ and means that when the real interest rate is 2%, growth is decreasing at the rate 0.036% per each percentage increase in real interest rates;

83. $5.62L$ **85.** $0.24(L + 0.0039L^2)$

87. $\frac{292.6T^{0.75}}{(273)^{1.75}P} \approx \frac{0.016}{P}T^{0.75}$

89. **a.** −1.36 and 0.92.
b. −1.36 and 0.92.
c. Solutions in part (b) are equal to values in part (a).
d. Exact solutions cannot be found.

91. L_1: $y = -4x - 3$, L_2: $y = 4x - 3$. Solve $4x - 3 = -4x - 3$ and obtain $x = 0$.

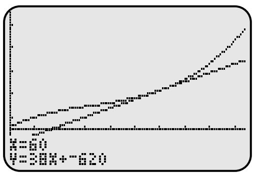
93. No. For example,

$$f(x) = \begin{cases} 1 & x \le 0 \\ 0 & x > 0 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 0 & x \le 0 \\ 1 & x > 0 \end{cases}$$

Notice that $f(x) + g(x) = 1$

95. $3x^2 - 4\sin x - 3\cos x$

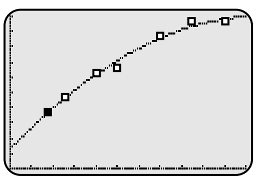
97. **a.** $0.03x^2 - 2x + 50$;
b. 22, the cost of the 21st item is approximately \$22; 18, the cost of the 41st item is approximately \$18; 38, the cost of the 41st item is approximately \$38.
c.



$[0, 94] \times [-1500, 5000]$

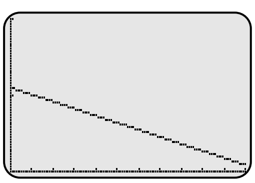
Marginal cost first decreases, and then at some point, begins to increase.

99. **a.** $y = -0.0023x^2 + 1.1x + 19.437$, $r^2 = 0.9893$



$[0, 220] \times [0, 150]$

b.



$[0, 220] \times [0, 2]$

Marginal cost decreases.

Exercise 4.2

1. $3x^2e^x + x^3e^x$

3. $\dfrac{\ln x + 2}{2\sqrt{x}}$

5. $e^x(x^2 + 2x - 3)$

7. $(4e^x - 5x^4)\ln x + (4e^x - x^5)/x$

9. $e^x(\sqrt{x} + 1) + \dfrac{(e^x + 1)}{2\sqrt{x}}$

11. $(2x - 3\ln x)(1 - x^{-2}) + \left(2 - \dfrac{3}{x}\right)(x + x^{-1})$

13. $\dfrac{1/x - \ln x}{e^x}$

15. $-\dfrac{1}{x(\ln x)^2}$

17. $\dfrac{2}{(x+2)^2}$

19. $\dfrac{8}{(x+5)^2}$

21. $\dfrac{e^x(x-3)}{(x-2)^2}$

23. $\dfrac{-1}{(2x-1)^2}$

25. $\dfrac{3e^u(u-1)^2}{(u^2+1)^2}$

27. $\dfrac{-u^2 - 2u + 2}{(u^2+2)^2}$

29. $\dfrac{u^4 + 9u^2 + 2u}{(u^2+3)^2}$

31. $\dfrac{-8u^3 + (3u-1)e^u - 1}{3\sqrt[3]{u^2}(u^3 - e^u - 1)^2}$

33. $\dfrac{1 - x^2}{(1+x^2)^2}$

35. $\dfrac{1}{10(10 - 0.1x)^2}$

37. $p'(x) = -145499(138570 + x)^{-2} < 0$. This indicates that the demand curve slopes down, as it should.

39. $\dfrac{-6t^2 + 15}{(2t^2+5)^2}$

41. $\dfrac{2(I^2 + 100{,}000I)}{5(I + 50{,}000)^2}$

43. $\dfrac{7.12(2x - 13.74)}{(13.74x - x^2)^2}$

45. **a.** $2e^{2x}$; **b.** $3e^{3x}$; **c.** ne^{nx}

47. $f'(x) = (x-a)[2g(x) + (x-a)g'(x)]$ implies $f'(a) = 0$. The equation of the tangent line to $y = f(x)$ at $x = a$ is $y - f(a) = f'(a)(x-a)$ or $y = 0$, which is the x-axis.

49. No. Letting $f(x) = |x|$ and $g(x) = x$ and $h(x) = f(x) \cdot g(x)$, we recall that $f'(0)$ does not exist. So the product rule cannot be used.

51. $\sin x + x\cos x$

53. $e^x(\cos x - \sin x)$

55. $\dfrac{x\cos x - \sin x}{x^2}$

57. $1 + \tan^2 x$

59. **a.** $2.05x^2 - 36.51x + 415.14$. $r^2 = 0.9996$.
b. -11.91 dollars per ton. When 6000 tons are produced, the average total cost is dropping by a rate of \$11.91 per ton.
c. $C'(x) = 1000[q(x) + x \cdot q'(x)]$. $C'(6) = 198,600$ is the rate of change of total cost when 6000 tons are produced.
d. $Q(x) = 79.365x^2 + 201,214x + 408771.1$. $Q'(6) = 202,167$.

61. **a.** $428.29x^{-0.25536}$. The square of the correlation coefficient is 0.9948.
b. -11.53 dollars per ton. When 6000 tons are produced, the average total cost is dropping by a rate of \$11.53 per ton.
c. $C'(x) = 1000[q(x) + x \cdot q'(x)]$. $C'(6) = 201,860$ is the rate of change of total cost when 6000 tons are produced.
d. $Q(x) = 428,290x^{0.744639}$. $Q'(6) = 201,825$.

Section 4.3

1. $14(2x+1)^6$

3. $8x(x^2+1)^3$

5. $3e^x\sqrt{2e^x - 3}$

7. $\dfrac{1}{2}\sqrt{e^x}$

9. $\dfrac{1}{\sqrt{2x+1}}$

11. $4x(x^2+1)^{-2/3}$

13. $-\dfrac{e^x}{(e^x+1)^2}$

15. $\dfrac{6x^2}{(x^3+1)^2}$

17. $\dfrac{x^2(1-\ln x)+1}{x(x^2+1)^{3/2}}$ **19.** $(3x-2)^4(36x+11)$

21. $\dfrac{4(\ln x)^3}{x}$ **23.** $e^x(x^2+1)^7(x^2+16x+1)$

25. $(1-x)^3(2x-1)^4(14-18x)$

27. $\dfrac{(4x+3)^2}{2\sqrt{x}}(28x+3)$ **29.** $e^x \ln x\left(\ln x+\dfrac{2}{x}\right)$

31. $-12(x+1)^{-5}$ **33.** $\dfrac{2xe^x-3e^x-6}{(x+3)^4}$

35. $\dfrac{1}{4\sqrt{x(\sqrt{x}+1)}}$ **37.** $-x(1+x^2)^{-3/2}$

39. $g(x)>0$ for $0<x<3.2$. $g(x)=0$ if $x=3.2$. The biomass will be greater next year if the current biomass is less than 3.2 million tons and will be less if the current biomass is greater than 3.2 million tons. Apparently, too many fish result in overuse of the finite resources, causing the total mass to decrease the next year.

$$g'(x)=0.30355\left(1-\frac{x}{3.2}\right)^{-0.6135}\left[1-\frac{1.35865}{3.2}x\right]$$

$g'(x)>0$ when $0<x<\frac{3.2}{1.35865}\approx 2.355$ and $g'(x)<0$ when $x>2.355$. The rate of increase of the additional mass of fish next year increases until $x=2.355$ and then begins to decrease.

41. $\dfrac{4\pi r^2 c}{3n^{2/3}}\left(1+\dfrac{c}{r}n^{1/3}\right)^2$ **43.** $\dfrac{a(3n-2b)}{2\sqrt{n-b}}$

45. $\dfrac{1}{8\sqrt{\sqrt{\sqrt{x}+1}+1}\cdot\sqrt{\sqrt{x}+1}\cdot\sqrt{x}}$

47. Write as a product $(e^x+1)(x+3)^{-3}$

49. $3\sin^2 x\cos x$ **51.** $-\dfrac{\sin x}{2\sqrt{\cos x}}$

53. $r'(t)=\dfrac{0.0001}{\sqrt{36-t}}\left[-1008+186t-5t^2\right]$

Section 4.4

1. $4e^{4x}$ **3.** $-3e^{-3x}$

5. $4xe^{2x^2+1}$ **7.** $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

9. $2xe^x+x^2e^x$ **11.** $e^x x^4(5+x)$

13. $(1-x)e^{-x}$ **15.** $2xe^{x^2}(1+x^2)$

17. $\dfrac{e^x}{2\sqrt{e^x+1}}$ **19.** $\dfrac{1}{2\sqrt{x}}e^{\sqrt{x}}+\dfrac{1}{2}e^{\sqrt{x}}$

21. $\dfrac{1+e^x-xe^x}{(1+e^x)^2}$ **23.** $\dfrac{8}{(e^{2x}+e^{-2x})^2}$

25. $\dfrac{3e^{3x}}{2\sqrt{e^{3x}+2}}$ **27.** $5^x\ln 5$

29. $\dfrac{3^{\sqrt{x}}\ln 3}{2\sqrt{x}}$ **31.** $3^x+x3^x\ln 3$

33. $\dfrac{3x^2+2x}{x^3+x^2+1}$ **35.** $4x\ln|x|+2x$

37. $\dfrac{2x+1}{2(x^2+x+1)\sqrt{\ln|x^2+x+1|}}$

39. $\dfrac{1}{x(x+1)}$ **41.** $-4xe^{-x^2}\ln|x|+\dfrac{2}{x}e^{-x^2}$

43. $2x\ln x+x$ **45.** $\dfrac{1}{x(x^2+1)}-\dfrac{2x\ln x}{(x^2+1)^2}$

47. $\dfrac{11}{x}(\ln x)^{10}$ **49.** $\dfrac{3}{x}$

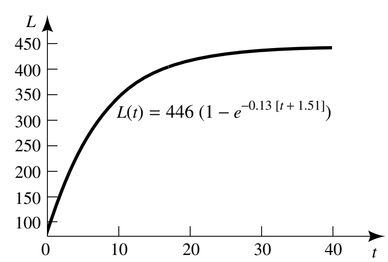
51. $e^{-3x}(1-3x),\ \dfrac{1}{3}$ **53.** $-2xe^{-x^2},\ 0$

55. $xe^x(2+x),\ 0,\ -2$ **57.** $xe^{-x}(2-x),\ 0,\ 2$

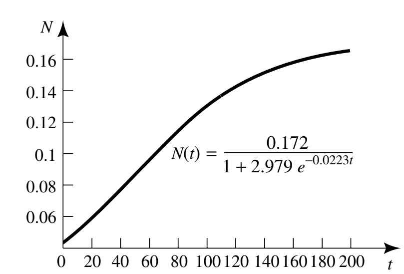
59. $\ln x^2+2,\ \dfrac{1}{e}$ **61.** $\dfrac{1-2\ln x}{x^3},\ \sqrt{e}$

63. $e^{-2x}(1-2x),\ \dfrac{1}{2}$ **65.** $\dfrac{2x}{x^2+5},\ (0,\infty)$

67. $L'(t)=(0.13)(446)e^{-0.13[t+1.51]}>0$. The fact that $L'(t)>0$ just says that the fish get bigger with age.



69. $N'(t)=(0.172)(2.979)(0.0223)(1+2.979e^{-0.0223t})^{-2}$ $e^{-0.0223t}>0$. The fact that $N'(t)>0$ indicates that the nitrogen is increasing in these fields. Indeed, that is why fields are rested. As t becomes large, we see from the graph that the tangent becomes small. Therefore, $N'(t)$ must become small. This indicates that the nitrogen restoration rate slows in time.



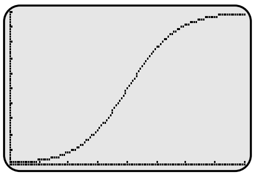
71. $\dfrac{3e^{3x}+2x}{(e^{3x}+x^2)\cdot\ln 2}$

73. $e^{e^{e^x}}\cdot e^{e^x}\cdot e^x$

75. **a.** 500;
b. $1488e^{-0.024t}(1+124e^{-0.024t})^{-2}$;

c. $p'(t) > 0$;

d.



[0, 400] × [0, 500]

The graph increases and is asymptotic to $y = 500$;

e. The two graphs are about the same for the first 50 years.

77. Let $u = f(x)$; then

$$\begin{aligned}\frac{d}{dx}(\sin f(x)) &= \frac{d}{dx}(\sin u)\\ &= \frac{d}{du}(\sin u)\cdot\frac{du}{dx}\\ &= \cos u \cdot \frac{du}{dx}\\ &= (\cos f(x))\, f'(x)\\ \frac{d}{dx}(\cos f(x)) &= \frac{d}{dx}(\cos u)\\ &= \frac{d}{du}(\cos u)\cdot\frac{du}{dx}\\ &= -\sin u \cdot \frac{du}{dx}\\ &= -(\sin f(x))\, f'(x)\end{aligned}$$

79. $-3x^2 \sin x^3$ **81.** $\dfrac{\cos(\ln x)}{x}$

83. $-\sin(\sin x)\cos x$

85. $g'(x) = re^{-bx}[1 - bx]$. $g'(x) > 0$ when $0 < x < 1/b$. $g'(x) < 0$ when $x > 1/b$. This indicates that when the population is small, it will increase next year. However, when the population is large, it will be less next year. In this case the population is so large that the finite resources available cannot hold the large population, and this population must decrease.

87.

$$\begin{aligned}\frac{d}{dt}W(L(t)) &= \frac{dW}{dL}\cdot\frac{dL}{dt}\\ &\approx 3839(1 - e^{-0.12[t+0.45]})^{2.31} e^{-0.12[t+0.45]}\end{aligned}$$

89. $P'(D) = (0.06D - 0.4)10^{0.03D^2 - 0.4D + 1} \ln 10$.

91.

$$\begin{aligned}f(t) &= (1-t)^t\\ \ln f(t) &= \ln(1-t)^t\\ &= t \cdot \ln(1-t)\\ \frac{f'(t)}{f(t)} &= \ln(1-t) - \frac{t}{1-t}\end{aligned}$$

$$\begin{aligned}f'(t) &= f(t)\left[\ln(1-t) - \frac{t}{1-t}\right]\\ &= (1-t)^t\left[\ln(1-t) - \frac{t}{1-t}\right]\end{aligned}$$

93. $e(t) = 3927.29(1.0330)^t$. Square of correlation coefficient is 0.9998. $e'(3.5) \approx 143$. This means that half way into the year 1793, the United States was growing at a rate of 143,000 per year.

Section 4.5

1. The demand drops by 7.7%

3. The demand drops by 5.0%

5. The demand drops by 3.1%

7. The demand drops by 13.2%

9. The demand drops by 5.6%

11. 0.5

13. **a.** $\frac{1}{4}$ inelastic; **b.** 1 unit; **c.** 9 elastic

15. **a.** $\frac{1}{7}$ inelastic; **b.** 1 unit; **c.** 3 elastic

17. **a.** $\frac{1}{3}$ inelastic; **b.** $\frac{1}{2}$ inelastic; **c.** $\frac{10}{11}$ inelastic

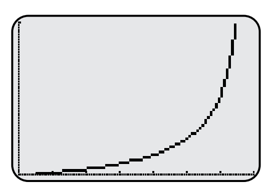
19. **a.** $\frac{2}{1.001}$ elastic; **b.** $\frac{2}{11}$ inelastic

21. **a.** 2 elastic; **b.** 2 elastic;

23. **a.** 15; **b.** 15

25. **a.** $\dfrac{1800p}{126.5 - 1800p}$

b. 0.74 inelastic, 2.47 elastic



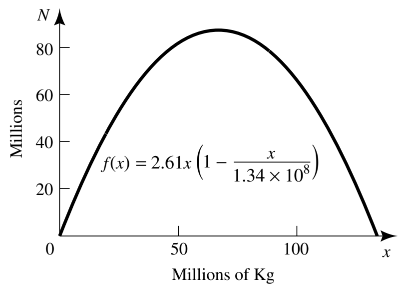
[0, 0.07] × [0, 10]

27. 2.0711, for every one percentage increase in the price of Grape Nuts, the demand decreases by 2.0711%.

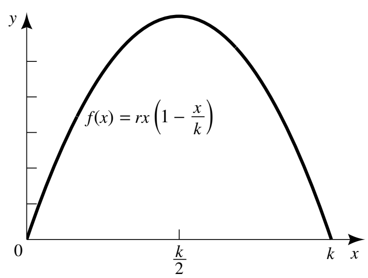
29. $E = -\frac{a}{x}\frac{dx}{da}$, 0.0229, for every one percent increase in the amount of coupons the demand for Shredded Wheat increases by 0.0229%.

Section 4.6

1. 1.34×10^8 kilograms. 0.67×10^8



3. $f(x) = 0$ implies $x = 0$ or $x = K$.
$f'(x) = r\left(1 - \dfrac{2x}{K}\right) = 0$ implies $x = \dfrac{K}{2}$.



5. 16,000 pounds, 6,912,000 pounds

7. msy = 243 million pounds, mvp ≈ 2.09 million pounds, cc ≈ 12.91 million pounds

17. $\dfrac{x^4 + 3x^2}{(x^2+1)^2}$

18. $\dfrac{3x^2 - 4x - 5}{(3x-2)^2}$

19. $\dfrac{-2x^5 + 4x^3 + 2x}{(x^4+1)^2}$

20. $30(3x+1)^9$

21. $40x(x^2+4)^{19}$

22. $\dfrac{5(\sqrt{x}-5)^{3/2}}{4\sqrt{x}}$

23. $\dfrac{2x+1}{2\sqrt{x^2+x+1}}$

24. $x^2(x^2+x+2)^4(13x^2+8x+6)$

25. $(x^4+x+1)^4(22x^5+60x^3+7x^2+2x+15)$

26. $\dfrac{-14x^5 - 16x^3 - 2x^2 + 2x - 4}{(x^4+x+1)^5}$

27. $\dfrac{3\sqrt{x}}{4\sqrt{x^{3/2}+1}}$

28. $\dfrac{1}{6\sqrt{x}\sqrt[3]{(\sqrt{x}+1)^2}}$

29. $-7e^{-7x}$

30. $6x^2e^{2x^3+1}$

31. $e^x(x^2+3x+3)$

32. $\dfrac{2x}{x^2+1}$

33. $2xe^{x^2}\ln x + \dfrac{1}{x}e^{x^2}$

34. $\dfrac{2x+e^x}{x^2+e^x}$

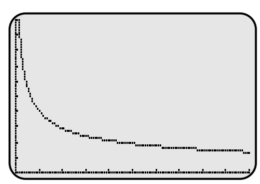
35. $\dfrac{1}{5}x^{-4/5}$

36. $\dfrac{2}{3}x^{-1/3}$

37. $-3x^{-4}$

38. $\dfrac{x-1}{2x^{3/2}}$

39.

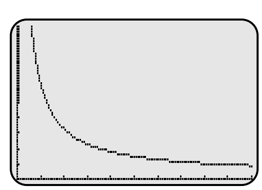


[0, 1000] × [0, 200]

x	100	200	300	400
$h'(x)$	−0.38	−0.14	−0.08	−0.05

$h'(100) \approx -0.38$ means that when the 100th camera was produced, direct labor hours were decreasing at the rate of 0.38 hour per camera. The rate is decreasing less, indicating that fewer additional hours are needed to make each additional camera.

40. a.



[0, 10] × [0, 1]

n	2	4	6	8
$C'(n)$	−0.1504	−0.0376	−0.0167	−0.0094

Chapter 4 Review Exercises

1. $6x^5 - 6x$

2. $\dfrac{24}{5}x^{1/5} + 2x^{-3/5}$

3. $-6x^{-4} - 2x^{-5/3}$

4. $2.6x^{0.3} - 7.2x^{-3.4}$

5. $2\pi^2x + 2$

6. $3\pi^3x^2 + 2e^2x$

7. $\dfrac{2}{3}x^{-1/3}$

8. $-\dfrac{5}{3}x^{-8/3}$

9. $(x^3+x+1) + (x+3)(3x^2+1)$

10. $2x(x^5-3x+2) + (x^2+5)(5x^4-3)$

11. $\dfrac{x^4-2x^2+3}{2\sqrt{x}} + \sqrt{x}(4x^3-4x)$

12. $\dfrac{x^3-3x+5}{3\sqrt[3]{x^2}} + \sqrt[3]{x}(3x^2-3)$

13. $\dfrac{3}{2}x^{1/2}(x^2+4x+7) + (x^{3/2})(2x+4)$

14. $-\dfrac{7}{2}x^{-9/2}(x^3+2x+1) + (x^{-7/2})(3x^2+2)$

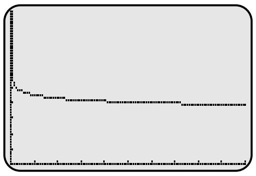
15. $\dfrac{2}{(x+1)^2}$

16. $\dfrac{1-2x}{2\sqrt{x}(2x+1)^2}$

$C'(2) \approx -0.1504$ means that when the number of hours worked per day was 2, the unit cost was decreasing at a rate of \$0.15 for each additional hour per day worked. The data indicates that the more hours worked per day, the less the unit costs, that is, the greater the efficiency.

b. 1.05 hours

41.

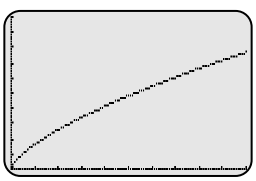


$[0, 1000] \times [0, 2]$

Q	100	200	400	800
$C'(Q)$	-0.00064	-0.00030	-0.00014	-0.00007

$C'(100) \approx -0.00064$ means that when 100,000 kg of milk was processed, the average cost was decreasing at the rate of \$0.00064 per 1000 kg increase in production. Average cost decreases less as the size of the plant increases, indicating economies of scale.

42.



$[0, 1000] \times [0, 2]$

D	100	200	400	800
$R'(D)$	0.0022	0.0018	0.0014	0.0011

$R'(100) \approx 0.0022$ means that when the distance was 100 miles, the rail transportation cost per ton was increasing at a rate of \$0.0022 per 1 mile increase in distance. This rate becomes less as the distance increases, indicating economies of scale.

43. **a.** 606.11, at $x = 2$ items sold, one more item sold results in a \$606.11 profit;
b. 329.14, at $x = 3$ items sold, one more item sold results in a \$329.14 profit;
c. -225.98, at $x = 4$ items sold, one more item sold results in a \$225.98 loss;
d. -1434.13, at $x = 5$ items sold, one more item sold results in a \$1434.13 loss;

44. 56, at $t = 1$, the particle is moving at 56 ft/sec in the positive direction; 0, the particle is at rest at the instant $t = 2$; -152, at $t = 3$, the particle is moving at 152 ft/sec in the negative direction.

45. $3.2572t^{1.395}$, the instantaneous rate of change of the number of wheat aphids at time t is given by $3.2572t^{1.395}$.

46. $7.08x^2 - 41.66x + 63.77$, at the age of x, the instantaneous rate of change of the number of the eggs is given by $7.08x^2 - 41.66x + 63.77$. Since the quadratic formula indicates that $f'(x)$ is never zero and since $f''(0) > 0$, $f'(x) > 0$ for all x. This means that *Ostrinia furnacalis* lays more eggs on older plants.

47. $f'(x) = -8.094 \ln(1.053) \cdot (1.053)^{-x} \approx -0.418(1.053)^{-x}$. $f'(x) < 0$ for all x means that more immature sweetpotato whiteflies will cause smaller tomato leaves.

48. 25

49. **a.** $\frac{2}{3}$ inelastic; **b.** 1 unit; **c.** $\frac{3}{2}$ elastic

50. $-\dfrac{900}{(3x+1)^2}$

51. For smaller outputs, average cost decreases as output increases (there are economies of scale), but for larger outputs, average costs increase as output increases.

52. For smaller percentages of industry capacity, average cost decreases (there are economies of scale) and then levels off as the percent of industry capacity increases. But for large percentages of industry capacity, average cost increases for increases of percentage of industry capacity.

53. For each 1% increase in household disposable income, dollars spent overseas increase by 0.7407%.

54. 2.174

55. The first, since $\dfrac{dx}{dp}$ is the larger in absolute value and $\dfrac{p_0}{x_0}$ is the same for both.

56. $E = n$

Chapter 5

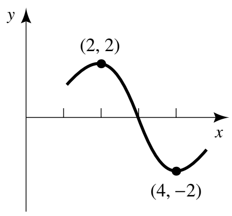
Section 5.1

1. Relative maximum on about May 30 and June 12; relative minimum on about June 5.

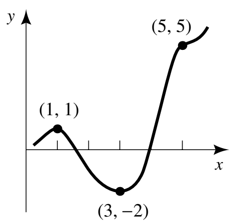
3. Relative maximum at about $x = 1300$, $x = 1850$, and $x = 1970$; relative minimum at about $x = 1400$ and $x = 1950$.

5. Relative maximum at about $x = 2500$ and $x = 3300$; relative minimum at about $x = 2800$.

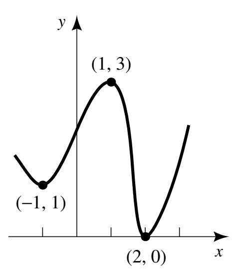
7. Critical values are $x = 2$ and $x = 4$
increasing on $(-\infty, 2)$, $(4, \infty)$
decreasing on $(2, 4)$
relative maximum at $x = 2$
relative minimum at $x = 4$



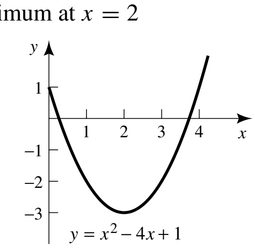
9. Critical values are $x = 1, 3, 5$
increasing on $(-\infty, 1)$, $(3, \infty)$
decreasing on $(1, 3)$.
relative maximum at $x = 1$
relative minimum at $x = 3$



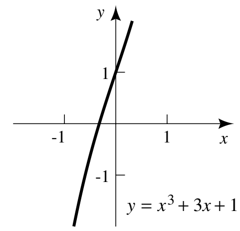
11. Critical values are $x = -1, 1, 2$
increasing on $(-1, 1)$, $(2, \infty)$
decreasing on $(-\infty, -1)$, $(1, 2)$.
relative maximum at $x = 1$
relative minima at $x = -1$ and $x = 2$



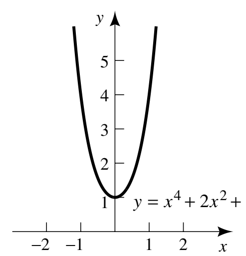
13. Critical value at $x = 2$
increasing on $(2, \infty)$
decreasing on $(-\infty, 2)$
no relative maxima
relative minimum at $x = 2$



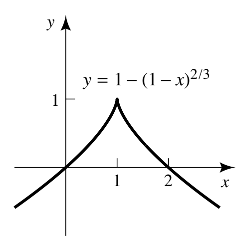
15. No critical values
increasing on $(-\infty, \infty)$
never decreasing
no relative maximum
no relative minimum



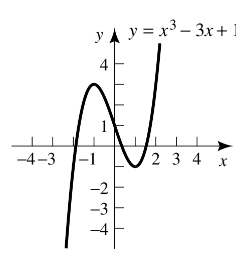
17. Critical value at $x = 0$
increasing on $(0, \infty)$
decreasing on $(-\infty, 0)$
no relative maximum
relative minimum at $x = 0$



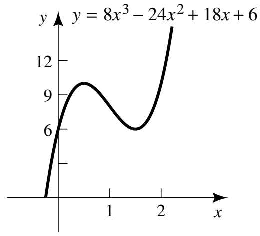
19. Critical value at $x = 1$
increasing on $(-\infty, 1)$
decreasing on $(1, \infty)$
relative maximum at $x = 1$
no relative minima



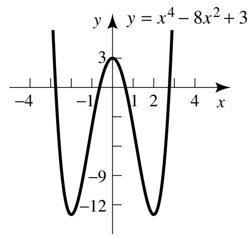
21. Critical values at $x = -1$ and $x = 1$
increasing on $(-\infty, -1)$, $(1, \infty)$
decreasing on $(-1, 1)$
relative maximum at $x = -1$
relative minimum at $x = 1$



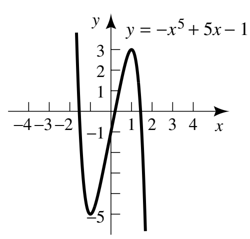
23. Critical values at $x = 1/2$ and $x = 3/2$
increasing on $(-\infty, 1/2)$, $(3/2, \infty)$
decreasing on $(1/2, 3/2)$
relative maximum at $x = 1/2$
relative minimum at $x = 3/2$



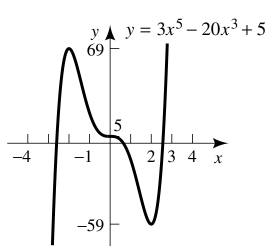
25. Critical values at $x = -2, 0, 2$
increasing on $(-2, 0)$, $(2, \infty)$
decreasing on $(-\infty, -2)$, $(0, 2)$
relative maximum at $x = 0$
relative minima at $x = -2, 2$



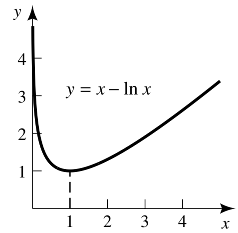
27. Critical values at $x = \pm 1$
increasing on $(-1, 1)$
decreasing on $(-\infty, -1)$, $(1, \infty)$
relative maximum at $x = 1$
relative minimum at $x = -1$



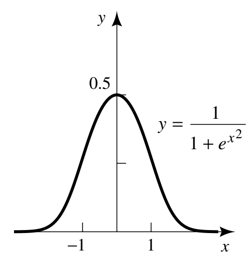
29. Critical values at $x = 0, \pm 2$
increasing on $(-\infty, -2)$, $(2, \infty)$
decreasing on $(-2, 2)$
relative maximum at $x = -2$
relative minimum at $x = 2$



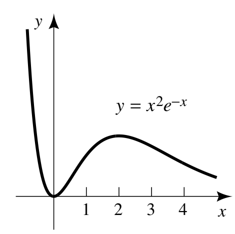
31. Critical value at $x = 1$
increasing on $(1, \infty)$
decreasing on $(0, 1)$
no relative maxima
relative minimum at $x = 1$



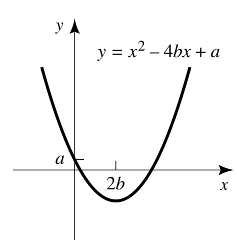
33. Critical value at $x = 0$
increasing on $(-\infty, 0)$
decreasing on $(0, \infty)$
relative maximum at $x = 0$
no relative minima



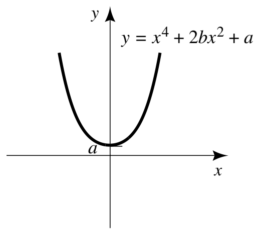
35. Critical values at $x = 0, 2$
increasing on $(0, 2)$
decreasing on $(-\infty, 0)$, $(2, \infty)$
relative maximum at $x = 2$
relative minimum at $x = 0$



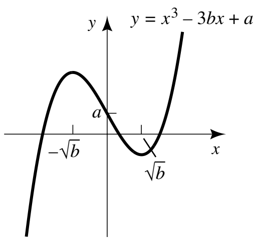
37. Critical value at $x = 2b$
increasing on $(2b, \infty)$
decreasing on $(-\infty, 2b)$
no relative maxima
relative minimum at $x = 2b$



39. Critical value at $x = 0$
increasing on $(0, \infty)$
decreasing on $(-\infty, 0)$
no relative maxima
relative minimum at $x = 0$



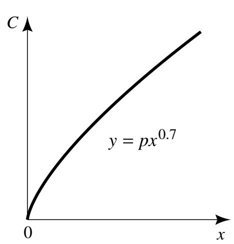
41. Critical values at $x = \pm\sqrt{b}$
increasing on $(-\infty, -\sqrt{b})$, $(\sqrt{b}, \infty)$
decreasing on $(-\sqrt{b}, \sqrt{b})$
relative maxima at $x = -\sqrt{b}$
relative minimum at $x = \sqrt{b}$



43. $f'(x) = 3ax^2 + b$ is positive if both $a > 0$ and $b > 0$ are positive and negative if both $a < 0$ and $b < 0$ are negative.

49. 500 **51.** $x = 20$ **53.** $x = 3$

55. $(0, \infty)$

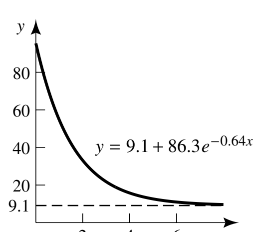


57. $x = 10.309052$ **59.** 33.95

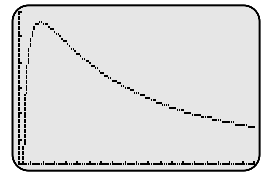
61. 1.044 **63.** 30. 24,400

65. $I/2$

67. $f'(x) = -55.232e^{-0.64x} < 0$ for all real x, $y \to 9.1$

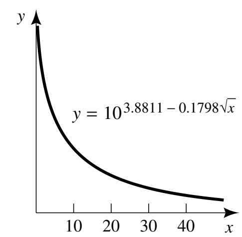


69. $x = 36.1$

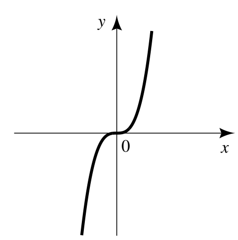


$[0, 400] \times [0, 60]$

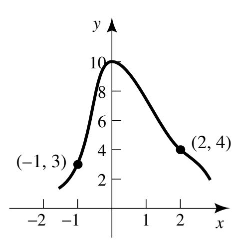
71. $f'(x) = \dfrac{-0.0899 \ln 10 \cdot 10^{3.8811 - 0.1798\sqrt{x}}}{\sqrt{x}} < 0$ for all $x > 0$, $y \to 0$



73. Neither. Consider the interval $(0, 10)$. If $f'(x) \le 0$ on $(0, 10)$, then f is decreasing there and $f(10) \le f(0) = 0$. But, $f(10) = 100$. From this contradiction, we conclude that f' is positive somewhere on $(0, 10)$. If f' were negative somewhere on $(0, \infty)$, then f' would change sign, indicating a relative extrema. Since this is not the case, we must have $f'(x) \ge 0$ on $(0, \infty)$. A similar argument shows that $f'(x) \ge 0$ on $(-\infty, 0)$. So we have that $f'(x) \ge 0$ on $(-\infty, \infty)$. Therefore, $f(0)$ cannot be a relative extrema.



75. Relative maximum



77. Let $f(x) = x^{13} - x^{12} - 10$. Then,

$$\begin{aligned} f'(x) &= 13x^{12} - 12x^{11} \\ &= x^{11}(13x - 12) \end{aligned}$$

Thus, there are critical points at $x = 0$ and $x = 12/13$. Notice that $f'(x) > 0$ and f increasing on $(-\infty, 0)$ and $(12/13, \infty)$ and that $f'(x) < 0$ and f decreasing on $(0, 12/13)$. Note $f(0) = -10$.

We conclude that $f(x) < 0$ on $(-\infty, 12/13]$ and increasing on $[12/13, \infty)$. Therefore, the graph crosses the x-axis exactly once.

79. Let $g(x) = e^x - (1 + x)$, then $g'(x) = e^x - 1$. Then $g'(x) < 0$ on $(-\infty, 0)$. So $g(x) > g(0) = 0$ for any $x < 0$. That is, $g(x) > 0$ on $(-\infty, 0)$. Since $e^x > 1$ on $(0, \infty)$, $g'(x) > 0$ on $(0, \infty)$. So $g(x) > g(0) = 0$ for any $x > 0$. That is, $g(x) > 0$ on $(0, \infty)$. Therefore, $g(x) \geq 0$ on $(-\infty, \infty)$, that is, $e^x \geq 1 + x$ for all x.

81. $h'(x) = f'(x)g(x) + f(x)g'(x) < 0$ since $f' > 0$, $g < 0$, $f < 0$, and $g' > 0$

83. $b^2 < 3c$

85. **a.** $f(x)$ has a relative minimum at $x = 1$, since f decreases to the left of 1 ($f'(x) < 0$ there) and increases to the right of 1 ($f'(x) > 0$ there).

b. $g(x)$ has a relative maximum at $x = 1$, since g increases to the left of 1 ($g'(x) > 0$ there) and decreases to the right of 1 ($g'(x) < 0$ there).

87. Positive

89. Negative

91. Let

$$f(m) = 1 - \left(\frac{m}{b}\right)^{x-1} - \left(1 - \frac{k}{m}\right)(x - 1)$$

$$f'(m) = -(x-1)\left(\frac{m}{b}\right)^{x-2} - (x-1)\left(\frac{k}{m^2}\right)$$

$$= -(x-1)\left[\left(\frac{m}{b}\right)^{x-2} + \frac{k}{m^2}\right] < 0$$

since $x > 1$. Thus, $f(m)$ is a decreasing function. We have

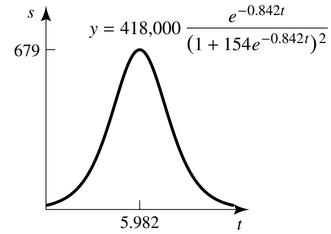
$$f(k) = 1 - \left(\frac{k}{b}\right)^{x-1} > 0$$

$$\text{and} \quad f(b) = -\left(1 - \frac{k}{b}\right)(x - 1) < 0$$

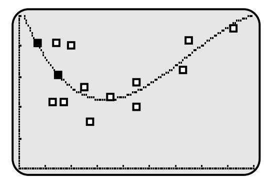
since $\frac{k}{b} < 1$ and $x - 1 > 0$. Therefore, by the constant sign theorem, $f(m)$ has at least one root in (k, b). Since f is also monotonic, f has exactly one root in (k, b), that is, the equation has a unique solution for m in (k, b).

93. 0.32

95. 5.982



97. $y = -0.00000217x^3 + 0.000425x^2 - 0.021x + 1.034$, $r^2 = 0.5356$.
Decreasing on $(0, 33.218567)$.
Increasing on $(33.218567, 90)$.



$[0, 90] \times [0, 1]$

99. $-1.10964x^2 + 44.71013x - 361.72713$. Approximately 20.146.

Section 5.2

1. $80x^3 + 2$

3. $2(x + 1)^{-3}$

5. $-(2x + 1)^{-3/2}$

7. $240x^2, 480x$

9. $-6x^{-4}, 24x^{-5}$

11. $\dfrac{8}{27}x^{-7/3}, -\dfrac{56}{81}x^{-10/3}$

13. No inflection value; concave down on $(0, 4)$

15. Inflection value at $x = 2$; concave up on $(2, 4)$; concave down on $(0, 2)$

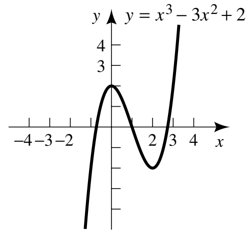
17. Inflection value at $x = 2$; concave up on $(2, 4)$; concave down on $(0, 2)$

19. No inflection values; concave up on $(2, 4)$; concave down on $(0, 2)$;

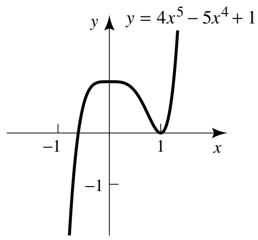
21. 1, 3, 5. Concave up on $(1, 3)$ and $(5, \infty)$, concave down on $(-\infty, 1)$ and $(3, 5)$

23. -2. Concave up on $(-2, \infty)$, concave down on $(-\infty, -2)$

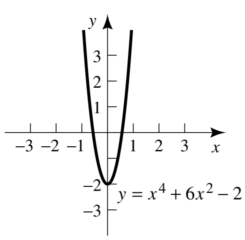
25. Critical values at $x = 0, 2$
relative maximum at $x = 0$
relative minimum at $x = 2$
inflection value at $x = 1$
increasing on $(-\infty, 0)$ and $(2, \infty)$
decreasing on $(0, 2)$
concave up on $(1, \infty)$
concave down on $(-\infty, 1)$



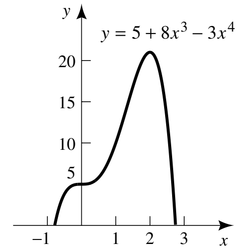
27. Critical values at $x = 0, 1$
relative maximum at $x = 0$
a relative minimum at $x = 1$
inflection value at $x = 0.75$
increasing on $(-\infty, 0)$ and $(1, \infty)$
decreasing on $(0, 1)$
concave up on $(0.75, \infty)$
concave down on $(-\infty, 0.75)$



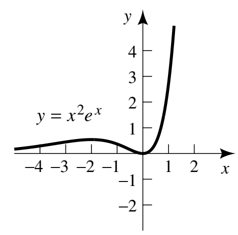
29. Critical value at $x = 0$
relative minimum at $x = 0$
no inflection values
increasing on $(0, \infty)$
decreasing on $(-\infty, 0)$
concave up on $(-\infty, \infty)$



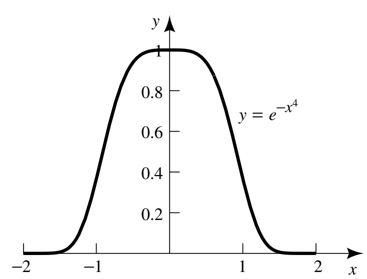
31. Critical values at $x = 0, 2$
relative maximum at $x = 2$
inflection values at $x = 0, \dfrac{4}{3}$
increasing on $(-\infty, 2)$
decreasing on $(2, \infty)$
concave up on $(0, 4/3)$
concave down on $(-\infty, 0)$, $(4/3, \infty)$



33. Critical values at $x = -2, 0$
relative maximum at $x = -2$
relative minimum at $x = 0$
inflection values at $x = -2 \pm \sqrt{2}$
increasing on $(-\infty, -2)$, $(0, \infty)$
decreasing on $(-2, 0)$
concave up on $(-\infty, -2 - \sqrt{2})$, $(-2 + \sqrt{2}, \infty)$
concave down on $(-2 - \sqrt{2}, -2 + \sqrt{2})$



35. Critical value at $x = 0$
relative maximum at $x = 0$
no relative minimum
inflection values at $x = \pm\sqrt[4]{0.75}$
increasing on $(-\infty, 0)$
decreasing on $(0, \infty)$
concave up on $(-\infty, -\sqrt[4]{0.75})$, $(\sqrt[4]{0.75}, \infty)$
concave down on $(-\sqrt[4]{0.75}, \sqrt[4]{0.75})$
asymptote: $y = 0$

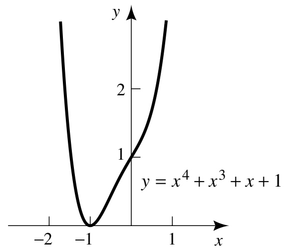


37. Since $f''(x) = 2a$, the sign of $f''(x)$ and a are the same.

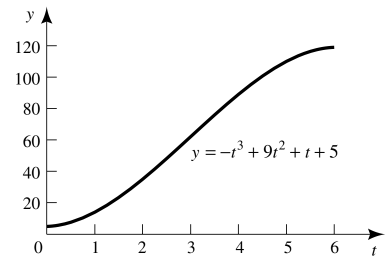
39. $f''(x) = 6ax + 2b = 0$ only if $x = -\dfrac{b}{3a}$, where f'' changes sign.

41. (i) c; (ii) a; (iii) d; (iv) b

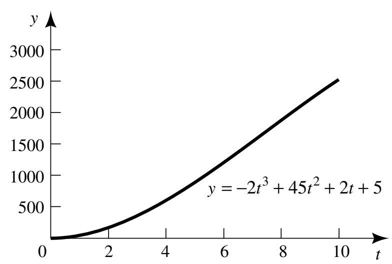
43. -1, relative minimum



45. $t = 3$. Marginal profit is increasing on $(0, 3)$ and decreasing on $(3, 6)$



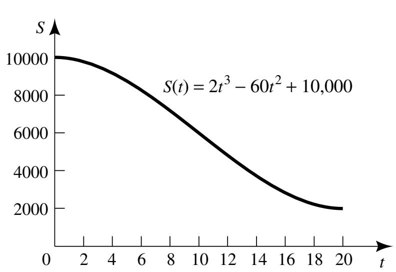
47. $t = 7.5$. Marginal profit is increasing on $(0, 7.5)$ and decreasing on $(7.5, 10)$



49. Yes. The graph is concave down, indicating that marginal costs are decreasing.

51. No. $C''(x) = 0.2 > 0$ so marginal cost $C'(x)$ is increasing.

53. Marginal sales decrease on $(0, 10)$ and increase for $x > 10$. Sales drop off ever more rapidly during the first 10 months, but afterward the drop off slows.



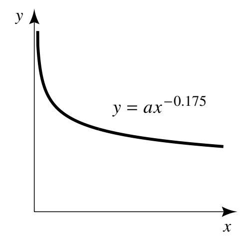
55. Decreasing. Concave up. The rate of decrease is slowing to zero. The economies of size become almost nil for larger schools.

57. Concave down. Since the graph is concave down, the rate of change of yield loss is decreasing.

59. 2

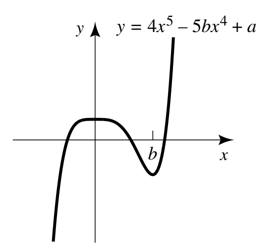
61. **a.** $y' = -0.175ax^{-1.175}$, $y'' \approx 0.206ax^{-2.175}$.

b. The average amount of needed labor decreases by a factor of 88.6%. Less labor is needed for production since workers become more efficient in performing the repeated manual tasks.

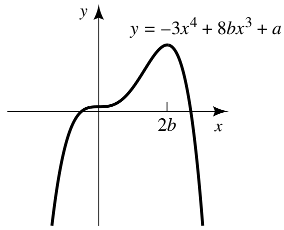


63. In parts (a), (b), and (c), $f''(0) = 0$. In part (d), $f''(0)$ does not exist. Thus, the second derivative test cannot be applied in any of these examples. In part (a), $f'(x) = 4x^3$ and is negative on $(-\infty, 0)$ and positive on $(0, \infty)$. Thus, $x = 0$ is a relative minimum. In part (b), $f'(x) = -4x^3$ and is positive on $(-\infty, 0)$ and negative on $(0, \infty)$. Thus, $x = 0$ is a relative maximum. In part (c), $f'(x) = 3x^2$ and is positive on $(-\infty, 0)$ and positive on $(0, \infty)$. Thus, there is no relative extrema at $x = 0$. In part (d), $f'(x) = 12x^{1/3}$ and is negative on $(-\infty, 0)$ and positive on $(0, \infty)$. Thus, $x = 0$ is a relative minimum.

65. Critical values at $x = 0, b$
relative maximum at $x = 0$
relative minimum at $x = b$
inflection value at $x = 3b/4$
increasing on $(-\infty, 0)$, (b, ∞)
decreasing on $(0, b)$
concave up on $(3b/4, \infty)$
concave down on $(-\infty, 3b/4)$

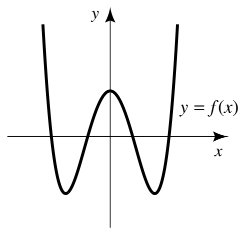


67. Critical values at $x = 0, 2b$
relative maxima at $x = 2b$
no relative minima
inflection value at $x = 0, 4b/3$
increasing on $(-\infty, 2b)$
decreasing on $(2b, -\infty)$
concave up on $(0, 4b/3)$
concave down on $(-\infty, 0)$, $(4b/3, \infty)$

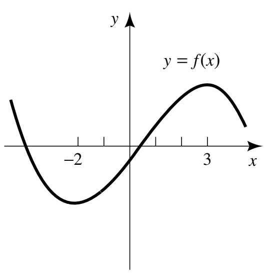


69. Recall that if $g'(a)$ exists, then $g(x)$ is continuous at $x = a$. Since $f''(a)$ exists, $f'(x)$ is continuous at $x = a$. This certainly means that $f'(a)$ exists. This in turn, implies that $f(x)$ is continuous at $x = a$.

71. **a.** **b.** 4 **c.** 2



73. **a.**



b. Yes. Since $f(x)$ is a polynomial, $f''(x)$ is also a polynomial. As a polynomial, $f''(x)$ is continuous everywhere. Since $f''(x)$ changes sign going from -2 to 3, $f''(x)$ must be zero on the interval $(-2, 3)$. So there must be an inflection point on this interval, since $f''(x)$ changes sign.

75. $g''(x) = \dfrac{-f''(x)[f(x)]^2 + 2f(x)[f'(x)]^2}{[f(x)]^4} < 0,$

since $f''(x) > 0$ and $f(x) < 0$ for all x.

77. We have $f'(0) = \lim\limits_{h\to 0} \dfrac{f(0+h) - f(0)}{h}$. To evaluate this limit we need to take the limit from the left and the limit from the right. We have

$$\lim_{h\to 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0^-} \frac{0-0}{h}$$

$$\begin{aligned}\lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h} &= \lim_{h\to 0^+} \frac{h^3 - 0}{h}\\ &= \lim_{h\to 0^+} h^2\\ &= 0\end{aligned}$$

This shows that $f'(0) = 0$. So we now have that $f'(x) = 0$ when $x \le 0$ and $f'(x) = 3x^2$ when $x > 0$.

We have $f''(0) = \lim\limits_{h\to 0} \dfrac{f'(0+h) - f'(0)}{h}$. To evaluate this limit we need to take the limit from the left and the limit from the right. We have

$$\lim_{h\to 0^-} \frac{f'(0+h) - f'(0)}{h} = \lim_{h\to 0^-} \frac{0-0}{h}$$

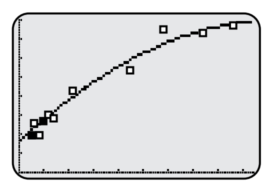
$$\begin{aligned}\lim_{h\to 0^+} \frac{f'(0+h) - f'(0)}{h} &= \lim_{h\to 0^+} \frac{3h^2 - 0}{h}\\ &= \lim_{h\to 0^+} 3h\\ &= 0\end{aligned}$$

This shows that $f''(0) = 0$.

79. $-\sin x, -\cos x$

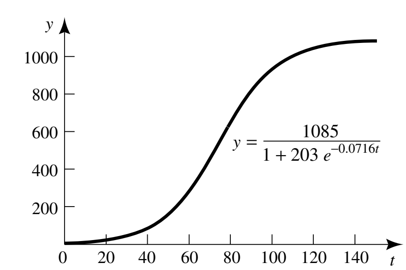
81. Critical values at $x = 1, 3$;
increasing on $(1, 3)$;
decreasing on $(-\infty, 1), (3, \infty)$;
concave up on $(-\infty, 2)$;
concave down on $(2, \infty)$;
inflection value at $x = 2$

83. **a.** $y = -0.0254x^2 + 2.5787x + 14.292$, $r^2 = 0.9720$
b. Yes, since the derivative, marginal cost, is decreasing.



$[0, 47] \times [0, 80]$

85. Logistic: $y = \dfrac{1085}{1 + 203e^{-0.0716t}}$. The graph is concave up at first, then turns concave down. As $t \to \infty$, $y \to 1085$.

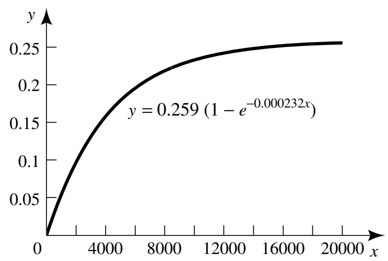


Section 5.3

1. $\frac{2}{3}$	**3.** 0
5. ∞	**7.** $-\infty$
9. 1	**11.** 5
13. 1	**15.** 0
17. 2	**19.** $r \to c$

21. 100%

23. Since $f'(x) > 0$ and $f''(x) < 0$ for positive x, f increases and is concave down. As $x \to \infty$, $f(x) \to 0.259$.



25. Since $L'(t) > 0$, $L(t)$ is always increasing. As $t \to \infty$, $L(t) \to 0.26e^{2.876}$, so this number is a bound for the length of the larva.

27. **a.** As $t \to \infty$, $T(t) \to 70$. Thus, the temperature of the milk returns ultimately to the temperature of the room.
b. About 1.01 hours.

29. As $x \to \infty$, $y \to 1.27$.

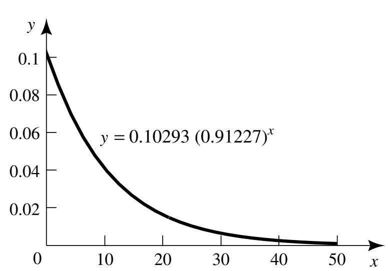
31. For large t, $P(t) \approx 4$, so the profit is approximately \$4 million.

33. 0

35. **a.** ii
b. iii
c. iv
d. i

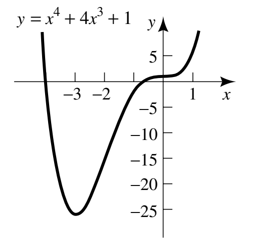
37. As x becomes large without bound, any polynomial (other than a constant) must become either positively or negatively large without bound. Thus, a nonconstant polynomial cannot be asymptotic to any horizontal line. Obviously, the indicated polynomial is not a constant.

39. **a.** $w = 0.10293(0.91227)^x$
b. As $t \to \infty$, $w \to 0$, as you would expect.

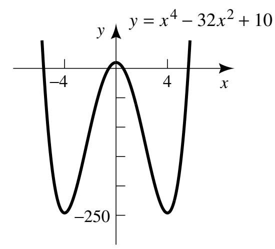


Section 5.4

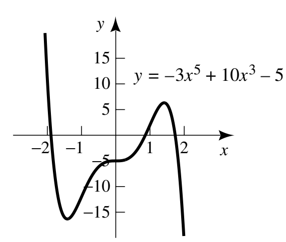
1. Critical values at $x = 0, -3$
no relative maximum
relative minimum at $x = -3$
inflection values at $x = 0, -2$
increasing on $(-3, \infty)$
decreasing on $(-\infty, -3)$
concave up on $(-\infty, -2)$, $(0, \infty)$
concave down on $(-2, 0)$



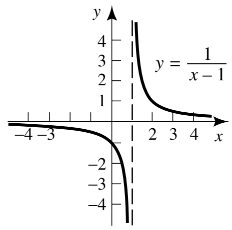
3. Critical values at $x = -4, 0, 4$
relative maximum at $x = 0$
relative minimum at $x = -4, 4$
inflection values at $x = \pm 4/\sqrt{3}$
increasing on $(-4, 0)$, $(4, \infty)$
decreasing on $(-\infty, -4)$, $(0, 4)$
concave up on $(-\infty, -4/\sqrt{3})$, $(4/\sqrt{3}, \infty)$
concave down on $(-4/\sqrt{3}, 4/\sqrt{3})$



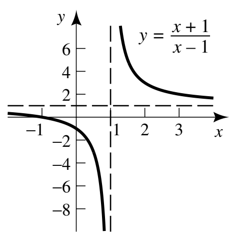
5. Critical values at $x = 0, \pm\sqrt{2}$
relative maximum at $x = \sqrt{2}$
relative minimum at $x = -\sqrt{2}$
inflection values at $x = 0, \pm 1$
increasing on $(-\sqrt{2}, \sqrt{2})$
decreasing on $(-\infty, -\sqrt{2})$ and $(\sqrt{2}, \infty)$
concave up on $(-\infty, -1)$ and $(0, 1)$
concave down on $(-1, 0)$ and $(1, \infty)$



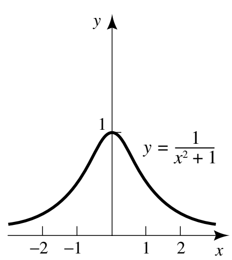
7. No critical values
no relative maximum
no relative minimum
no inflection values
nowhere increasing
decreasing on $(-\infty, 1)$, $(1, \infty)$
concave up on $(1, \infty)$
concave down on $(-\infty, 1)$
asymptotes: $x = 1$, $y = 0$



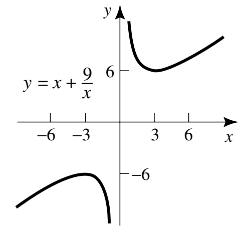
9. No critical or inflection values
no maxima or minima
increasing nowhere
decreasing on $(-\infty, 1)$, $(1, \infty)$
concave up on $(1, \infty)$
concave down on $(-\infty, 1)$
asymptotes: $x = 1$, $y = 1$



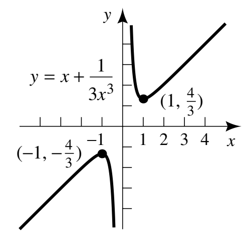
11. Critical value at $x = 0$
relative maximum at $x = 0$
no relative minimum
inflection values at $x = \pm\dfrac{1}{\sqrt{3}}$
increasing on $(-\infty, 0)$
decreasing on $(0, \infty)$
concave up on $(-\infty, -1/\sqrt{3})$, $(1/\sqrt{3}, \infty)$
concave down on $(-1/\sqrt{3}, /\sqrt{3})$
asymptote: $y = 0$



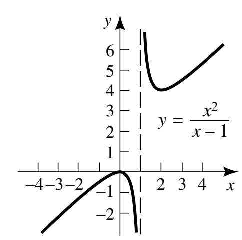
13. Critical values at $x = \pm 3$
relative maximum at $x = -3$
relative minimum at $x = 3$
no inflection values
increasing on $(-\infty, -3)$, $(3, \infty)$
decreasing on $(-3, 0)$, $(0, 3)$
concave up on $(0, \infty)$
concave down on $(-\infty, 0)$
asymptote: $x = 0$



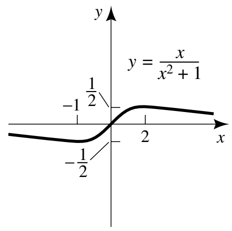
15. Critical values at $x = \pm 1$
relative maximum at $x = -1$
relative minimum at $x = 1$
no inflection values
increasing on $(-\infty, -1)$, $(1, \infty)$
decreasing on $(-1, 0)$, $(0, 1)$
concave up on $(0, \infty)$
concave down on $(-\infty, 0)$
asymptote: $x = 0$



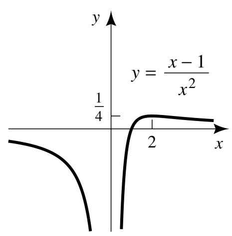
17. Critical values at $x = 0, 2$
relative maximum at $x = 0$
relative minimum at $x = 2$
no inflection values
increasing on $(-\infty, 0)$, $(2, \infty)$
decreasing on $(0, 1)$, $(1, 2)$
concave up on $(1, \infty)$
concave down on $(-\infty, 1)$
asymptote: $x = 1$



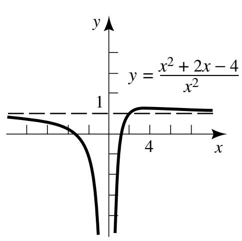
19. Critical values at $x = \pm 1$
relative maximum at $x = 1$
relative minimum at $x = -1$
inflection values at $x = 0, \pm\sqrt{3}$
increasing on $(-1, 1)$
decreasing on $(-\infty, -1)$, $(1, \infty)$
concave up on $(-\sqrt{3}, 0)$, $(\sqrt{3}, \infty)$
concave down on $(-\infty, -\sqrt{3})$, $(0, \sqrt{3})$
asymptote: $y = 0$



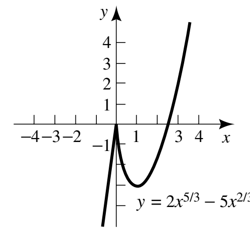
21. Critical value at $x = 2$
relative maximum at $x = 2$
no relative minima
inflection value at $x = 3$
increasing on $(0, 2)$
decreasing on $(-\infty, 0)$, $(2, \infty)$
concave up on $(3, \infty)$
concave down on $(-\infty, 0)$, $(0, 3)$
asymptotes: $x = 0$, $y = 0$



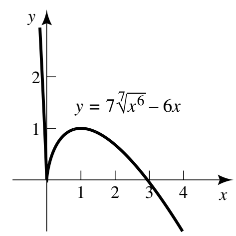
23. Critical value at $x = 4$
relative maximum at $x = 4$
no relative minima
inflection value at $x = 6$
increasing on $(0, 4)$
decreasing on $(-\infty, 0)$, $(4, \infty)$
concave up on $(6, \infty)$
concave down on $(-\infty, 0)$, $(0, 6)$
asymptotes: $x = 0$, $y = 1$



25. Critical values at $x = 0, 1$
relative maximum at $x = 0$
relative minima at $x = 1$
inflection value at $x = -\dfrac{1}{2}$
increasing on $(-\infty, 0)$, $1, \infty)$
decreasing on $(0, 1)$
concave up on $\left(-\frac{1}{2}, 0\right)$, $(0, \infty)$
concave down on $(-\infty, -1/2)$
no asymptotes

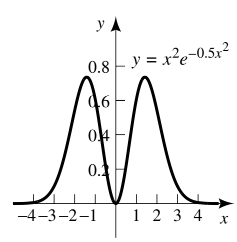


27. Critical values at $x = 0, 1$
relative maximum at $x = 1$
relative minimum at $x = 0$
no inflection value
increasing on $(0, 1)$
decreasing on $(-\infty, 0)$, $(1, \infty)$
concave up nowhere
concave down on $(-\infty, 0)$, $(0, \infty)$



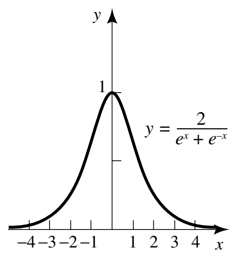
29. Critical values at $x = 0, \pm\sqrt{2}$
relative maxima at $x = \pm\sqrt{2}$
relative minimum at $x = 0$
inflection values at $x = \pm\sqrt{\dfrac{5 \pm \sqrt{17}}{2}}$
increasing on $(-\infty, -\sqrt{2})$, $(0, \sqrt{2})$
decreasing on $(-\sqrt{2}, 0)$, $(\sqrt{2}, \infty)$
concave up on $\left(-\infty, -\sqrt{\frac{5+\sqrt{17}}{2}}\right)$, $\left(-\sqrt{\frac{5-\sqrt{17}}{2}}, \sqrt{\frac{5-\sqrt{17}}{2}}\right)$
and $\left(\sqrt{\frac{5+\sqrt{17}}{2}}, \infty\right)$

concave down on $\left(-\sqrt{\frac{5+\sqrt{17}}{2}}, -\sqrt{\frac{5-\sqrt{17}}{2}}\right)$ and
$\left(\sqrt{\frac{5-\sqrt{17}}{2}}, \sqrt{\frac{5+\sqrt{17}}{2}}\right)$
asymptote: $y = 0$

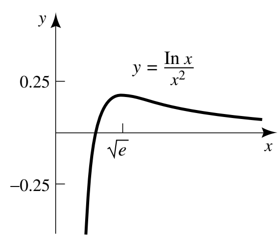


31. Critical value at $x = 0$
relative maximum at $x = 0$
no relative minimum

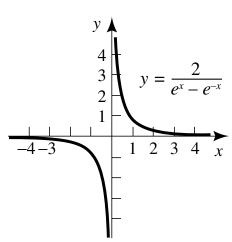
inflection values at $x = \ln(\sqrt{2} \pm 1)$
increasing on $(-\infty, 0)$
decreasing on $(0, \infty)$
concave up on $(-\infty, \ln(\sqrt{2}-1)), (\ln(\sqrt{2}+1), \infty)$
concave down on $(\ln(\sqrt{2}-1), \ln(\sqrt{2}+1))$
asymptote: $y = 0$



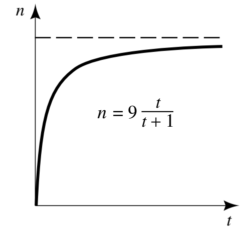
33. Critical value at $x = \sqrt{e}$
relative maximum at $x = \sqrt{e}$
no relative minimum
inflection values at $x = e^{5/6}$
increasing on $(0, \sqrt{e})$
decreasing on $(\sqrt{e}, \infty)$
concave up on $(e^{5/6}, \infty)$
concave down on $(0, e^{5/6})$
asymptote: $x = 0$, $y = 0$



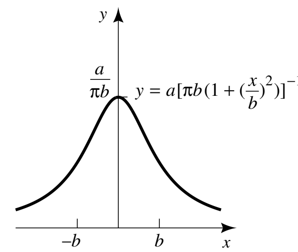
35. No critical value
no relative maximum
no relative minimum
no inflection values
nowhere increasing
decreasing on $(-\infty, 0), (0, \infty)$
concave up on $(0, \infty)$
concave down on $(-\infty, 0)$
asymptote: $x = 0$, $y = 0$



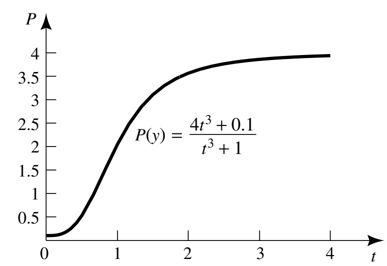
37. $n(t)$ is increasing and concave down on $(0, \infty)$ and $n(t) \to 9$, as $t \to \infty$



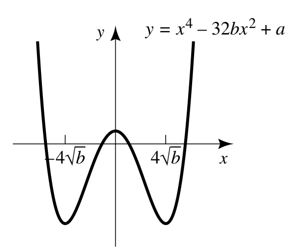
39. Critical value at $x = 0$
inflection values at $x = \pm b/\sqrt{3}$
asymptote: $y = 0$



41. $\sqrt[3]{0.5}$

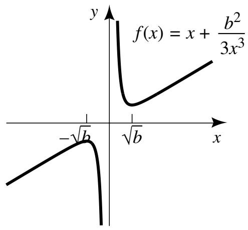


43. Critical values at $x = 0, \pm 4\sqrt{b}$
relative maximum at $x = 0$
relative minimum at $x = \pm 4\sqrt{b}$
inflection values at $\pm 4\sqrt{b/3}$
increasing on $(-4\sqrt{b}, 0), (4\sqrt{b}, \infty)$
decreasing on $(-\infty, -4\sqrt{b}), (0, 4\sqrt{b})$
concave up on $(-\infty, -4\sqrt{b/3}), (4\sqrt{b/3}, \infty)$
concave down on $(-4\sqrt{b/3}, 4\sqrt{b/3})$

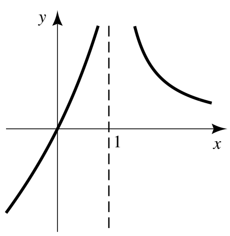


45. Critical values at $x = \pm\sqrt{b}$
relative maximum at $x = -\sqrt{b}$
relative minimum at $x = \sqrt{b}$
no inflection values
increasing on $(-\infty, -\sqrt{b}), (\sqrt{b}, \infty)$

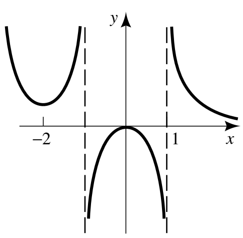
decreasing on $(-\sqrt{b}, 0)$, $(0, \sqrt{b})$
concave up on $(0, \infty)$
concave down on $(-\infty, 0)$
asymptote: $x = 0$



47.



49.



51. $C(t) = k(e^{-at} - e^{-bt}) = ke^{-at}[1 - e^{(a-b)t}]$
Since $a - b < 0$, $1 - e^{(a-b)t}$ is approximately 1 for large t.
Therefore, $C(t) \approx ke^{-at}$.

Section 5.5

1. Absolute minimum in June.
absolute maximum in November.

3. Absolute minimum in September.
absolute maximum in April.

5. Absolute minimum in 1934,
absolute maximum in 1910.

7. **a.** Absolute minimum at $x = 2$,
absolute maximum at $x = 4$;
b. absolute minimum at $x = 1$,
absolute maximum at $x = 2$;
c. absolute minimum at $x = -2, 1$,
absolute maximum at $x = -1, 2$.

9. **a.** Absolute minimum at $x = 0$,
absolute maximum at $x = -2$;
b. absolute minimum at $x = 2$,
absolute maximum at $x = 1$;
c. absolute minimum at $x = 2$,
absolute maximum at $x = -2$.

11. Absolute minimum at $x = 1$,
absolute maximum at $x = -1$.

13. Absolute minimum at $x = 0$,
absolute maximum at $x = 2$.

15. Absolute minimum at $x = 1$,
absolute maximum at $x = 3$.

17. Absolute minimum at $x = -3, 0$,
absolute maximum at $x = 2$.

19. Absolute minimum at $x = -2, 2$,
absolute maximum at $x = -3, 3$.

21. Absolute minimum at $x = 2$,
absolute maximum at $x = 8$.

23. Absolute minimum at $x = \frac{1}{2}$,
no absolute maximum.

25. Absolute minimum at $x = 0$,
no absolute maximum.

27. Absolute minimum at $x = 0$,
no absolute maximum.

29. **a.** Absolute minimum at $x = 1$,
absolute maximum at $x = \frac{1}{2}$;
b. no absolute minimum, no absolute maximum;
c. absolute minimum at $x = 1$, no absolute maximum;
d. no absolute minimum, absolute maximum at $x = -1$.

31. 8, 8

33. 10, 10

35. 5, 20

37. 10, −10

39. $2a^2$

41. $\left(\frac{c}{2a}, \frac{c}{2b}\right)$

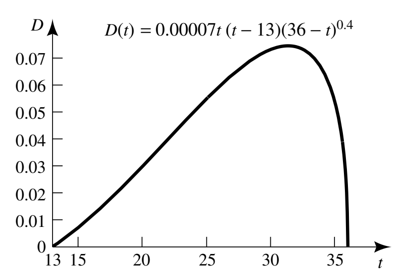
43. $x = 5$

45. $r/2$

47. 200 pounds. 145 bushels.

49. b/c. $D = 200$ corresponds to about July 21.

51. $D'(t) = 0.00007(36 - t)^{-0.6}[-2.4t^2 + 90.2t - 468]$
Maximum when $D \approx 31.4$.



53. **a.** $0.0008227x^2 - 0.1981x + 21.04$
b. 120

55. $0.03321x^2 - 0.39969x + 1.01396$. 6.02.

57. $-0.0057x^2 + 0.297x - 2.8402$. About 26 degrees.

Section 5.6

1. 200×200 **3.** $x = 1.5$ **5.** 10×20

7. 100×200 **9.** 40 **11.** \$450

13. $4 \times 4 \times 2$, 48 **15.** $6 \times 6 \times 9$

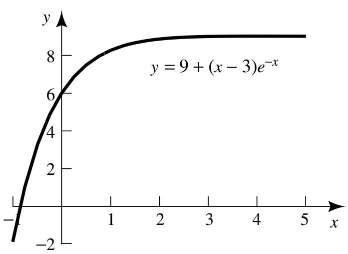
17. About 185 pounds **19.** Four runs of 4000 each

21. C is $10\sqrt{3}$ feet to the right of D

23. Cut at $\dfrac{48}{3\sqrt{3}+4} \approx 5.22$, and use the shorter piece for the square.

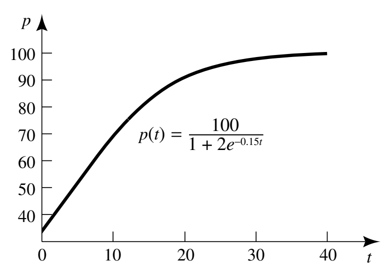
25. $\dfrac{500}{9+\sqrt{3}} \approx 46.6$ yards

27. Absolute minimum at $x = -1$, absolute maximum at $x = 4$

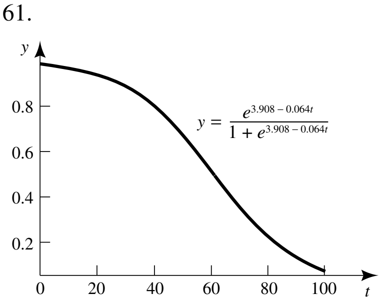


Section 5.7

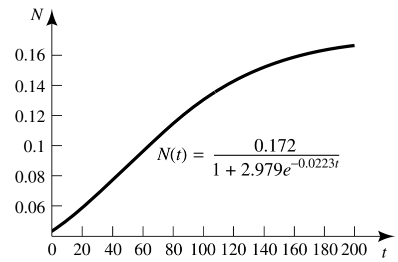
1. 100%



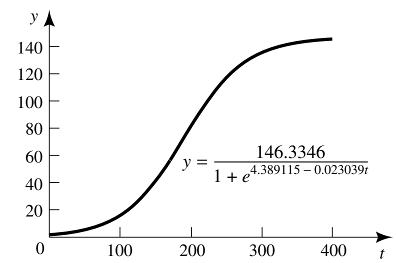
3. About 61.



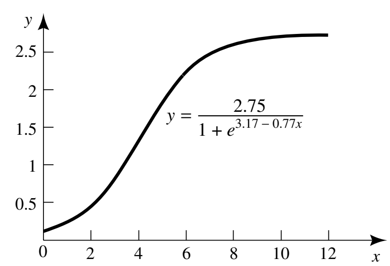
5. The graph is concave up to about $x = 49$ and concave down afterward. The rate of nitrogen accumulation increases until about 49 years later and then begins to decrease to zero. The level of nitrogen approaches 0.172.



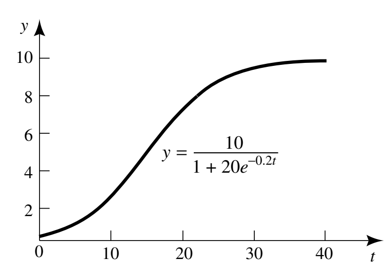
7. The graph is concave up to about $t = 190.5$ and concave down afterward. The rate of citrus growth increases until about 190.5 days and then begins to decrease to zero. The fruit surface area approaches 146.3346 cm^2.



9. The graph is concave up to about $x = 4$ and concave down afterward. The rate of head capsule width growth increases until about $x = 4$ (the fourth instar) and then begins to decrease to zero. The head capsule width approaches 2.75 mm.



11. $5 \ln 20 \approx 14.98$, 10



13. 5.4 years

15. $P'(t) = \dfrac{akLe^{-kt}}{(1 + ae^{-kt})^2} > 0$ for all $t \geq 0$

17. $P''(t) = ak^2 L \dfrac{e^{-kt}(ae^{-kt}-1)}{(1+ae^{-kt})^3} = 0$ if and only if $ae^{-kt} - 1 = 0$. This implies $t = \dfrac{1}{k}\ln a$.

19. Since $e^{-kc} = e^{-\ln a} = \dfrac{1}{a}$,
$P(c) = \dfrac{L}{1+ae^{-kc}} = \dfrac{L}{1+a/a} = \dfrac{L}{2}$

21. Since $P_0 = \dfrac{100}{1+a}$, $a = \dfrac{100}{P_0} - 1$. Since
$Q = P(T) = \dfrac{100}{1+ae^{-kT}} = \dfrac{100}{1+(100/P_0 - 1)e^{-kT}}$,
$e^{-kT} = \dfrac{P_0(100-Q)}{Q(100-P_0)}$. This implies that
$k = -\dfrac{1}{T}\ln\dfrac{P_0(100-Q)}{Q(100-P_0)} = \dfrac{1}{T}\ln\dfrac{Q(100-P_0)}{P_0(100-Q)}$

Section 5.8

1. -2
3. $-\dfrac{3}{5}$
5. $\dfrac{9}{2}$
7. $-\dfrac{2}{7}$
9. -2
11. $-\dfrac{1}{2}$
13. $\dfrac{16}{31}$
15. $-\dfrac{1}{2}$
17. -3
19. $-\dfrac{15}{7}$
21. $-\dfrac{3}{4}$
23. $-\dfrac{5}{4}$
25. $-\dfrac{\sqrt[4]{al}}{4}$
27. 6
29. 6
31. 16
33. 12
35. -1
37. -1
39. -2
41. -200
43. 400
45. 1.6 miles per hour
47. -3 feet per second
49. $\dfrac{125}{16\pi}$ yards per minute
51. 40 square feet per second

53. Since $f(g(x)) = x$, $\dfrac{df(g(x))}{dx} = 1$ and $f'(g(x))\cdot g'(x) = 1$. When $x = c$, $g(c) = y_0$, we have $f'(g(c))g'(c) = 1$. Then $f'(y_0) = \dfrac{1}{g'(c)}$, since $g'(c) \neq 0$.

55. $f(g(x)) = \sqrt[3]{g(x)} = \sqrt[3]{x^3} = x$, $c = 2$, $y_0 = g(c) = c^3 = 8$. Then $f'(y_0) = \dfrac{1}{3}y_0^{-2/3} = \dfrac{1}{12}$ and $g'(c) = 3c^2 = 12$. Therefore, $f'(y_0) = \dfrac{1}{g'(c)}$.

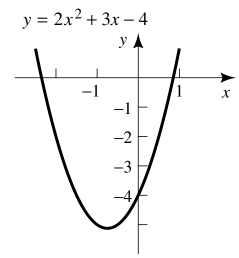
Chapter 5 Review Exercises

1. Critical values at $x = 3, 5, 7, 11, 13, 15$;
increasing on $(3, 7)$, $(11, 13)$, $(15, 18)$;
decreasing on $(1, 3)$, $(7, 11)$, $(13, 15)$

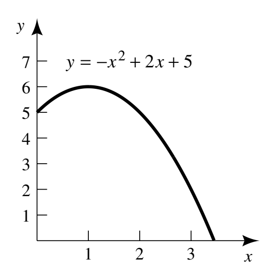
2. Critical values at $x = -2, 1$;
increasing on $(-\infty, -2)$, $(1, \infty)$;
decreasing on $(-2, 1)$;
relative maximum at $x = -2$;
relative minimum at $x = 1$

3. Critical values at $x = -4, 2, 5$;
increasing on $(-\infty, -4)$, $(5, \infty)$;
decreasing on $(-4, 5)$;
relative maximum at $x = -4$;
relative minimum at $x = 5$

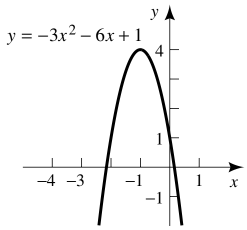
4. Critical value at $x = -\dfrac{3}{4}$
relative minimum at $x = -\dfrac{3}{4}$
increasing on $(-3/4, \infty)$
decreasing on $(-\infty, -3/4)$
concave up on $(-\infty, \infty)$
concave down nowhere



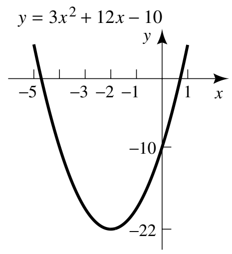
5. Critical value at $x = 1$
relative maximum at $x = 1$
increasing on $(-\infty, 1)$
decreasing on $(1, \infty)$
concave up nowhere
concave down on $(-\infty, \infty)$



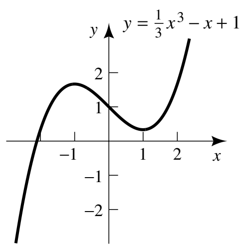
6. Critical value at $x = -1$
relative maximum at $x = -1$
increasing on $(-\infty, -1)$
decreasing on $(-1, \infty)$
concave up nowhere
concave down on $(-\infty, \infty)$



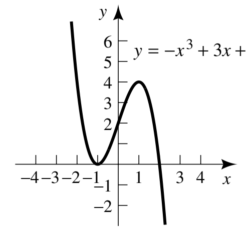
7. Critical value at $x = -2$
relative minimum at $x = -2$
increasing on $(-2, \infty)$
decreasing on $(-\infty, -2)$
concave up on $(-\infty, \infty)$
concave down nowhere



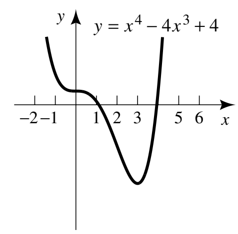
8. Critical values at $x = -1, 1$
relative maximum at $x = -1$
relative minimum at $x = 1$
increasing on $(-\infty, -1), (1, \infty)$
decreasing on $(-1, 1)$
concave up on $(0, \infty)$
concave down on $(-\infty, 0)$
inflection value at $x = 0$



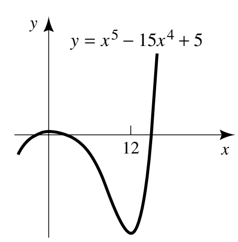
9. Critical values at $x = -1, 1$
relative maximum at $x = 1$
relative minimum at $x = -1$
increasing on $(-1, 1)$
decreasing on $(-\infty, -1), (1, \infty)$
concave up on $(-\infty, 0)$
concave down on $(0, \infty)$
inflection value at $x = 0$



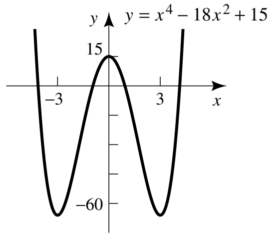
10. Critical values at $x = 0, 3$
relative minimum at $x = 3$
increasing on $(3, \infty)$
decreasing on $(-\infty, 3)$
concave up on $(-\infty, 0), (2, \infty)$
concave down on $(0, 2)$
inflection values at $x = 0, 2$



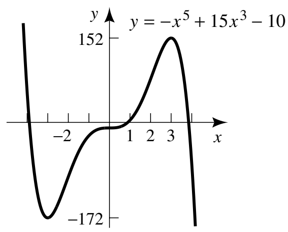
11. Critical values at $x = 0, 12$
relative maximum at $x = 0$
relative minimum at $x = 12$
increasing on $(-\infty, 0), (12, \infty)$
decreasing on $(0, 12)$
concave up on $(9, \infty)$
concave down on $(-\infty, 9)$
inflection value at $x = 9$



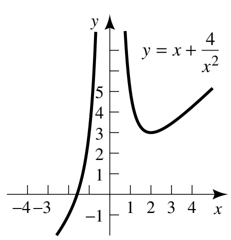
12. Critical values at $x = -3, 0, 3$
relative maximum at $x = 0$
relative minima at $x = -3, 3$
increasing on $(-3, 0), (3, \infty)$
decreasing on $(-\infty, -3), (0, 3)$
concave up on $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$
concave down on $(-\sqrt{3}, \sqrt{3})$
inflection value at $x = \pm\sqrt{3}$



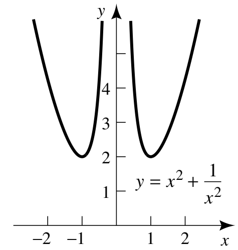
13. Critical values at $x = -3, 0, 3$
relative maximum at $x = 3$
relative minimum at $x = -3$
increasing on $(-3, 3)$
decreasing on $(-\infty, -3)$, $(3, \infty)$
concave up on $(-\infty, -3/\sqrt{2})$, $(0, 3/\sqrt{2})$
concave down on $(-3/\sqrt{2}, 0)$, $(3/\sqrt{2}, \infty)$
inflection value at $x = 0, \pm 3/\sqrt{2}$



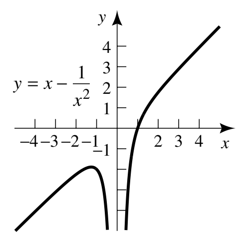
14. Critical value at $x = 2$
relative minimum at $x = 2$
increasing on $(-\infty, 0)$, $(2, \infty)$
decreasing on $(0, 2)$
concave up on $(-\infty, 0)$, $(0, \infty)$
concave down nowhere
no inflection values
asymptote at $x = 0$



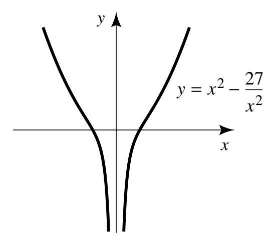
15. Critical values at $x = -1, 1$
relative maximum at $x = -1, 1$
increasing on $(-1, 0)$, $(1, \infty)$
decreasing on $(-\infty, -1)$, $(0, 1)$
concave up on $(-\infty, 0)$, $(0, \infty)$
concave down nowhere
no inflection values
asymptote at $x = 0$



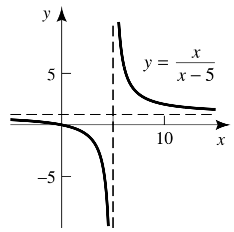
16. Critical value at $x = -\sqrt[3]{2}$
relative maximum at $x = -\sqrt[3]{2}$
increasing on $(-\infty, -\sqrt[3]{2})$, $(0, \infty)$
decreasing on $(-\sqrt[3]{2}, 0)$
concave up nowhere
concave down on $(-\infty, 0)$, $(0, \infty)$
no inflection values
asymptote at $x = 0$



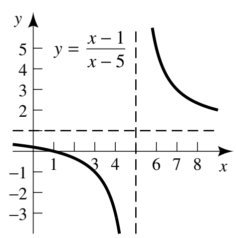
17. No critical values
increasing on $(0, \infty)$
decreasing on $(-\infty, 0)$
concave up on $(-\infty, -3)$, $(3, \infty)$
concave down on $(-3, 0)$, $(0, 3)$
inflection values at $x = \pm 3$
asymptote at $x = 0$



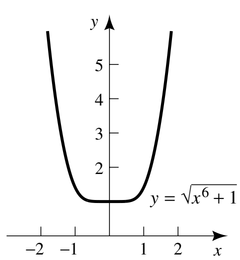
18. No critical values
increasing nowhere
decreasing on $(-\infty, 5)$, $(5, \infty)$
concave up on $(5, \infty)$
concave down on $(-\infty, 5)$
no inflection values
asymptotes at $x = 5$, $y = 1$



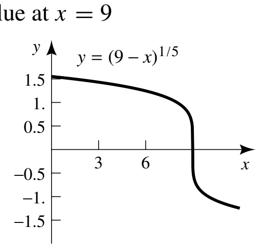
19. No critical values
increasing nowhere
decreasing on $(-\infty, 5)$, $(5, \infty)$
concave up on $(5, \infty)$
concave down on $(-\infty, 5)$
no inflection values
asymptotes at $x = 5$, $y = 1$



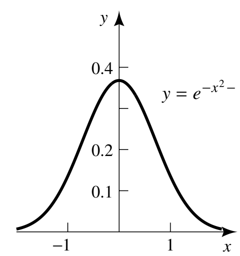
20. Critical values at $x = 0$
relative minimum at $x = 0$
increasing on $(0, \infty)$
decreasing on $(-\infty, 0)$
concave up on $(-\infty, \infty)$
concave down nowhere
no inflection values



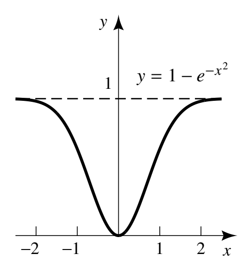
21. Critical value at $x = 9$
increasing nowhere
decreasing on $(-\infty, \infty)$
concave up on $(9, \infty)$
concave down $(-\infty, 9)$
inflection value at $x = 9$



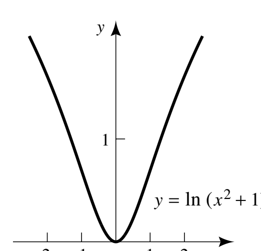
22. Critical value at $x = 0$
relative maximum at $x = 0$
increasing on $(-\infty, 0)$
decreasing on $(0, \infty)$
concave up on $(-\infty, -1/\sqrt{2})$, $(1/\sqrt{2}, \infty)$
concave down $(-1/\sqrt{2}, 1/\sqrt{2})$
inflection values at $x = \pm 1/\sqrt{2}$
asymptote at $y = 0$



23. Critical value at $x = 0$
relative minimum at $x = 0$
increasing on $(0, \infty)$
decreasing on $(-\infty, 0)$
concave up on $(-1/\sqrt{2}, 1/\sqrt{2})$
concave down $(-\infty, -1/\sqrt{2})$, $(1/\sqrt{2}, \infty)$
inflection values at $x = \pm 1/\sqrt{2}$
asymptote at $y = 1$

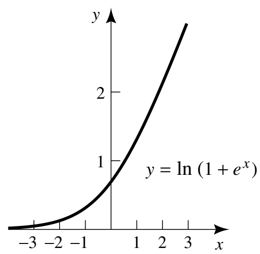


24. Critical value at $x = 0$
relative minimum at $x = 0$
increasing on $(0, \infty)$
decreasing on $(-\infty, 0)$
concave up on $(-1, 1)$
concave down $(-\infty, -1)$, $(1, \infty)$
inflection values at $x = \pm 1$



25. No critical values
increasing on $(-\infty, \infty)$
decreasing nowhere

concave up on $(-\infty, \infty)$
concave down nowhere
no inflection values
asymptotes at $y = 0$



26. Absolute maximum at $x = 1$;
absolute minimum at $x = -1$

27. Absolute maximum at $x = -2$;
absolute minimum at $x = 2$

28. No absolute maximum;
absolute minimum at $x = 1$

29. Absolute maximum at $x = 1$;
no absolute minimum

30. Absolute maximum at $x = 2$;
absolute minimum at $x = -1$

31. Absolute maximum at $x = 3$;
absolute minimum at $x = 0$

32. Absolute maximum at $x = 0$;
absolute minimum at $x = 2$

33. Absolute maximum at $x = 1$;
absolute minimum at $x = 2$

34. Absolute maximum at $x = 1$;
absolute minimum at $x = 2$

35. absolute maximum at $x = 4$;
absolute minimum at $x = 1$

36. $-\dfrac{11}{30}$ **37.** $-\dfrac{5}{16}$

38. -1 **39.** 0

40. $f''(x) = 12ax^2 + 2b$. $f'' > 0$ if $a, b > 0$ and $f'' < 0$ if $a, b < 0$.

41. Relative minimum at $x = a$; relative maximum at $x = b$

42. $g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right] \geq 0$, since $e^{f(x)} > 0$ and $f''(x) > 0$ for all x.

43. $g''(x) = \dfrac{f''(x)f(x) - [f'(x)]^2}{[f(x)]^2} < 0$, since $f > 0$ and $f'' < 0$ for all x.

44. Increasing on $(\sqrt{a/b}, \infty)$; decreasing on $(0, \sqrt{a/b})$

45. 10

46. Solving $\bar{C} = C'$ for x, we obtain $x = 10$, which is the value where the average cost is minimized.

47. $\bar{C}$ is minimized at the value where $\bar{C}' = 0$, that is, $\bar{C}'(x) = \dfrac{xC'(x) - C(x)}{x^2} = 0$. This implies that $C'(x) = \dfrac{C(x)}{x} = \bar{C}(x)$

48. $\dfrac{N}{k}\dfrac{1-a}{2-a}$ **49.** $\sqrt[3]{\dfrac{a}{2b}}$

50. Build only a square enclosure.

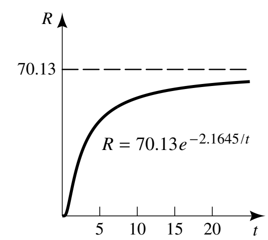
51. \$70 **52.** \$72

53. $r = \dfrac{3}{\sqrt[3]{\pi}}, h = \dfrac{12}{\sqrt[3]{\pi}}$ **54.** One run of 1000 cases each

55. When $P'(x) = 0$, $x = \dfrac{1}{|p'(x)|}\ (p(x) - a)$.
Thus, increasing a decreases x.

56. 180 **57.** $\dfrac{\sqrt{6}}{3} \approx 0.82$ ft/sec

58. 0, 70.13, everywhere



59. Everywhere **60.** $bk \leq a$ **61.** 0.82

62. Concave down. As advertising increases, sales increase less and less, indicating diminishing returns.

63. Concave up. As assets increase, average cost increases less and less, indicating economies of scale.

64. Concave down on $(0, 20000/36)$, concave up on $(20000/36, \infty)$ On the interval $(0, 20000/36)$ the rate of change of the earnings of the greatest earner with respect to the secondary worker is decreasing. On the interval $(20000/36, \infty)$ the rate of change of the earnings of the greatest earner with respect to the secondary worker is increasing.

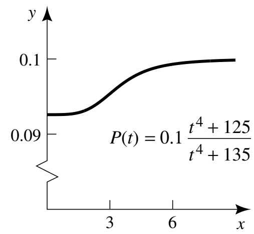
65. **a.** (ii) **b.** (vi) **c.** (i)
d. (v) **e.** (iv) **f.** (iii)

66. 2 **67.** 0

68. Does not exist (∞) **69.** 0

70. 1 **71.** 0

72. Concave up on $(0, 3)$
concave down on $(3, \infty)$
inflection point at $t = 3$
point of diminishing returns at $t = 3$



Chapter 6

Section 6.1

1. $\frac{1}{100}x^{100} + C$

3. $-\frac{1}{98}x^{-98} + C$

5. $5x + C$

7. $-\frac{5}{2}y^{-2} + C$

9. $\frac{\sqrt{2}}{5}y^{5/2} + C$

11. $\frac{3}{5}u^{5/3} + C$

13. $2x^3 + 2x^2 + C$

15. $\frac{1}{3}x^3 + \frac{1}{2}x^2 + x + C$

17. $\frac{10\sqrt{2}}{11}u^{1.1} - \frac{1}{21}u^{2.1} + C$

19. $\frac{1}{3}t^3 - t + C$

21. $\frac{2}{3}t^{3/2} - \frac{3}{8}t^{8/3} + C$

23. $t^6 - t^4 + t + C$

25. $x + 3\ln|x| + C$

27. $\pi x + \ln|x| + C$

29. $\frac{2}{3}t^{3/2} + 2t^{1/2} + C$

31. $e^x - \frac{3}{2}x^2 + C$

33. $5e^x + C$

35. $5e^x - 4x + C$

37. $x + \ln|x| + C$

39. $R(x) = 30x - 0.25x^2$

41. $p(x) = 2x^{-1/2} + 1$

43. $C(x) = 100x - 0.1e^x + 1000.1$

45. $C(x) = 20x^{3/2} - 2x^3$

47. $s(t) = -16t^2 + 30t + 15$

49. $s(t) = -16t^2 + 10t + 6$

51. Approximately \$3,200,000

53. $C(x) = x - 0.25x^2 + 0.2$

55. $L(x) = 0.28x + .025x^2 - \frac{1}{30000}x^3$

57. $\frac{1}{2}e^{2x} + C$

59. $\frac{1}{3}\sin 3x + C$

Section 6.2

1. $\frac{2}{11}(3x+1)^{11} + C$

3. $-\frac{1}{16}(3-x^2)^8 + C$

5. $\frac{4}{5}(2x^2+4x-1)^{5/2} + C$

7. $\frac{1}{48}(3x^4+4x^3+6x^2+12x+1)^4 + C$

9. $\frac{4}{3}(x+1)^{3/2} + C$

11. $\frac{1}{3}(x^2+1)^{3/2} + C$

13. $\frac{3}{4}(x^2+1)^{2/3} + C$

15. $2(x^{1/3}+1)^{3/2} + C$

17. $\frac{2}{3}(\ln x)^{3/2} + C$

19. $-e^{1-x} + C$

21. $-\frac{1}{2}e^{1-x^2} + C$

23. $2e^{\sqrt{x}} + C$

25. $\frac{1}{3}\ln|3x+5| + C$

27. $-\frac{1}{6}(x^2+3)^{-3} + C$

29. $-\ln(e^{-x}+1) + C$

31. $\frac{1}{2}\ln(e^{2x}+e^{-2x}) + C$

33. $\frac{1}{2}\ln|\ln|x|| + C$

35. $p(x) = 4(3x^2+1)^{-1} + 9$

37. $R(x) = 20x^2 - x^4$

39. $C(x) = 2\ln(x^2+1) + 1000$

41. 6 tons

43. $20{,}000e^{2.2} + 80{,}000 \approx 260{,}500$

45. $\frac{1}{2}\ln 7 \approx 0.97$

47. \$16,703.20

49. Let $u = x^5 + x^4 + 1$; then $du = x(5x^3+4x^2)\,dx$, and

$$\int (x^5+x^4+1)^2(5x^3+4x^2)\,dx = \int u^2 \cdot \frac{du}{x}$$

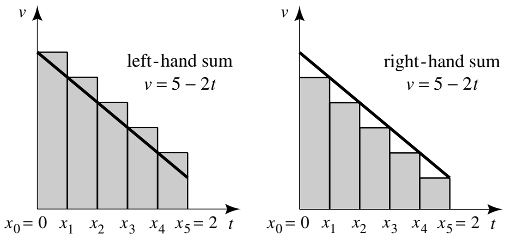
There is no way to eliminate the factor $\frac{1}{x}$; therefore, the method fails.

51. $-\cos x^2 + C$

53. $\frac{1}{3}\sin x^3 + C$

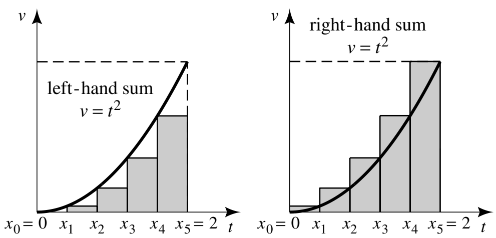
Section 6.3

1. a.



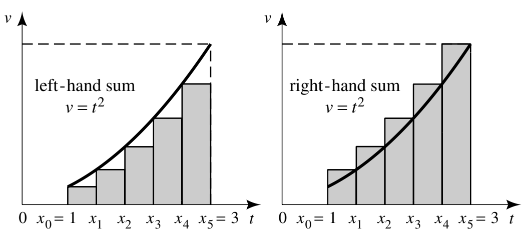
b. 6.8, 5.2, 1.6, 6 **c.** 6.4, 5.6, 0.8, 6

3. a.



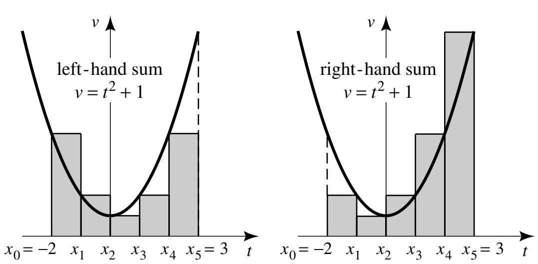
b. 1.92, 3.52, 1.6, 2.72 **c.** 2.28, 3.08, 0.8, 2.68

5. a.



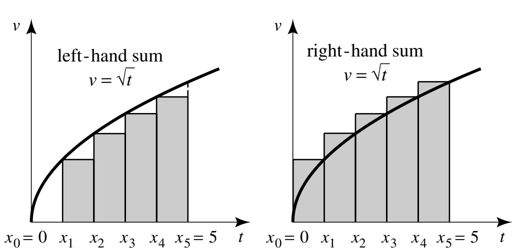
b. 7.12, 10.32, 3.2, 8.72 **c.** 7.88, 9.48, 1.6, 8.68

7. a.



b. 15, 20, 5, 17.5 **c.** 15.625, 18.125, 2.5, 16.875

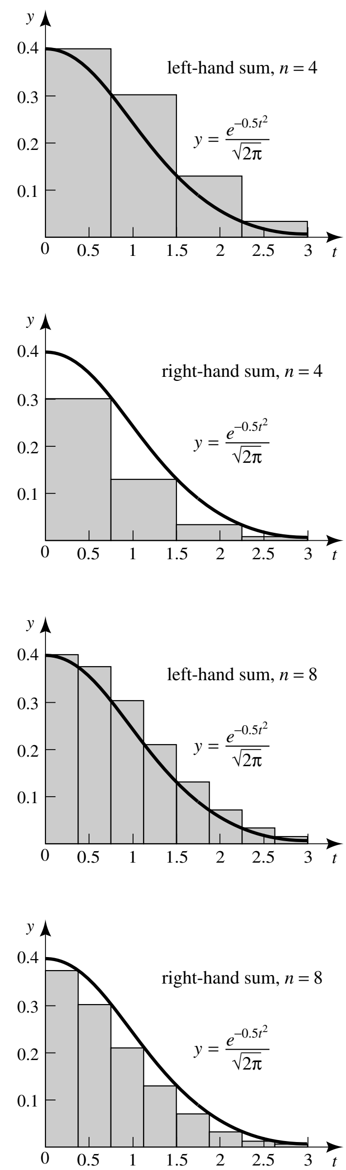
9. a.

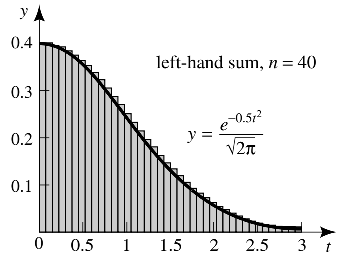


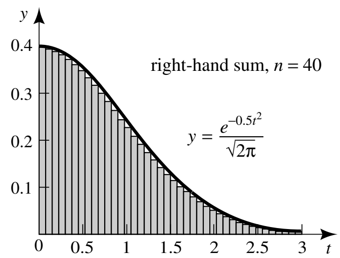
b. 6.15, 8.38, 2.23, 7.265 **c.** 6.83, 7.94, 1.11, 7.385

11.

n	Left-Hand Sum	Right-Hand Sum	Average of Sums
4	0.646003	0.350120	0.498062
8	0.572467	0.424526	0.498497
40	0.513438	0.483850	0.498644

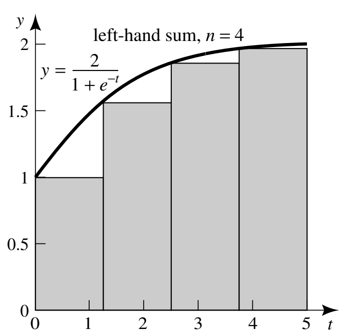


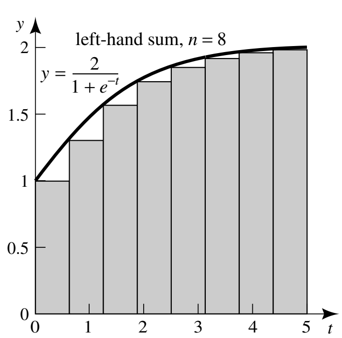


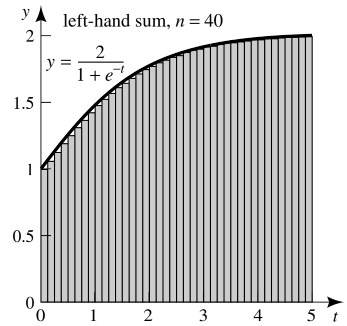


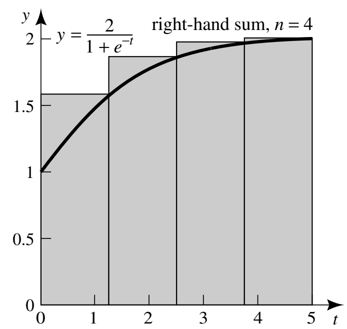
13.

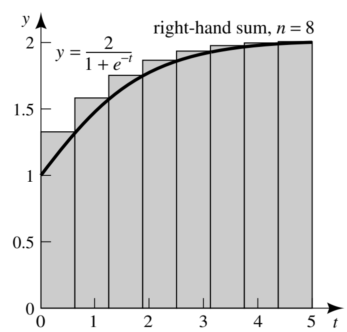
n	Left-Hand Sum	Right-Hand Sum	Average of Sums
4	7.946161	9.179429	8.562795
8	8.302919	8.919553	8.611236
40	8.564839	8.688166	8.626503

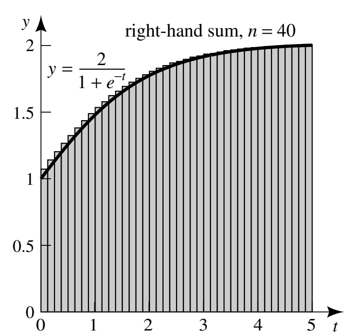






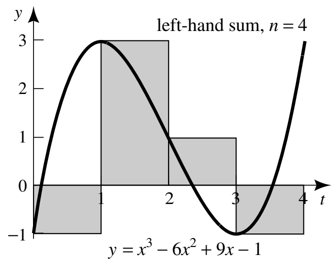


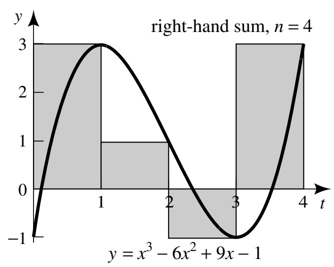


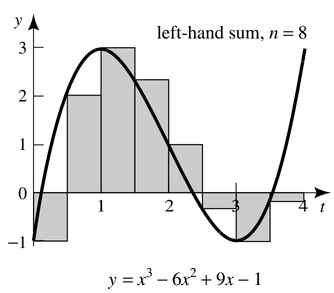


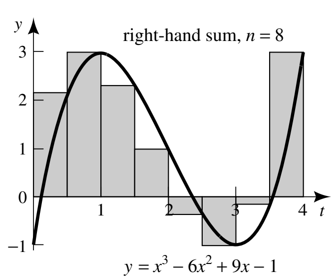
15.

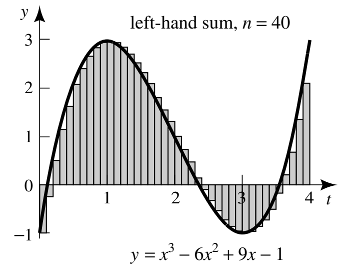
n	Left-Hand Sum	Right-Hand Sum	Average of Sums
4	2.000000	6.000000	4.000000
8	3.000000	5.000000	4.000000
16	3.800000	4.200000	4.000000

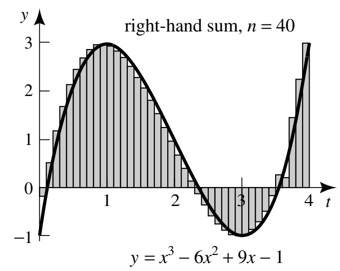












17. 44. Taking $n = 10$, the left-hand sum is 43, and the right-hand sum is 45. The average is 44.

19. Use rectangles with base of length 1 unit and obtain a lower bound of 15 and an upper bound of 24.

21. 135, 95

23. 0.01, 0.001

25. We know that right-hand sum − left-hand sum $= [f(b) - f(a)]\Delta x$. If $f(a) = f(b)$, then $f(b) - f(a) = 0$ and the difference of the two sums is zero.

27. 10, 100

Exercises 6.4

1.

n	Left-Hand Sum	Right-Hand Sum	Average of Sums
10	0.610509	0.710509	0.660509
100	0.661463	0.671463	0.666463
1000	0.666160	0.667160	0.666667

3.

n	Left-Hand Sum	Right-Hand Sum	Average of Sums
10	0.202500	0.302500	0.252500
100	0.245025	0.255025	0.250025
1000	0.249500	0.250500	0.250000

5.

n	Left-Hand Sum	Right-Hand Sum	Average of Sums
10	4.920000	5.720000	5.320000
100	5.293200	5.373200	5.333200
1000	5.329332	5.337332	5.333332

7.

n	Right-Hand Sum	Left-Hand Sum	Average of Sums
10	1.034896	1.168229	1.101563
100	1.091975	1.105309	1.098642
1000	1.097946	1.099279	1.098613

9.

n	Left-Hand Sum	Right-Hand Sum	Average of Sums
10	1.633799	1.805628	1.719714
100	1.709705	1.726888	1.718297
1000	1.717423	1.719141	1.718282

11.

n	Left-Hand Sum	Right-Hand Sum	Average of Sums
10	0.675611	0.710268	0.692939
100	0.691412	0.694878	0.693145
1000	0.692974	0.693321	0.693147

13.

n	Left-Hand Sum	Right-Hand Sum	Average of Sums
80	0.506046	0.491251	0.498649
160	0.502348	0.494951	0.498650
320	0.500499	0.496801	0.498650
640	0.499575	0.497725	0.498650

15.

n	Left-Hand Sum	Right-Hand Sum	Average of Sums
80	8.596146	8.657810	8.626978
160	8.611681	8.642513	8.627097
320	8.619419	8.634834	8.627127
640	8.623280	8.630988	8.627134

17.

n	Left-Hand Sum	Right-Hand Sum	Average of Sums
80	3.900000	4.100000	4.000000
160	3.950000	4.050000	4.000000
320	3.975000	4.025000	4.000000
640	3.987500	4.012500	4.000000

19.

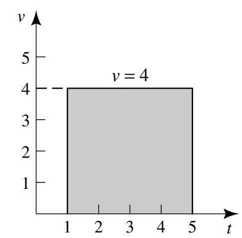
n	Left-Hand Sum	Right-Hand Sum
10	10.560000	10.560000
100	10.665600	10.665600
1000	10.666656	10.666656

21.

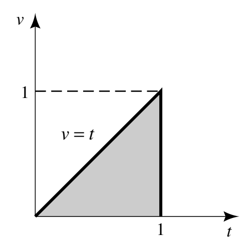
n	Left-Hand Sum	Right-Hand Sum
10	2.622043	2.700019
100	2.636816	2.644614
1000	2.639529	2.640309

23. 20 **25.** 6 **27.** $\frac{9}{2}$

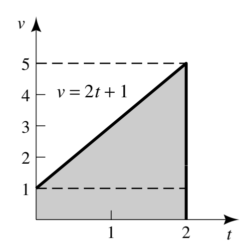
29. $\frac{\pi}{2}$ **31.** $\frac{\pi}{2}$ **33.** 16



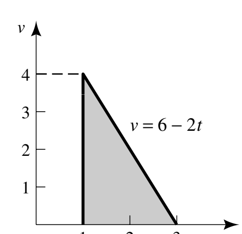
35. 0.5



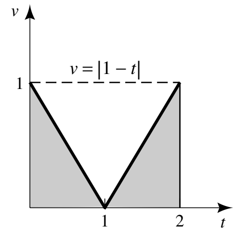
37. 6



39. 4



41. 1



43. On $[1, 4]$, $1 \le \sqrt{x} \le 2$. Take $m = 1$, $M = 2$. Then $\int_1^4 \sqrt{x}\, dx \le M(b-a) = 2(4-1) = 6$. $\int_1^4 \sqrt{x}\, dx \ge m(b-a) = 1(4-1) = 6$.

45. On $[1, 2]$, $0.5 \le 1/x \le 1$. Take $m = 0.5$, $M = 1$. Then $\int_1^2 \frac{1}{x}\, dx \le M(b-a) = 1(2-1) = 1$. $\int_1^2 \frac{1}{x}\, dx \ge m(b-a) = 0.5(2-1) = 0.5$.

47. On $[0, 1]$, $x^3 \le x$. Then $\int_0^1 x^3\, dx \le \int_0^1 x\, dx = 0.5$

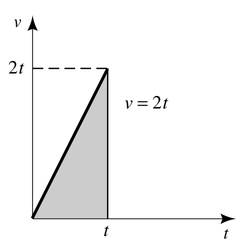
49. On $[-1, 1]$, $\sqrt{3} \le \sqrt{3 + x^2} \le 2$. Take $m = \sqrt{3}$ and $M = 2$. Then

$$\int_{-1}^1 \sqrt{1 + x^2}\, dx \le M(b-a) = 2[1 - (-1)] = 4$$

$$\int_{-1}^1 \sqrt{1 + x^2}\, dx \ge m(b-a) = \sqrt{3}[1 - (-1)] = 2\sqrt{3}$$

51. Left-hand sum is 0.747140; right-hand sum is 0.746508; left-hand sum must be greater since the function is decreasing on $[0, 1]$.

53. t^2



55. $n = 10$, left-hand sum gives a lower estimate of 0.687405, right-hand sum gives an upper estimate of 0.787405

57. $n = 160$, left-hand sum gives a lower estimate of 3.950156, right-hand sum gives an upper estimate of 4.050156

59. $n = 80$, left-hand sum gives a lower estimate of 5.283125, right-hand sum gives an upper estimate of 5.383125

61. No difference

63. 1.09861214. Correct answer to eight decimal places is 1.09861229. The left-hand sum is 1.09927925. The right-hand sum is 1.09794592.

65. $N = 22$

67. a. The first runner is ahead after the first minute, since the first runner has a larger velocity during every moment of the first minute.

b. The second runner is ahead after the first 5 minutes. The second runner gets behind the first runner at the 1-minute mark a distance equal to the area A_1 between the two curves on $[0, 1]$. From the 1-minute mark to the end of the fifth minute the second runner gains a distance equal to the area A_2 between the two curves on $[1, 5]$. Notice that $A_2 > A_1$. Thus, at the end of the fifth minute the second runner is ahead of the first runner by a distance equal to $A_2 - A_1$.

Section 6.5

1. 15	**3.** 0	**5.** 2
7. 14	**9.** $\frac{11}{8}$	**11.** $\frac{1}{2}(e^{-2} - e^{-4})$
13. $\frac{1}{2}\ln 3$	**15.** 0	**17.** $-\frac{1}{16}$
19. $\frac{26}{3}$	**21.** 0	**23.** $\frac{1}{2}\ln\frac{5}{3}$
25. 6	**27.** 5	**29.** 0
31. 0	**33.** $\frac{1}{3}$	**35.** $\int_1^5 f(x)\, dx$

37. $\int_8^{10} f(x)\, dx$

39. a. 5; **b.** 7

41. The lower estimate is 13; the upper estimate is 21. Take $\Delta t = 1$.

43. 948, 928

45. $2(e^2 - e) \approx 9.342$ million barrels.

47. 71

49. About 11 years

51. About 8535 gal

53. a. $V(0) = I$. Thus,

$$\begin{aligned} V(T) &= I + \int_0^T -k(T - t)\, dt \\ &= I - kT \int_0^T 1\, dt + k \int_0^T t\, dt \\ &= I - kT \cdot t\Big|_0^T + \frac{k}{2}t^2\Big|_0^T \\ &= I - kT^2 + \frac{k}{2}T^2 \\ &= I - \frac{1}{2}kT^2 \end{aligned}$$

b. Suppose the equipment is worth $\frac{1}{2}I$ after T^* years. Then

$$V(T^*) - V(0) = \int_0^{T^*} -k(T-t)\,dt$$
$$\frac{1}{2}I - I = -kT\int_0^{T^*} 1\,dt + k\int_0^{T^*} t\,dt$$
$$-\frac{1}{2}I = -kT\cdot t\Big|_0^{T^*} + \frac{1}{2}kt^2\Big|_0^{T^*}$$
$$= -kTT^* + \frac{1}{2}kT^{*2}$$
$$\frac{1}{2}k(T^*)^2 - kT(T^*) + \frac{1}{2}I = 0$$

$$T^* = \frac{kT \pm \sqrt{k^2T^2 - 4\cdot\frac{1}{2}k\cdot\frac{1}{2}I}}{2\cdot\frac{1}{2}k}$$
$$= T \pm \sqrt{T^2 - \frac{I}{k}}$$

Since $T^* \le T$, $T^* = T - \sqrt{T^2 - \frac{I}{k}}$.

55. $\frac{74}{3}$ thousand **57.** \$3194.53

59. \$8 million **61.** About 10.23 **63.** 129,744

65. **a.** First store. Revenue from the first store during the first year is

$$R(1) - R(0) = \int_0^1 R_1'(t)\,dt,$$

which is also the area between the graph of the curve $y = R_1'(t)$ and the x-axis on the interval [0, 1]. This is greater than the area under the graph of $y = R_2$ and the x-axis on [0, 1].

b. Second store. Sales for the second store less sales of the first store during the three-week period is
$[R_2(3) - R_2(0)] - [R_1(3) - R_1(0)]$ equals
$\int_0^3 R_2'(t)\,dt - \int_0^3 R_1'(t)\,dt$ equals
$\int_1^3 [R_2'(t) - R_1'(t)]\,dt - \int_1^3 [R_1'(t) - R_2'(t)]\,dt$
The first integral on the right-hand side of the preceding equation is the area under the graph of $y = R_2'(t)$ and above the graph of $y = R_1'(t)$ on [1, 3] and is greater than the second integral, which is the area under the graph of $y = R_1'(t)$ and above the graph of $y = R_2'(t)$ on [0, 1].

67. $\int_a^b f(x)\,dx$ is a real number. $\int f(x)\,dx$ is a function of x. For example, $\int_0^1 x\,dx = 0.5$ and $\int x\,dx = \frac{1}{2}x^2 + C$.

69. $\frac{a}{t}\left(\frac{1}{m} - \frac{1}{m+tb}\right)$

71.

$$\int_a^b cf(x)\,dx = \lim_{n\to\infty}\sum_{k=1}^n cf(x_i)\Delta x$$
$$= \lim_{n\to\infty} c\sum_{k=1}^n f(x_i)\Delta x$$
$$= c\lim_{n\to\infty}\sum_{k=1}^n f(x_i)\Delta x$$
$$= c\int_a^b f(x)\,dx$$

Section 6.6

1. 3.75 **3.** $10e - 20$

5. $\frac{1}{6}$ **7.** $\frac{1}{2}\left(e^4 - e^2\right) - 1$

9. $\frac{9}{2}$ **11.** $\frac{1}{2}$

13. $\frac{16}{3}$ **15.** $\frac{8}{3}$

17. $\frac{125}{6}$ **19.** $\frac{125}{6}$

21. $e^2 + e - 2$ **23.** 16

25. 1.5 **27.** $\frac{1}{2}$

29. 1 **31.** 8

33. $\frac{23}{2}$ **35.** 20.75

37. x-coordinates of intercepts are $x = -1.53$, $x = 0$, $x = 1.28$. The upper estimate is 5.5821; the lower estimate is 5.5815.

39. x-coordinates of intercepts are $x = -1.80$, $x = 0$, $x = 1.19$. The lower estimate is 5.7052; the upper estimate is 5.7069.

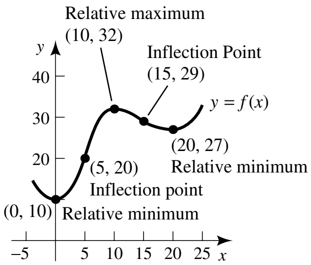
41. **a.** 10 **b.** -6
c. 4 **d.** 6

43. $\frac{1}{3}$ **45.** $\frac{1}{12}$

47. 0.1 **49.** $\frac{1}{300}$

51. \$4.8 billion **53.** $\frac{26}{3}$

55.



57. The coefficient of inequality has become larger.

59. 15

61. 22, 15

63. 0.382 **65.** 0.361

Section 6.7

1. a. \$100;
b. $\$200(1 - e^{-0.50}) = \$78.69$;
c. $\$200(e^{0.50} - 1) = \$129.74$

3. a. $\$200(e - 1) = \$3436.56$;
b. \$2000;
c. $\$2000e = \$5436.56$

5. a. $\$400(1 - e^{-0.50}) = \$157.39$;
b. $\$2000\dfrac{1 - e^{-1.5}}{15} = \$103.58$;
c. $\$2000\dfrac{e - e^{-0.5}}{15} = \$281.57$

7. a. \$50; **b.** \$26.42; **c.** \$71.83

9. a. \$333.33; **b.** \$160.60; **c.** \$436.56

11. a. Yes, since $P_V(10) = \$948{,}180.84$;
b. no, since $P_V(10) = \$776{,}869.84$

13. a. Yes, since $P_V(5) \approx \$316{,}000$;
b. no, since $P_V(5) \approx \$303{,}000$

15. $\dfrac{128}{3}$ **17.** $0.9 - 0.1 \ln 10$

19. $\dfrac{1}{2}$ **21.** $\dfrac{2}{9}$

23. 8, 16, where equilibrium point is (4, 8)

25. $\dfrac{128}{3}$, 8, where equilibrium point is (4, 4)

27. \$1416.40

Chapter 6 Review Exercises

1. $\dfrac{1}{10}x^{10} + C$ **2.** $\dfrac{2}{3}x^{1.5} + C$

3. $-2y^{-1} + C$ **4.** $\dfrac{\sqrt{3}}{3}y^3 + C$

5. $2x^3 + 4x^2 + C$ **6.** $x^2 - 3\ln|x| + C$

7. $\dfrac{2}{5}t^{5/2} + 2t^{1/2} + C$ **8.** $\dfrac{1}{3}e^{3x} + C$

9. $-e^{-x} - \dfrac{1}{5}e^{-5x} + C$ **10.** $\dfrac{2}{21}(2x + 1)^{21} + C$

11. $(2x^3 + 1)^{10} + C$ **12.** $\dfrac{2}{3}\sqrt{x^3 + 1} + C$

13. $\dfrac{1}{2}\ln|x^2 + 2x| + C$ **14.** $\dfrac{1}{2}e^{x^2+5} + C$

15. $-\dfrac{5}{7}(x^2 + 1)^{-7} + C$ **16.** $-2e^{-\sqrt{x}} + C$

17. $\ln(e^x + 1) + C$ **18.** $\dfrac{1}{4}(\ln x)^4 + C$

19. $n = 10$: 1.77, 1.57; $n = 100$: 1.6767, 1.6567; $n = 1000$: 1.667667, 1.665667;

$$\int_0^1 (2x^2 + 1)\,dx$$

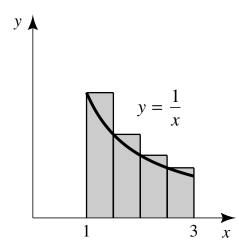
20.

n	Left-Hand Sum	Right-Hand Sum
10	1.2025	1.3025
100	1.2450	1.2550
1000	1.2495	1.2505

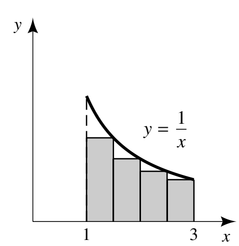
$$\int_0^1 (x^3 + 1)\,dx$$

21. $\dfrac{9\pi}{4}$

22. 1.283. The area of the rectangles is the left-hand sum and is greater than the area under the curve.



23. 0.95. The area of the rectangles is the right-hand sum and is less than the area under the curve.



24. 52 **25.** −8 **26.** 8

27. $\dfrac{1}{2}e^4 - \dfrac{1}{2}e^2 + \ln 2 \approx 24.3$

28. $\dfrac{1}{5}(2^{10} - 1) = 204.6$ **29.** $\dfrac{1}{3}(e^9 - e^1) \approx 2700.1$

30. $\dfrac{1}{2}\ln 5 \approx 0.8$ **31.** $3 - \sqrt{3} \approx 1.26795$

32. $\frac{1}{2}\int_0^2 e^{t^4}\,dt$ **33.** 0 **34.** $\frac{1}{2}\ln 3$

35. 9 **36.** $\frac{8}{3}$ **37.** $\frac{8}{3}$

38. 1 **39.** $\frac{20}{3}$ **40.** 72

41. $\frac{32}{3}$ **42.** 9 **43.** $\frac{2}{3}$

44. 13 **45.** $4\ln 2 - 2\ln 3$

46. $\frac{38}{3}$ **47.** $x^2 - 10x$

48. $x^3 + x + 1000$ **49.** \$98,419.70

50. 1.96 thousand if $S(0) = 0$

51. 34, 24 million **52.** 6906

53. 10.82 **54.** 17,183

55. $x_0 = 2$, $p_0 = 6$ **56.** $\frac{16}{3}$

57. 6 **58.** $L = \frac{3}{5}$

59. There is a greater inequality of income distribution in the United Kingdom than there is in Norway.

60. There is a greater inequality of income distribution for females than there is for males.

61. $\$250(1 - e^{-0.4}) = \$82.42$

62. $\frac{\$250}{3}\left(1 - e^{-1.2}\right) = \$58.23$

63. $\frac{\$250}{3}\left(e^{0.8} - e^{-0.4}\right) = \$129.59$

64. $\frac{\$5000}{9}\left(1 - e^{-0.9}\right) = \$329.68$ thousand

65. \$1189.12

Chapter 7

Section 7.1

1. $\left(x - \frac{1}{2}\right)e^{2x} + C$

3. $\frac{3}{4}e^4 + \frac{1}{4}$ **5.** 2

7. $2x\sqrt{1+x} - \frac{4}{3}(1+x)^{3/2} + C$

9. $\frac{1}{11}x(1+x)^{11} - \frac{1}{132}(1+x)^{12} + C$

11. $-\frac{x}{x+2} + \ln|x+2| + C$

13. $\frac{2}{9}x^3(1+x^3)^{3/2} - \frac{4}{45}(1+x^3)^{5/2} + C$

15. $\frac{x^2}{22}\left(1+x^2\right)^{11} - \frac{1}{264}\left(1+x^2\right)^{12} + C$

17. $\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{2}e^{2x} + C$

19. $\frac{1}{2}(3e^4 + 1)$ **21.** 111 **23.** \$26,306.67

25. \$36.47 **27.** $\frac{1000}{e-1} \approx 582$

29. Let $u = (\ln x)^n$ and $dv = dx$. Then $du = \frac{n}{x}(\ln x)^{n-1}\,dx$, $v = x$, and

$$\begin{aligned}\int (\ln x)^n\,dx &= \int u\,dv\\ &= uv - \int v\,du\\ &= x(\ln x)^n - \int nx(\ln x)^{n-1}\,dx\end{aligned}$$

31. Let $u = f(x)$ and $dv = dx$. Then $du = f'(x)\,dx$, $v = x$, and

$$\begin{aligned}\int f(x)\,dx &= \int u\,dv\\ &= uv - \int v\,du\\ &= xf(x) - \int xf'(x)\,dx\end{aligned}$$

Section 7.2

1. $\frac{2}{9}\left[\frac{1}{5}(3x+2)^{5/2} - \frac{2}{3}(3x+2)^{3/2}\right] + C$

3. $\frac{1}{2}x\sqrt{x^2-9} - \frac{9}{2}\ln\left|x + \sqrt{x^2-9}\right| + C$

5. $\ln\left|\frac{\sqrt{1-x}-1}{\sqrt{1-x}+1}\right| + C$ **7.** $\frac{1}{3}\ln\left|\frac{2x+1}{2x+4}\right| + C$

9. $\frac{1}{3}(x^2-9)^{3/2} + C$ **11.** $\frac{1}{3x} + \frac{1}{9}\ln\left|\frac{x-3}{x}\right| + C$

13. $-\frac{x}{\sqrt{x^2+16}} + \ln\left|x + \sqrt{x^2+16}\right| + C$

15. $\frac{1}{3}\ln\left|\frac{x^3}{2x^3+1}\right| + C$ **17.** $-\frac{\sqrt{1-4x^2}}{x} + C$

19. $\frac{1}{18}x\sqrt{9x^2-1} + \frac{1}{54}\ln\left|x + \frac{1}{3}\sqrt{9x^2-1}\right| + C$

21. $x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x - 6x + C$

23. $\frac{1}{2}(e^x+1)^2 - 2(e^x+1) + \ln|e^x+1| + C$

25. $-\frac{1}{2}e^{-2x}\left(x^2+x+\frac{1}{2}\right)+C$

27. $R(x)=\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}+\frac{4}{15}$

29. $\frac{10t}{9\sqrt{t^2+9}}$ **31.** 4.8 **33.** 327

35. $y_3(x)=-1$. Thus, $y_1(x)=y_2(x)-1$.

Section 7.3

1. $T_2=5$, $S_2=4$, exact $=4$

3. To four decimal places, $T_2=1.1667$, $S_2=1.1111$, exact $=1.0986$

5. To four decimal places, $T_2=0.6830$, $S_2=0.7440$, exact $=0.7854$

7. To six decimal places, $T_2=1.571583$, $S_2=1.475731$, exact $=1.462652$

9. $T_4=4.25$, $S_2=S_4=4$, exact $=4$

11. To four decimal places, $T_4=1.1167$, $S_4=1.1$, exact $=1.0986$

13. To four decimal places, $T_4=0.7489$, $S_4=0.7709$, exact $=0.7854$

15. To six decimal places, $T_4=1.490679$, $S_4=1.463711$, exact $=1.462652$

17. $T_{100}=4.0004$, $S_{100}=4$, exact $=4$

19. $T_{100}=1.098642$, $S_{100}=1.098612294$, exact to nine decimal places $=1.098612289$

21. $T_{100}=0.785104$, $S_{100}=0.785283$, exact to six decimal places $=0.785398$

23. $T_{100}=1.462697$, $S_{100}=1.462652$, exact to six decimal places $=1.462652$

25. a. $T_4=5.8$; **b.** $S_4\approx 5.87$

27. a. $T_4=50$; **b.** $S_4=52$

29. a. $T_6=4.325$; **b.** $S_6=4.35$

31. a. $\frac{1}{75}$; **b.** 0

33. a. 0.32; **b.** 0.00043

35. $78,818

37. Yes. When the graph of the function is concave down, the line segment connecting the two endpoints of any two subintervals lies below the curve. Thus, the trapezoid rule always gives an underestimate. When the graph of the function is concave up, the line segment connecting the two endpoints of any subinterval lies above the curve. Thus, the trapezoid rule always gives an overestimate.

39. We have $\Delta x=\frac{b-0}{2}=\frac{b}{2}$. Then

$$\begin{aligned} S_2&=\frac{b}{6}\left[f(0)+4f\left(\frac{b}{2}\right)+f(b)\right]\\ &=\frac{b}{6}\left[0+4\cdot\frac{b^3}{8}+b^3\right]\\ &=\frac{b^4}{4}\end{aligned}$$

And

$$\int_0^b x^3\,dx=\frac{1}{4}x^4\Big|_0^b=\frac{1}{4}(b^4-0)=\frac{b^4}{4}$$

Thus, $S_2=\int_0^b x^3\,dx$.

41. Left-hand sum is $f(x_0)\Delta x+f(x_1)\Delta x+\cdots+f(x_{n-1})\Delta x$. Right-hand sum is $f(x_1)\Delta x+f(x_2)\Delta x+\cdots+f(x_n)\Delta x$. $\frac{(\text{left-hand sum}+\text{right-hand sum})}{2}$ is $\frac{f(x_0)+f(x_1)}{2}\Delta x+\frac{f(x_1)+f(x_2)}{2}\Delta x+\cdots+\frac{f(x_{n-1})+f(x_n)}{2}\Delta x=T_n$

Section 7.4

1. $\frac{1}{2}$ **3.** Diverges

5. 100 **7.** Diverges

9. 1 **11.** $-\frac{1}{2}$

13. $\frac{1}{3}$ **15.** $-\frac{1}{2}$

17. 0 **19.** 0

21. Approximately 667 engine manifolds

23. $20 million **25.** 200 million

27. 2000 tons **29.** 1000

31. $1250 **33.** $555.56

35. a. No, since $P_V(\infty)=\$100{,}000$; **b.** yes, since $P_V(\infty)=\$100{,}000$

37. a. No, since $P_V(\infty)=\$600{,}000$; **b.** yes, since $P_V(\infty)=\$600{,}000$

39. $\int_{-\infty}^{\infty}x\,dx=\int_{-\infty}^{0}x\,dx\int_{0}^{+\infty}x\,dx$. Since $\int_{-\infty}^{0}x\,dx$ diverges, $\int_{-\infty}^{\infty}x\,dx$ diverges. Notice that

$$\begin{aligned}&\lim_{a\to-\infty}\int_a^0 f(x)\,dx+\lim_{b\to\infty}\int_0^b f(x)\,dx\\ &\neq\lim_{a\to\infty}\int_{-a}^{a} f(x)\,dx\end{aligned}$$

41. Since

$$\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \lim_{b\to\infty}\int_1^b x^{-1/2}\,dx$$
$$= \lim_{b\to\infty}(2\sqrt{b}-2)$$

diverges, and $f(x) \geq \frac{1}{\sqrt{x}}$ on $[1, \infty)$, we have

$$\int_1^\infty f(x)\,dx \geq \int_1^\infty \frac{1}{\sqrt{x}}\,dx$$

Thus, $\int_1^\infty f(x)\,dx$ diverges.

43.

$$\int_{-\infty}^c f(x)\,dx + \int_c^\infty f(x)\,dx$$
$$= \lim_{a\to-\infty}\int_a^c f(x)\,dx + \lim_{b\to\infty}\int_c^b f(x)\,dx$$
$$= \lim_{a\to-\infty}\int_a^0 f(x)\,dx + \int_0^c f(x)\,dx$$
$$+ \int_c^0 f(x)\,dx + \lim_{b\to\infty}\int_0^b f(x)\,dx$$
$$= \lim_{a\to-\infty}\int_a^0 f(x)\,dx + \lim_{b\to\infty}\int_0^b f(x)\,dx$$
$$= \int_{-\infty}^\infty f(x)\,dx$$

Chapter 7 Review Exercises

1. $\frac{1}{25}e^{5x}(5x-1)+C$

2. $\frac{2}{3}x\sqrt{5+3x} - \frac{4}{27}(5+3x)^{3/2}+C$

3. $\frac{1}{6}x^6\ln x - \frac{1}{36}x^6 + C$

4. $-\frac{x}{x+1} + \ln|x+1| + C$

5. $-\frac{1}{5}\left(9-x^2\right)^{3/2}\left(x^2+6\right)+C$

6. $2\sqrt{x}\ln x - 4\sqrt{x}+C$

7. $\frac{x}{2} - \frac{3}{4}\ln|2x+3|+C$

8. $\frac{1}{4(x+4)} - \frac{1}{16}\ln\left|\frac{x+4}{x}\right|+C$

9. $-\frac{\sqrt{4-x^2}}{4x}+C$

10. $\sqrt{9-x^2} - 3\ln\left|\frac{3+\sqrt{9-x^2}}{x}\right|+C$

11. $\frac{1}{2}x\sqrt{x^2+1} + \frac{1}{2}\ln|x+\sqrt{x^2+1}|+C$

12. $x - \ln(1+e^x)+C$

13. 10 **14.** Diverges

15. $\frac{2}{3}$ **16.** Diverges

17. a. 45; **b.** $\frac{100}{3}$; **c.** correct $= 32$

18. To four decimal places:
a. 1.5375; **b.** 1.425;
c. correct $= 1.3863$

19. To four decimal places:
a. 35.3125; **b.** 32.0833; **c.** correct $= 32$

20. To four decimal places:
a. 1.4281; **b.** 1.3916;
c. correct $= 1.3863$

21. \$555,600

Chapter 8

Section 8.1

1. a. 13; **b.** 10; **c.** 12

3. a. 2; **b.** 13; **c.** 2

5. a. $\frac{1}{3}$; **b.** -1; **c.** $\frac{1}{3}$

7. a. $\sqrt{6}$; **b.** 3; **c.** 2

9. a. 1; **b.** $\frac{1}{2}$; **c.** $\frac{1}{2}$

11. All x and y **13.** $\{(x, y) | x \neq -y\}$

15. $\{(x, y) | x^2 + y^2 \leq 16\}$ **17.** 3

19. Plane with intercepts (0, 0, 4), (0, 6, 0), (12, 0, 0)

21. Plane with intercepts (0, 0, 2), (0, 4, 0), (10, 0, 0)

23. Horizontal plane three units above xy-plane

25. Sphere of radius 6 centered at (0, 0, 0)

27. (f) **29.** (c) **31.** (d)

33. a. $z_0 = 0$: $x = 0 = y$;
b. $z_0 = 1$: $x^2 + y^2 = 1$;
c. $z_0 = 4$: $x^2 + y^2 = 4$;
d. $z_0 = 9$: $x^2 + y^2 = 9$;
e. surface is a bowl

35. **a.** $z_0 = 1$: $x = 0 = y$;
b. $z_0 = 0$: $x^2 + y^2 = 1$;
c. $z_0 = -3$: $x^2 + y^2 = 4$;
d. $z_0 = -8$: $x^2 + y^2 = 9$;
e. surface is a mountain

37. $S(x, y) = 2x^2 + 4xy$, $S(3, 5) = 78$

39. $R(x, y) = 1400x + 802y - 12x^2 - 4xy - 0.5y^2$, 80,400

41. $P(x, y) = -15000 + 1350x + 801.5y - 12x^2 - 4xy - 0.5y^2$, 60,300

43. $A(m, t) = 1000\left(1 + \dfrac{0.08}{m}\right)^{mt}$, \$1489.85

45. **a.** $\{(x, y) | y \neq 0\}$; **b.** 200; **c.** 50

47. **a.** $\{(d, P) | P \geq 0\}$; **b.** 24π; **c.** 8π

49. When $y = mx$,

$$\begin{aligned}\lim_{x\to 0} f(x, mx) &= \lim_{x\to 0} \frac{mx^3}{x^4 + m^2x^2}\\ &= \lim_{x\to 0} \frac{mx}{x^2 + m^2}\\ &= \frac{0}{0 + m^2} = 0\end{aligned}$$

for all m. But this does not imply that f is continuous at $(0, 0)$.

Section 8.2

1. $f_x = 2x$, $f_y = 2y$, 2, 6

3. $f_x = 2xy - 3x^2y^2$, $f_y = x^2 - 2x^3y$, -8, -3

5. $f_x = \dfrac{\sqrt{y}}{2\sqrt{x}}$, $f_y = \dfrac{\sqrt{x}}{2\sqrt{y}}$, $\dfrac{1}{2}$, $\dfrac{1}{2}$

7. $f_x = \dfrac{xy^2}{\sqrt{1 + x^2y^2}}$, $f_y = \dfrac{x^2y}{\sqrt{1 + x^2y^2}}$, 0, 0

9. $f_x = 2e^{2x+3y}$, $f_y = 3e^{2x+3y}$, $2e^5$, $3e^5$

11. $f_x = ye^{xy} + xy^2e^{xy}$, $f_y = xe^{xy} + x^2ye^{xy}$, $2e$, $2e$

13. $f_x = \dfrac{1}{x + 2y}$, $f_y = \dfrac{2}{x + 2y}$, 1, 2

15. $f_x = ye^{xy}\ln x + \dfrac{1}{x}e^{xy}$, $f_y = xe^{xy}\ln x$, 1, 0

17. $f_x = \dfrac{-1}{x^2y}$, $f_y = \dfrac{-1}{xy^2}$, $-\dfrac{1}{2}$, $-\dfrac{1}{4}$

19. $f_x = \dfrac{y^2 - x^2 + 2xy}{(x^2 + y^2)^2}$, $f_y = \dfrac{y^2 - x^2 - 2xy}{(x^2 + y^2)^2}$, $\dfrac{1}{25}$, $-\dfrac{7}{25}$

21. $f_{xx} = 2y^4$, $f_{xy} = f_{yx} = 8xy^3$, $f_{yy} = 12x^2y^2$

23. $f_{xx} = 4e^{2x-3y}$, $f_{xy} = f_{yx} = -6e^{2x-3y}$, $f_{yy} = 9e^{2x-3y}$

25. $f_{xx} = -\dfrac{1}{4}\dfrac{\sqrt{y}}{x^{3/2}}$, $f_{xy} = f_{yx} = \dfrac{1}{4\sqrt{xy}}$, $f_{yy} = -\dfrac{1}{4}\dfrac{\sqrt{x}}{y^{3/2}}$

27. $f_{xx} = 2e^y$, $f_{xy} = f_{yx} = 2xe^y$, $f_{yy} = x^2e^y$

29. $f_{xx} = \dfrac{y^2}{(x^2 + y^2)^{3/2}}$, $f_{xy} = f_{yx} = \dfrac{-xy}{(x^2 + y^2)^{3/2}}$, $f_{yy} = \dfrac{x^2}{(x^2 + y^2)^{3/2}}$

31. $f_x = yz$, $f_y = xz$, $f_z = xy$

33. $f_x = \dfrac{x}{\sqrt{x^2 + y^2 + z^2}}$, $f_y = \dfrac{y}{\sqrt{x^2 + y^2 + z^2}}$, $f_z = \dfrac{z}{\sqrt{x^2 + y^2 + z^2}}$

35. $f_x = e^{x+2y+3z}$, $f_y = 2e^{x+2y+3z}$, $f_z = 3e^{z+2y+3z}$

37. $f_x = \dfrac{1}{x + 2y + 5z}$, $f_y = \dfrac{2}{x + 2y + 5z}$, $f_z = \dfrac{5}{x + 2y + 5z}$

39. Complementary **41.** Competitive

43. Competitive

45. $A_r = 1000t(1 + r/12)^{12t-1}$. This gives the rate of change of the amount with respect to changes in the interest rate r.

47. $\dfrac{\partial V}{\partial r} = 4\pi r^2\left(1 + \dfrac{c}{r}N^{1/3}\right)^2$. This gives the rate of change of the detection volume with respect to changes in the detection radius r. $\dfrac{\partial V}{\partial N} = \dfrac{4\pi cr^2}{3N^{2/3}}\left(1 + \dfrac{c}{r}N^{1/3}\right)^2$. This gives the rate of change of the detection volume with respect to changes in the number of prey N.

49. $\dfrac{\partial L}{\partial r} = -2p\pi r\left(1 - \dfrac{p\pi r^2}{n}\right)^{n-1}$. This gives the rate of change of light with respect to changes in the radius r of the leaves. $\dfrac{\partial L}{\partial p} = -\pi r^2\left(1 - \dfrac{p\pi r^2}{n}\right)^{n-1}$. This gives the rate of change of light with respect to changes in the total density p of the leaves.

51. $\dfrac{\partial A}{\partial S}(S, D) = 0.11297S^{0.43}D^{-0.73}$, $\dfrac{\partial A}{\partial D}(S, D) = -0.05767S^{1.43}D^{-1.73}$

53. $\dfrac{\partial x}{\partial A} > 0$ means that there is more demand when advertising is at a higher level. $\dfrac{\partial x}{\partial P} < 0$ means that there is less demand when advertising is at a higher price.

55. **a.** The instantaneous rate of change of f with respect to x at the point $x = 1$ and $y = 2$ is 3.
b. The instantaneous rate of change of f with respect to y at the point $x = 1$ and $y = 2$ is -5.

Section 8.3

1. Relative minimum at $(5, -2)$

3. Relative maximum at $(2, 1)$

5. Saddle at (1, 1)

7. Relative maximum at $(-1, -2)$

9. Relative minimum at (0, 0)

11. Saddle at (0, 2)

13. Saddle at (0, 0), relative minimum at (1, 1)

15. Saddle at (0, 0), relative maximum at (1/6, 1/12)

17. Relative maximum at (1, 1)

19. Saddle at (0, 0)

21. Since $f(x, y) = x^4y^4 \geq 0 = f(0, 0)$ for all x and all y, f has a relative minimum at (0, 0). Since $f_{xx} = 12x^2y^4$, $f_{yy} = 12x^4y^3$, and $f_{xy} = 16x^3y^3$, $\Delta(0, 0) = 0$.

23. If $y = x$, then $f(x, y) = x^3x^3 = x^6$, and $f(x, x)$ increases as x increases. But if $y = -x$, $f(x, -x) = -x^6$, and $f(x, -x)$ decreases as x increases. Since $f_{xx} = 6xy^3$, $f_{yy} = 6x^3y$, and $f_{xy} = 9x^2y^2$, $\Delta(0, 0) = 0$.

25. Increasing in all cases.

27. Increasing in the third case and decreasing in all the others.

29. $x = 1, y = 2, z = 4$ **31.** Base 4×4 and height 6

33. $x = 200, y = 100$ **35.** $x = 18, y = 8$

37. $x = 9, y = 4$

39. $x = 200, y = 100, z = 700$

41. $x = 1600, y = 1000$ **43.** $x = 2100, y = 0$

45. $N \approx 2.463$, $P \approx 2.399$

47. (1) $x = (P/a)^{1/b}y^{1-1/b}$; (2) $C = C(y) = p_1(P/a)^{1/b}y^{1-1/b} + p_2y$; (3) $C'(y) = 0$ implies that $y = \left(\frac{p_1}{p_2}\right)^b \left(\frac{1}{b} - 1\right)^b \frac{P}{a}$; (4) $x = \left(\frac{p_1}{p_2}\right)^{b-1} \left(\frac{1}{b} - 1\right)^{b-1} \frac{P}{a}$; (5) $C = C(P) = \left(\frac{1}{b} - 1\right)^b \left(\frac{p_1}{p_2}\right)^b \left(\frac{p_2}{1-b}\right) \frac{P}{a}$

Section 8.4

1. Minimum at $x = 1, y = 3$

3. Maximum at $x = -3, y = -1$

5. Minimum at $x = 1, y = 1$

7. Maximum at $x = -1, y = -3$

9. Minimum at $x = 2, y = 2$

11. Maximum at $x = 4, y = 4$

13. Minimum at $x = 4, y = 2, z = 1$

15. Maximum at $x = 2, y = 2, z = 1$

17. 10 and 10 **19.** 12, 12, and 12

21. $r = 2, h = 4$ **23.** $x = 200, y = 100$

25. $6 \times 6 \times 9$ **27.** $x = 2000, y = 4000$

29. $x = 1, y = 2, z = 4$

31. $x = 200, y = 100, z = 700$

33. Let $F(x, y, \lambda) = p_1x + p_2y + \lambda f(x, y) - \lambda P_1$; then

$$\begin{cases} 0 = F_x = p_1 + \lambda f_x \\ 0 = F_y = p_2 + \lambda f_y \end{cases}$$

which implies that

$$\lambda = -\frac{p_1}{f_x} = -\frac{p_2}{f_y}$$

That is, $\dfrac{f_x}{p_1} = \dfrac{f_y}{p_2}$.

Section 8.5

1. 1 **3.** -6 **5.** $\frac{1}{15}$

7. $\frac{-11}{1250}$ **9.** $\frac{1}{20}$ **11.** $\frac{-1}{20}$

13. 0.05 **15.** $\frac{e}{10} \approx 0.2718$

17. 6.05 **19.** 5.036 **21.** 3.11

23. 8.9 **25.** 24.57 **27.** 1.08

29. \$2000 **31.** 118.75 **33.** 0.70 ft^3

35. 0.9573 m^2 **37.** 0.7116

Section 8.6

1. $\frac{15}{4}$ **3.** $\frac{3}{4}$ **5.** 8

7. 2.5 **9.** 1 **11.** 35

13. $\frac{4}{15}(2\sqrt{2} - 1)$ **15.** $\ln 2 \ln 3$

17. $(e^2 - 1)(e^3 - 1)$ **19.** $\frac{7}{24}$

21. $\frac{e^4 - 1}{2}$ **23.** $\frac{1}{2}$ **25.** $\frac{1}{3}$

27. $\frac{1}{8}$ **29.** $\frac{1}{4}$ **31.** $\frac{1}{40}$

33. $\frac{1}{24}$ **35.** $-\frac{2}{3}$ **37.** $\frac{7}{6}$

39. $\frac{3}{2} - \ln 2$ **41.** 4 **43.** 19,000

45. $\frac{3627}{532}$ **47.** $e - 1$ **49.** 10,000

51.

n	LLSUM	URSUM
20	0.9025	1.1025
40	0.950625	1.050625
100	0.9801	1.0201

Chapter 8 Review Exercises

1. $-1, 7, 4$ **2.** $0, \frac{1}{5}, -\frac{8}{5}$

3. (1) All x and y; (2) $\{(x, y) | y \neq 0\}$

4. $2\sqrt{11}$

5. $z_0 = 16$: $x^2 + y^2 = 0$ or the single point $(0, 0)$; $z_0 = 12$: $x^2 + y^2 = 4$, circle of radius 2 centered at $(0, 0)$; $z_0 = 7$: $x^2 + y^2 = 9$, circle of radius 3 centered at $(0, 0)$.

6. $f_x(x, y) = 1 + 2xy^5$, $f_y(x, y) = 5x^2y^4$

7. $f_x(2, 1) = 5$, $f_y(2, 1) = 20$

8. $f_x = -ye^{-xy} + \dfrac{1}{x}$, $f_y = -xe^{-xy} + \dfrac{1}{y}$

9. $f_{xx} = 2y^5$, $f_{yy} = 20x^2y^3$, $f_{xy} = 10xy^4$

10. $f_{xx} = y^2e^{-xy} - \dfrac{1}{x^2}$, $f_{yy} = x^2e^{-xy} - \dfrac{1}{y^2}$, $f_{xy} = -e^{-xy} + xye^{-xy}$

11. $f_x = -yze^{-xyz} + y^2z^3$, $f_y = -xze^{-xyz} + 2xyz^3$, $f_z = -xye^{-xyz} + 3xy^2z^2$

12. Relative minimum at $(0, 0)$

13. Relative maximum at $(0, 0)$

14. Relative maximum at $(-2, -1)$

15. Relative minimum at $(2, -3)$

16. Saddle at $(0, 0)$, relative maximum at $(-1, -1)$

17. Saddle at $(-1, -1)$, relative minimum at $(3, 3)$

18. Saddle at $(5, 3)$, relative maximum at $(5, -1)$

19. Saddle at $(0, 0)$, relative maximum at $(2/3, 1/9)$

20. Since $f_{xx}(0, 0) = 2a$ and $\Delta(0, 0) = 4(ac - b^2)$, the result follows.

21. Minimum at $(4, -3)$ 22. Maximum at $(1/4, 1/2)$

23. 2.8 24. 3.2 25. 4.99

26. 24 27. 80 28. $\dfrac{5}{12}$

29. $\dfrac{7}{24}$ 30. $\dfrac{4}{5}$ 31. $\dfrac{3}{20}$

32. $\dfrac{1}{2}$ 33. $\dfrac{13}{20}$ 34. $\dfrac{13}{20}$

35. $\dfrac{1}{6}$ 36. 80

37. $Q_L = 2.014(0.99)L^{-0.01}E^{0.47}$, rate of change of GNP with respect to employment. $Q_E = 2.014(0.47)L^{0.99}E^{-0.53}$, rate of change of GNP with respect to energy consumption

38. $\dfrac{\partial Q}{\partial T} = (0.02)(5868)X^{0.17}Y^{0.13}T^{-0.98}$
The instantaneous rate of change of onion production with respect to time. Units are 1000 cwt per year.

39. Approximately 436 sq. ft. and 100 trees

40. 1091

41. $C_x = 2x - y$, $C_y = 2y - x$

42. $C_{xx} = 2$, $C_{yy} = 2$, $C_{xy} = -1$

43. $A_P = e^{rt}$, $A_r = tPe^{rt}$, $A_t = Pre^{rt}$

44. $2 \times 2 \times 4$ 45. $x = 10$, $y = 12$

46. $x = 11$, $y = 14$

47. 11,000 of regular and 9000 of mint

48. 500 of each 49. 170 cubic inches

50. $\dfrac{\partial h}{\partial e} > 0$ means that the individual firm has a higher harvest rate with a higher effort rate. $\dfrac{\partial h}{\partial N} < 0$ means that more participating firms result in less of a harvest. $\dfrac{\partial h}{\partial X} > 0$ means that larger resource stock size results in more of a harvest.

Appendix

Section A.1

1. x^{10} 3. x^{15} 5. $(x + 2)^5$

7. x^5 9. $(x + 1)^3$ 11. 1

13. 1/2 15. 10 17. 1

19. a^3/b 21. xyz 23. y^6/x^{12}

25. −4 27. −2 29. 7

31. −3 33. 5/2 35. −2/5

37. 2 39. 4 41. 0.1

43. 1/216 45. 16 47. 256

49. 10,000/81 51. 9/4 53. $2x^2$

55. ab 57. $6\sqrt{2}a^2b^{3/2}c^{5/2}$

59. $2a^2b^3$ 61. $2a^3b^2/c^4$ 63. a^2b^2

65. $x^{-1/2} + x^{1/2}$ 67. $(x + 1)^{2/3}$

69. $x^{2/3}$ 71. $(x/y)^{1/2}$ 73. $(x^2 + 1)^{17/12}$

75. $1/x^{1/3}$ 77. ab 79. b

81. $(x + 1)(x + 2)$ 83. $(x + 1)^5$

85. $\sqrt{5}/5$ 87. $\sqrt{2} - 1$

89. $\sqrt[3]{25}/5$ 91. $-(1 + \sqrt{3})^2/2$

93. $1/(\sqrt{5} + \sqrt{2})$ 95. $1/(\sqrt{10} + 2)$

Section A.2

1. $3x^2 + 3x + 2$ 3. $-3x^2 - x - 1$

5. $x^4 - x^3 + x^2 - x + 4$ 7. $x^3 - x^2 + x + 2$

9. x^3+3x^2+5x+3 **11.** $2x^3-3x^2+7x-3$

13. x^4+4x^2+3 **15.** $(x-3)(x^2+3x+9)$

17. $\left(x-\frac{1}{2}\right)\left(x^2+\frac{1}{2}x+\frac{1}{4}\right)$

19. $(3x-2)(9x^2+6x+4)$

21. $\left(\frac{1}{2}x-3\right)\left(\frac{1}{4}x^2+\frac{3}{2}x+9\right)$

23. $\left(\frac{1}{4}x-\frac{1}{3}\right)\left(\frac{1}{16}x^2+\frac{1}{12}x+\frac{1}{9}\right)$

25. $(ax-b)(a^2x^2+abx+b^2)$

27. $(x-4)(x+4)$ **29.** $(4x-1)(4x+1)$

31. $\left(x^2+\frac{1}{4}\right)\left(x+\frac{1}{2}\right)\left(x-\frac{1}{2}\right)$

33. $\left(\frac{1}{4}x^2+1\right)\left(\frac{1}{2}x+1\right)\left(\frac{1}{2}x-1\right)$

35. $(x^2+2)(x+\sqrt{2})(x-\sqrt{2})$

37. $\left(\frac{1}{2}x^2+1\right)\left(\frac{1}{\sqrt{2}}x+1\right)\left(\frac{1}{\sqrt{2}}x-1\right)$

39. $\frac{x+1}{x-1}$ **41.** $\frac{x}{1-x}$

43. $\frac{x-1}{x+2}$ **45.** $\frac{x-1}{x+4}$

47. $2x+1$ **49.** $\frac{x^2+1}{x}$

51. $\frac{x-1}{x+1}$ **53.** $\frac{1}{x+1}$

55. $-\frac{x}{(x-1)(x+1)}$ **57.** $\frac{4}{(x^2+4)(x+2)(x-2)}$

59. $\frac{\sqrt{x}-1}{x-1}$ **61.** $\frac{\sqrt{x}-\sqrt{y}}{x-y}$

63. $\sqrt{y}+3$ **65.** $(a+1)(\sqrt{a}+1)$

67. $\frac{1}{\sqrt{x}+1}$ **69.** $\frac{1}{\sqrt{x+h}+\sqrt{x}}$

Section A.3

1. -4 **3.** $11/7$ **5.** 2

7. $-19/10$ **9.** $13/5$ **11.** ± 2

13. $\pm 3/2$ **15.** ± 2 **17.** $\pm 1/2$

19. $\pm 3/5$ **21.** ± 6 **23.** $\pm 1/6$

25. $\pm b/a$ **27.** -2 **29.** $1, 4$

31. $1/2, -2$ **33.** $-1/3, -2$ **35.** $1/3, -1/2$

37. $1/5, 1$ **39.** $-2/3, -3/2$ **41.** $1/4, 2/3$

43. $1/3$ **45.** $-2/3$ **47.** $1/2$

49. $-1/4$ **51.** ± 2 **53.** $\pm 3/2$

55. $0, 1, -2$ **57.** $x=0, \pm 3$ **59.** -1

61. $\frac{7}{10}\pm\frac{3}{10}\sqrt{5}$ **63.** $\frac{1}{2}\pm\frac{1}{2}\sqrt{5}$

65. $-\frac{2}{3}\pm\frac{\sqrt{19}}{3}$ **67.** $2\pm\sqrt{3}$

69. $\frac{7}{6}\pm\frac{\sqrt{41}}{6}$ **71.** No real solution

73. $1, 0$ **75.** $2, -4/3$ **77.** $0, 4/3$

79. $-1, 9$ **81.** $-1/6, 7/6$ **83.** $5/3, 1$

Section A.4

1. $x<\frac{4}{3}$ **3.** $x<-6$ **5.** $x\le -1$

7. $x\le 7$ **9.** $x<\frac{3}{8}$ **11.** $x\le 1$

13. $-4<x<2$ **15.** $0\le x\le 1$

17. $x<-1$ or $x>3$ **19.** $x\le -\frac{4}{3}$ or $x\ge\frac{8}{3}$

21. $-\frac{19}{12}<x<-\frac{17}{12}$ **23.** $x\le -5$ or $x\ge 10$

25. $1<x<3$ **27.** $x\le -2$ or $x\ge 3$

29. $-2<x<2$

31. $x\le -3$ or $x\ge 1$ **33.** $\frac{1}{2}<x<2$

35. $x\le -3$ or $x\ge\frac{2}{3}$ **37.** $-2<x<1$

39. $x\le 1$ or $x>2$ **41.** $x<-2$ or $x>1$

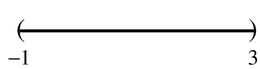
43. $-1\le x<1$ **45.** $-1\le x<0$ or $x\ge 1$

47. $x<-1$ or $0<x<1$ **49.** $x<1$ or $2<x<3$

51. $-1\le x\le 1$ or $x>2$ **53.** $x<0$

55. $-1<x<2$ with $x\ne 0$

Section A.5

1.


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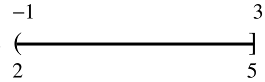

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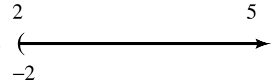

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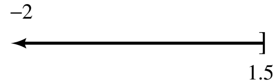
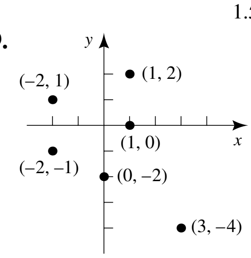


9–19.


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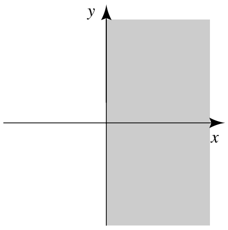

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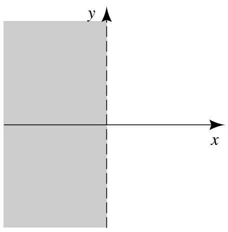

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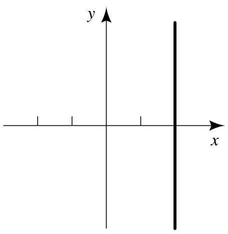

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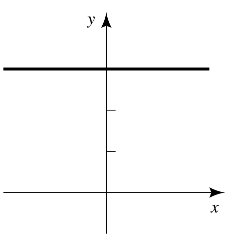

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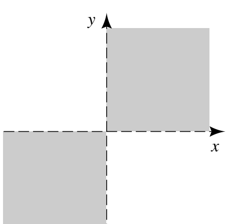

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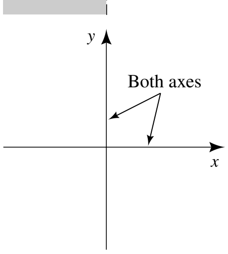
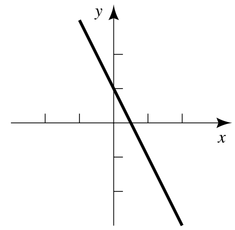


33. $\sqrt{5}$ **35.** $\sqrt{37}$

37. $5\sqrt{2}$ **39.** $\sqrt{2}|b-a|$

41.


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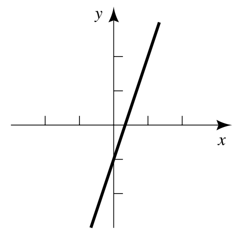

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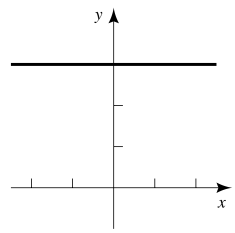

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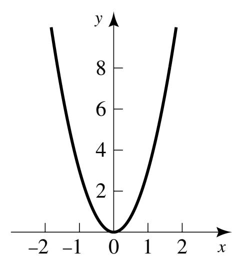

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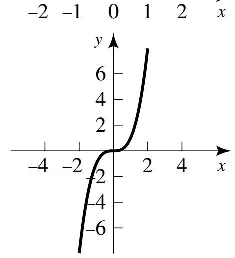


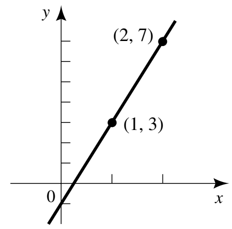
51. 25 miles **53.** 13 miles

55. 10 days **57.** $100x + 150y = 3000$

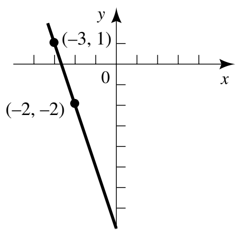
59. $125x + 190y = 800$ **61.** $c = \sqrt{a^2 - b^2}$

Section A.6

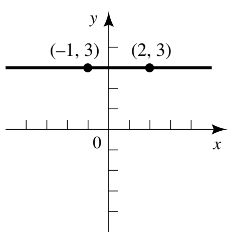
1. 4



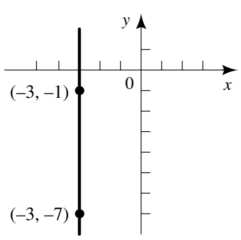
3. -3



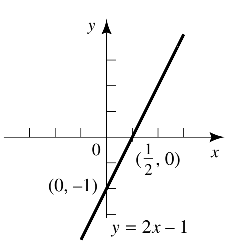
5. 0



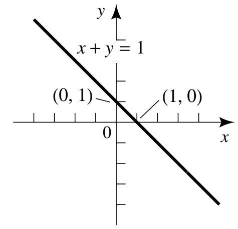
7. Undefined



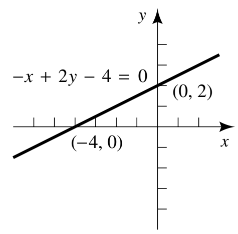
9. $2, \frac{1}{2}, -1$



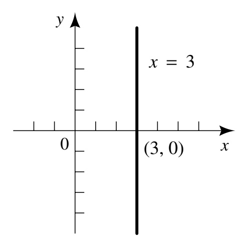
11. $-1, 1, 1$



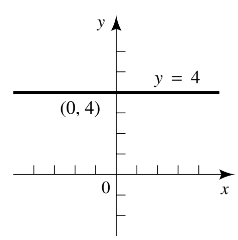
13. $\frac{1}{2}, -4, 2$



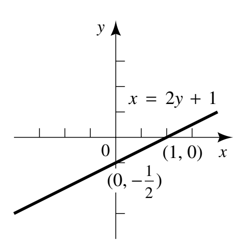
15. Undefined, 3, no y-intercept



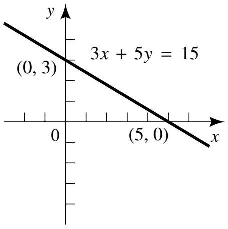
17. 0, no x-intercept, 4



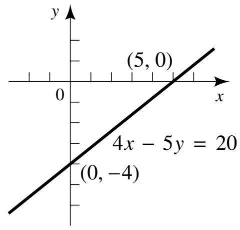
19. $\frac{1}{2}, 1, -\frac{1}{2}$



21. 5, 3



23. 5, −4



25. $y = -3x + 7$

27. $y = -3$

29. $y = -x + 2$

31. $x = -2$

33. $y = -\dfrac{b}{a}x + b$

35. $y = -\dfrac{2}{5}x + \dfrac{11}{5}$

37. $y = 0$

39. $x = 1$

41. $y = 2x + 5$

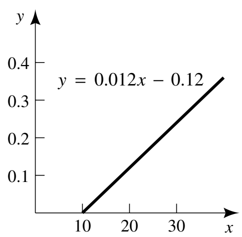
43. $y = -\dfrac{5}{3}x + 4$

45. $y = -\dfrac{2}{5}x + 4$

47. No

49. $0.08x + 0.05y = 576$, \$160

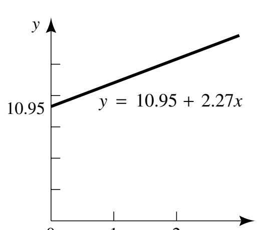
51. $y = 0.012x - 0.12$, 0.36 kilograms



53. Each additional pound of milk per cow per year requires an additional 3 cents in cost per cow per year. \$30.

55. $L - 140 = \dfrac{182}{79}(l - 60)$, $\dfrac{182}{9}$, each 79-mm of increase in the tail brings a 182-mm increase in the total length.

57. For each additional fly on the front legs of the bull, there are (on average) 2.27 additional flies on the bull: no flies on the front legs, implies 10.95 flies (on average) on the bull.

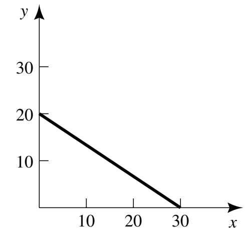


59. a. $100x + 150y = 3000$

b. $-\dfrac{2}{3}$

c. 18. If three chairs are sold, then 18 sofas need to be sold for the profit to be \$3000.

d. 30

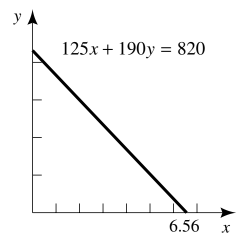


61. a. $125x + 190y = 820$

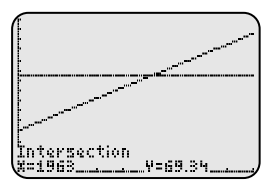
b. $-125/190$

c. 3. If 2 ounces of sardines are consumed, then 3 cups of broccoli need to be eaten to obtain 820 mg of calcium.

d. 6.56



63. 1963

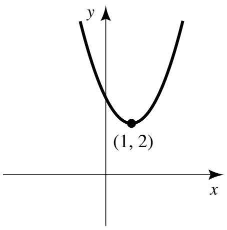


[1958, 1967] × [60, 75]

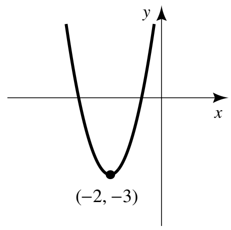
65. If $b \neq 0$, then $ax + by = c$ if and only if $y = -\dfrac{a}{b}x + \dfrac{c}{b}$. This is the straight line with slope $-\dfrac{a}{b}$ and y-intercept $\dfrac{c}{b}$. If $b = 0$, then by assumption $a \neq 0$, and we have $x = \dfrac{c}{a}$ which is a vertical line. Therefore, $ax + by = c$ is a straight line if a and b are not both zero. Conversely, every line is the graph of either the equation $y - y_1 = m(x - x_1)$ if and only if $-mx + y = y_1 - mx_1$ with $a = -m$, $b = 1$, and $c = y_1 - mx_1$, or the equation $x = x_0$ if m fails to exist, in which case $a = 1$, $b = 0$, and $c = x_0$. Therefore, every line is the graph of a linear equation.

Section A.7

1.



3.



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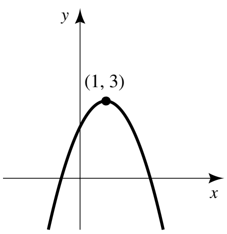

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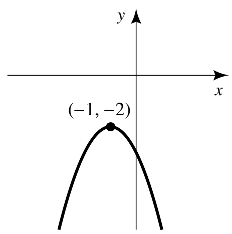

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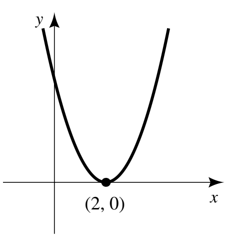
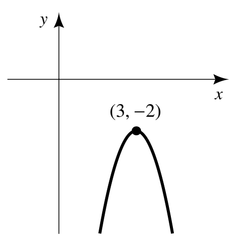
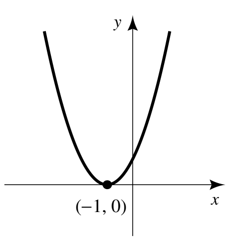


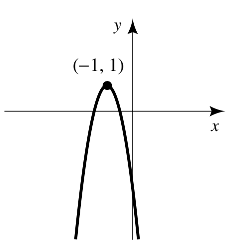
11.



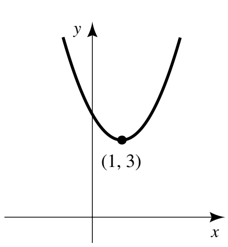
13. Zero at $x = -1$



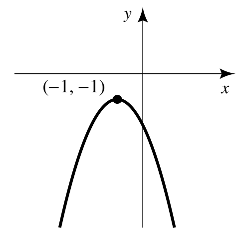
15. Zeros at $x = -1/2, x = -3/2$



17. No zeros



19. No zeros



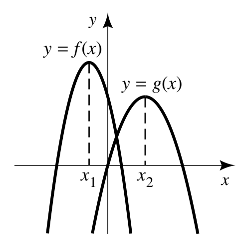
21. $(1, -1)$ **23.** $(2, 2)$

25. **a.** ii; **b.** iii; **c.** iv; **d.** i

27. 20.25

29. Both $p(x) = x^2$ and $q(x) = -x^2$ are quadratics, but $p(x) + q(x) = 0$ and is not a quadratic. Thus, the sum of two quadratics need not be a quadratic.

31. Both $f(x)$ and $g(x)$ are increasing for $x < x_1$; thus, the peak of $f + g$ must occur after x_1. Both $f(x)$ and $g(x)$ are decreasing for $x > x_2$; thus, the peak of $f + g$ must occur before x_2.



33. 0.9, 2.2

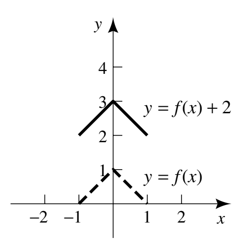
35. Approximately 1.9364358 and 7.6635642

Section A.8

1. 8, $(-\infty, \infty)$, 1 **3.** 9, $(-\infty, \infty)$, 1000

5. $x \neq 3$ **7.** All x **9.** $x \neq 0$

11.



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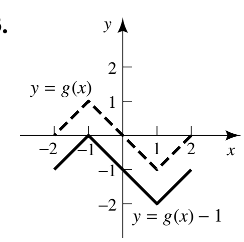

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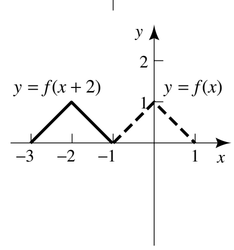

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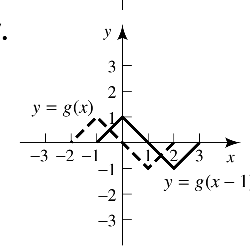

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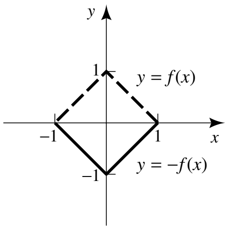

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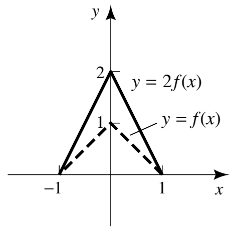

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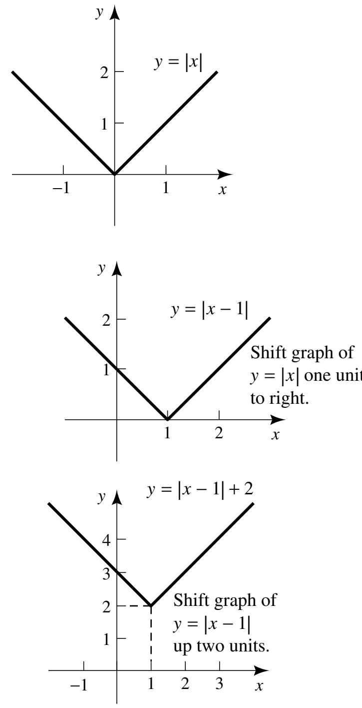

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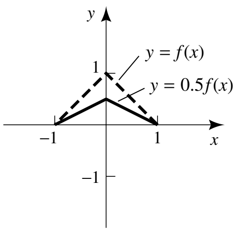

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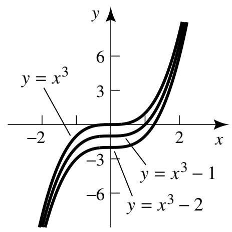

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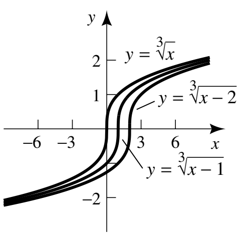

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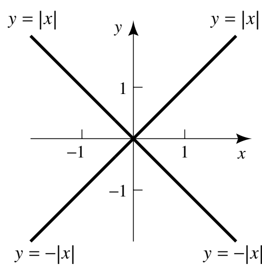

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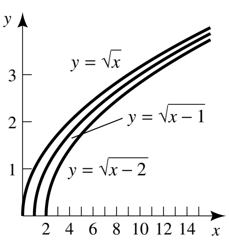
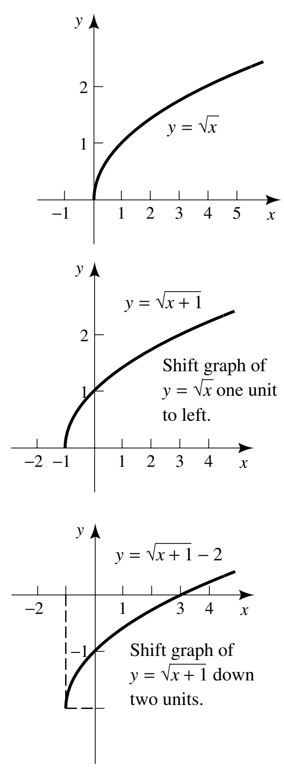


35.



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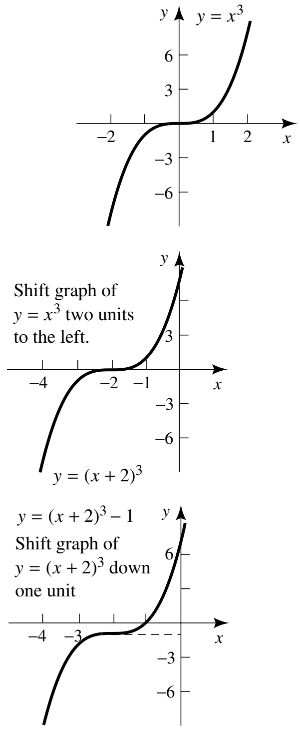

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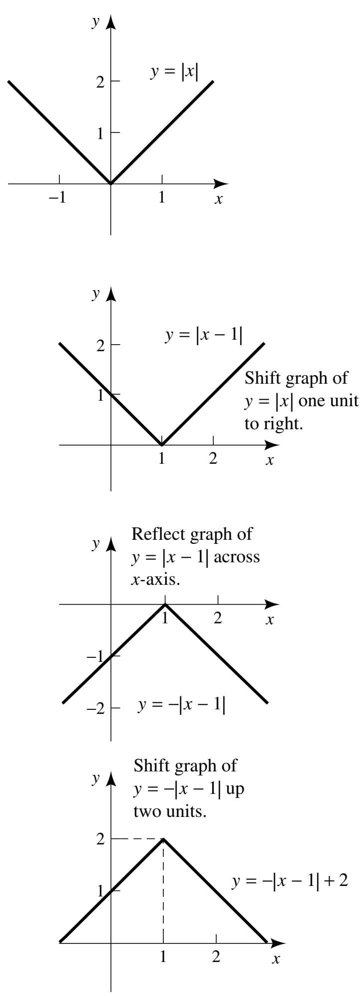

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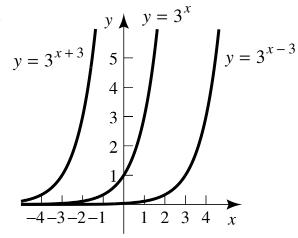

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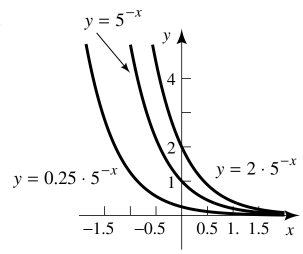

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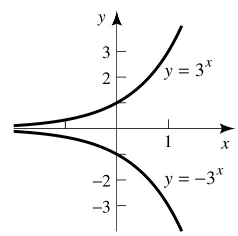

47. $y = 0.5|x| + 3$ **49.** $y = 2|x^3| - 3$

51. $y = -|x - 2| + 3$ **53.** No

55. $x^2 < x^3 < x^4 < x^5$, $x^m < x^n$ if $m < n$

57. $x^{1/2} > x^{1/3} > x^{1/4} > x^{1/5}$, $x^{1/m} > x^{1/n}$ if $m < n$

59. **a.** $x = 4$ and $x \approx -0.7666647$;
b. $2^x < x^2$ on $(-\infty, -0.7666647)$ and $(2, 4)$ and $x^2 < 2^x$ on $(-0.7666647, 2)$ and $(4, \infty)$;
c. When x is a large negative number, 2^x is very close to zero, whereas x^2 is very large. For large values of x, 2^x is much larger than x^2.

61. **a.** y_2;
b. $(1.1)^x$ eventually becomes larger than x^3.

63. **a.** is the graph of $y = x^{12}$ since $(-x)^{12} = x^{12}$. (The graph is symmetrical about the y-axis.)
b. is the graph of $y = x^{11}$.