AC Circuits

TOPICS AND FILES

E&M Topic

LRC circuit and alternating current

Capstone File

81 AC Circuit.cap

EQUIPMENT LIST

Qty	Items	Part Numbers
1	PASCO 750 Interface	
1	Voltage Sensor	CI-6503
1	AC/DC Electronics Laboratory	EM-8656
2	Banana Plug Patch Cord	SE-9750 or SE-9751
1	Resistors 10 Ω	
1	Resistors 39 Ω	
1	Capacitor 100 μF	

INTRODUCTION

The purpose of this activity is to study AC circuits with a resistor, a capacitor, and inductor. You will be able to do that by examining the current through the circuit as a function of the frequency of the applied voltage. Determine what happens to the resistance and the current when we change the parameters of voltage and frequency. Use the 'OUTPUT' feature of the PASCO 750 Interface to apply voltage to the circuit. Use the voltage sensor and *Capstone* to measure the voltage across the resistor in the circuit as the frequency of the voltage is changed. Also, investigate the phase relationship between the applied voltage and the resistor voltage as you vary the frequency.

BACKGROUND

An AC circuit consists of circuit elements and a power source that provides alternating voltage Δv. This time-varying voltage from the source is described by

$$\Delta v = \Delta V_{\max} \sin \omega t \tag{1}$$

where $\Delta V_{\max}$ is the maximum output voltage of the source, or the *voltage amplitude*.

In resistors, if $\Delta V_{\max}$ is the emf supplied by the generator, the Kirchhoff's loop rule gives

$$\begin{gathered}\Delta v - i_R \cdot R = 0 \\ \text{where } i_R = \frac{\Delta v}{R} = \frac{\Delta V_{\max}}{R} = I_{\max} \cdot \sin(\omega t) \\ \text{where } I_{\max} = \frac{\Delta V_{\max}}{R} \\ \text{and } \omega = 2 \cdot \pi \cdot f = 2 \cdot \frac{\pi}{f}\end{gathered} \tag{2}$$

So, the instantaneous voltage across the resistor is:

$$\Delta v_R = i_R \cdot R = I_{\max} \cdot R \cdot \sin(\omega t). \tag{3}$$

It means that for such a current, the current and voltage are in phase: both of them are zero at the same instant, both of them pass through their maximum values at the same instant.

For capacitors, the Kirhhoffs's loop rule gives us:

$$\Delta v - \left(\frac{q}{C}\right) = 0. \tag{4}$$

Substituting $\Delta V_{\max} \cdot \sin(\omega t)$ for Δv and rearranging gives:

$$q = C \cdot \Delta V_{\max} \cdot \sin(\omega t). \tag{5}$$

So, the instantaneous voltage across the resistor is:

$$i_C = \frac{dq}{dt} = \omega \cdot C \cdot \Delta V_{\max} \cdot \cos(\omega t). \tag{6}$$

And for inductors, again, Kirchhoff's loop rule gives us

$$\Delta v - L \cdot \frac{di_L}{dt} = 0. \tag{7}$$

Substituting $\Delta V_{\max} \cdot \sin(\omega t)$ for Δv and rearranging gives:

$$\Delta v = L \cdot \frac{di_L}{dt} = \Delta V_{\max} \cdot \sin(\omega t). \tag{8}$$

And solving for di_L gives:

$$di_L = \frac{\Delta V_{\max}}{L} \cdot \sin(\omega t) \cdot dt. \tag{9}$$

Integrating this expression gives the instantaneous current i_L in the inductor as a function of time:

$$i_L = \frac{\Delta V_{\max}}{\omega L} \cdot \sin\left(\omega t - \frac{\pi}{2}\right). \tag{10}$$

The instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are *out* of the phase by $\frac{\pi}{2}$ rad $= 90°$.