10.1 Equation Stoichiometry

Chapter 9 asked you to pretend to be an industrial chemist at a company that makes phosphoric acid, H_3PO_4. The three-step "furnace method" for producing this compound is summarized by these three equations:

$$2Ca_3(PO_4)_2 + 6SiO_2 + 10C \rightarrow 4P + 10CO + 6CaSiO_3$$

$$4P(s) + 5O_2(g) \rightarrow P_4O_{10}(s)$$

$$P_4O_{10}(s) + 6H_2O(l) \rightarrow 4H_3PO_4(aq)$$

Following the strategy demonstrated in Example 9.6, we calculated the maximum mass of tetraphosphorus decoxide, P_4O_{10}, that can be made from 1.09×10^4 kilograms of phosphorus in the second of these three reactions. The answer is 2.50×10^4 kg P_4O_{10}. We used the following steps:

1.09×10^4 kg P → g P → mol P → mol P_4O_{10} → g P_4O_{10} → kg P_4O_{10}

$$? \text{ kg } P_4O_{10} = 1.09 \times 10^4 \cancel{\text{kg P}} \left(\frac{10^3 \cancel{\text{g}}}{1 \cancel{\text{kg}}}\right)\left(\frac{1 \cancel{\text{mol P}}}{30.9738 \cancel{\text{g P}}}\right)\left(\frac{1 \cancel{\text{mol } P_4O_{10}}}{4 \cancel{\text{mol P}}}\right)\left(\frac{283.889 \cancel{\text{g}}\ P_4O_{10}}{1 \cancel{\text{mol } P_4O_{10}}}\right)\left(\frac{1 \text{ kg}}{10^3 \cancel{\text{g}}}\right)$$

$$= 2.50 \times 10^4 \text{ kg } P_4O_{10}$$

The ratio of moles of P_4O_{10} to moles of P (which came from the subscripts in the chemical formula, P_4O_{10}) provided the key conversion factor that allowed us to convert from units of phosphorus to units of tetraphosphorus decoxide.

Now let's assume that you have been transferred to the division responsible for the final stage of the process, the step in which tetraphosphorus decoxide is converted into phosphoric acid in the third reaction in the list displayed above. Your first assignment there is to calculate the mass of water in kilograms that would be necessary to react with 2.50×10^4 kg P_4O_{10}. The steps for this conversion are very similar to those in Example 9.6:

2.50×10^4 kg P_4O_{10} → g P_4O_{10} → mol P_4O_{10} → mol H_2O → g H_2O → kg H_2O

As part of our calculation, we convert from moles of one substance (P_4O_{10}) to moles of another (H_2O), so we need a conversion factor that relates the numbers of particles of these substances. The coefficients in the balanced chemical equation provide us with information that we can use to build this conversion factor. They tell us that six molecules of H_2O are needed to react with one molecule of P_4O_{10} in order to produce four molecules of phosphoric acid:

$$P_4O_{10}(s) + 6H_2O(l) \rightarrow 4H_3PO_4(aq)$$

Thus the ratio of amount of H_2O to amount of P_4O_{10} is

$$\left(\frac{6 \text{ molecules } H_2O}{1 \text{ molecule } P_4O_{10}}\right)$$

We found in Chapter 9 that it is convenient to describe numbers of molecules in terms of moles. If the reaction requires six molecules of water for each molecule of P_4O_{10}, it would require six dozen H_2O molecules for each dozen P_4O_{10} molecules, or six moles of H_2O for each mole of P_4O_{10} (Table 10.1).

$$\left(\frac{6 \text{ dozen } H_2O}{1 \text{ dozen } P_4O_{10}}\right) \quad \text{or} \quad \left(\frac{6 \text{ mol } H_2O}{1 \text{ mol } P_4O_{10}}\right)$$

Table 10.1
Information Derived from the Coefficients in the Balanced Equation for the Reaction That Produces Phosphoric Acid

OBJECTIVE 2

$P_4O_{10}(s)$ +	$6H_2O(l)$ →	$4H_3PO_4(aq)$
1 molecule P_4O_{10}	6 molecules H_2O	4 molecules H_3PO_4
1 dozen P_4O_{10} molecules	6 dozen H_2O molecules	4 dozen H_3PO_4 molecules
6.022×10^{23} molecules P_4O_{10}	$6(6.022 \times 10^{23})$ molecules H_2O	$4(6.022 \times 10^{23})$ molecules H_3PO_4
1 mole P_4O_{10}	6 moles H_2O	4 moles H_3PO_4

Example 10.1 shows how the coefficients in a balanced chemical equation provide a number of conversion factors that allow us to convert from moles of any reactant or product to moles of any other reactant or product.

EXAMPLE 10.1 - Equation Stoichiometry

OBJECTIVE 2

Write three different conversion factors that relate moles of one reactant or product in the reaction below to moles of another reactant or product in this reaction.

$$P_4O_{10}(s) + 6H_2O(l) \rightarrow 4H_3PO_4(aq)$$

Solution

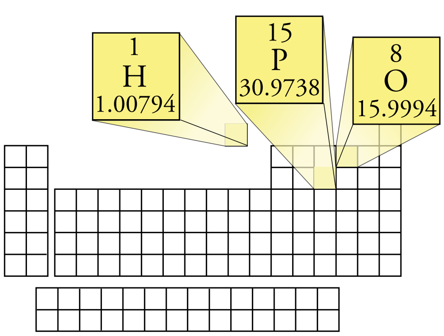
Any combination of two coefficients from the equation leads to a conversion factor.

$$\left(\frac{1\text{ mol } P_4O_{10}}{6\text{ mol } H_2O}\right) \quad \left(\frac{1\text{ mol } P_4O_{10}}{4\text{ mol } H_3PO_4}\right) \quad \left(\frac{6\text{ mol } H_2O}{4\text{ mol } H_3PO_4}\right)$$

OBJECTIVE 3
OBJECTIVE 4

Let's return to our conversion of 2.50×10^4 kg P_4O_{10} to kilograms of water. Like so many chemistry calculations, this problem can be worked using the unit analysis thought process and format. We start by identifying the unit that we want to arrive at (kg H_2O) and a known value that can be used to start the unit analysis setup (2.50×10^4 kg P_4O_{10}). We have already decided that we will convert from amount of P_4O_{10} to amount of H_2O using the molar ratio derived from the balanced equation, but before we can convert from moles of P_4O_{10} to moles of H_2O, we need to convert from mass of P_4O_{10} to number of moles of P_4O_{10}.

mass P_4O_{10} → mol P_4O_{10} → mol H_2O → mass H_2O



P_4O_{10} is a molecular compound, and we discovered in Section 9.3 that we can convert from mass of a molecular substance to moles using its molar mass, which comes from its molecular mass. Because we are starting with a mass measured in kilograms, our equation also needs a conversion factor for converting kilograms to grams. We can convert from moles of H_2O to grams of H_2O using the molar mass of water (which we determine from the molecular mass of H_2O). We then convert from grams to kilograms to complete the calculation.

$$? \text{ kg } H_2O = 2.50 \times 10^4 \text{ kg } P_4O_{10} \left(\frac{10^3 \text{ g}}{1 \text{ kg}}\right)\left(\frac{1 \text{ mol } P_4O_{10}}{283.889 \text{ g } P_4O_{10}}\right)\left(\frac{6 \text{ mol } H_2O}{1 \text{ mol } P_4O_{10}}\right)\left(\frac{18.0153 \text{ g } H_2O}{1 \text{ mol } H_2O}\right)\left(\frac{1 \text{ kg}}{10^3 \text{ g}}\right)$$

$$= 9.52 \times 10^3 \text{ kg } H_2O$$

There is a shortcut for this calculation. We can collapse all five of the conversion factors above into one. The reaction equation tells us that there are six moles of H_2O for each mole of P_4O_{10}. The molecular masses of these substances tell us that each mole of H_2O weighs 18.0153 g, and each mole of P_4O_{10} weighs 283.889 g. Thus the mass ratio of H_2O to P_4O_{10} is six times 18.0153 g to one times 283.889 g.

Objective 4

$$\left(\frac{6 \times 18.0153 \text{ g } H_2O}{1 \times 283.889 \text{ g } P_4O_{10}}\right)$$

We can describe this mass ratio using any mass units we want.

$$\left(\frac{6 \times 18.0153 \text{ g } H_2O}{1 \times 283.889 \text{ g } P_4O_{10}}\right)$$

or

$$\left(\frac{6 \times 18.0153 \text{ kg } H_2O}{1 \times 283.889 \text{ kg } P_4O_{10}}\right)$$

or

$$\left(\frac{6 \times 18.0153 \text{ lb } H_2O}{1 \times 283.889 \text{ lb } P_4O_{10}}\right)$$

Thus our setup for this example can be simplified to the following.

Objective 4

$$? \text{ kg } H_2O = 2.50 \times 10^4 \cancel{\text{kg } P_4O_{10}} \left(\frac{6 \times 18.0153 \text{ kg } H_2O}{1 \times 283.889 \cancel{\text{kg } P_4O_{10}}}\right)$$
$$= 9.52 \times 10^3 \text{ kg } H_2O$$

Calculations like this are called **equation stoichiometry** problems, or just stoichiometry problems. Stoichiometry, from the Greek words for "measure" and "element," refers to the quantitative relationships between substances. Calculations such as those in Chapter 9, in which we convert between an amount of compound and an amount of element in the compound, are one kind of stoichiometry problem, but the term is rarely used in that context. Instead, it is usually reserved for calculations such as the one above, which deal with the conversion of the amount of one substance in a chemical reaction into the amount of a different substance in the reaction.

The following is a sample study sheet for equation stoichiometry problems.

Sample Study Sheet 10.1

Basic Equation Stoichiometry–Converting Mass of One Substance in a Reaction to Mass of Another

OBJECTIVE 3
OBJECTIVE 4

TIP-OFF The calculation calls for you to convert from an amount of one substance in a given chemical reaction to the corresponding amount of another substance participating in the same reaction.

GENERAL STEPS Use a unit analysis format. Set it up around a mole-to-mole conversion in which the coefficients from a balanced equation are used to generate a mole ratio. (See Figure 10.1 for a summary.) The general steps are

STEP 1 If you are not given it, write and balance the chemical equation for the reaction.

STEP 2 Start your unit analysis in the usual way.

You want to calculate amount of substance 2, so you set that unknown equal to the given amount of substance 1. (In this section, the given units will be mass of an element or compound, but as you will see later in this chapter and in Chapter 13, the given units might instead be the volume of a solution or the volume of a gas.)

STEP 3 If you are given a unit of mass other than grams for substance 1, convert from the unit that you are given to grams. This may require one or more conversion factors.

STEP 4 Convert from grams of substance 1 to moles of substance 1, using the substance's molar mass.

STEP 5 Convert from moles of substance 1 to moles of substance 2 using their coefficients from the balanced equation to create a molar ratio to use as a conversion factor.

STEP 6 Convert from moles of substance 2 to grams of substance 2, using the substance's molar mass.

STEP 7 If necessary, convert from grams of substance 2 to the desired unit for substance 2. This may require one or more conversion factors.

STEP 8 Calculate your answer and report it with the correct significant figures (in scientific notation, if necessary) and with the correct unit.

The general form of the unit analysis setup follows.

Molar mass of substance 2

$$? \text{ (unit) } 2 = \text{(given) (unit) } 1 \left(\frac{\text{--- g}}{\text{--- (unit)}}\right)\left(\frac{\text{1 mol 1}}{\text{--- g 1}}\right)\left(\frac{\text{(coefficient 2) mol 2}}{\text{(coefficient 1) mol 1}}\right)\left(\frac{\text{--- g 2}}{\text{1 mol 2}}\right)\left(\frac{\text{--- (unit)}}{\text{--- g}}\right)$$

One or more conversion factors convert the given unit to grams.

Molar mass of substance 1

One or more conversion factors convert grams to the given unit.

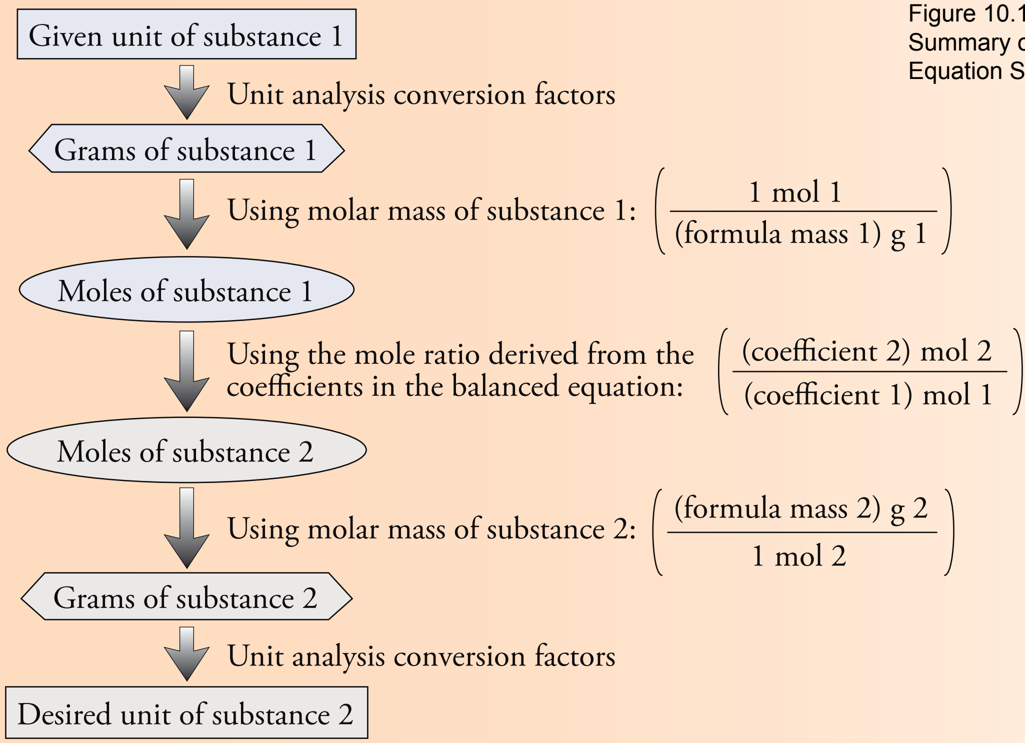


Figure 10.1
Summary of Steps for Mass-Mass Equation Stoichiometry Problems.

OBJECTIVE 4

OBJECTIVE 4

SHORTCUT STEPS - If the mass unit desired for substance 2 is the same mass unit given for substance 1, the general steps described above can be condensed into a shortcut. (See Figure 10.2 for a summary.)

STEP 1 If you are not given it, write and balance the chemical equation for the reaction.

STEP 2 Start your unit analysis set-up in the usual way.

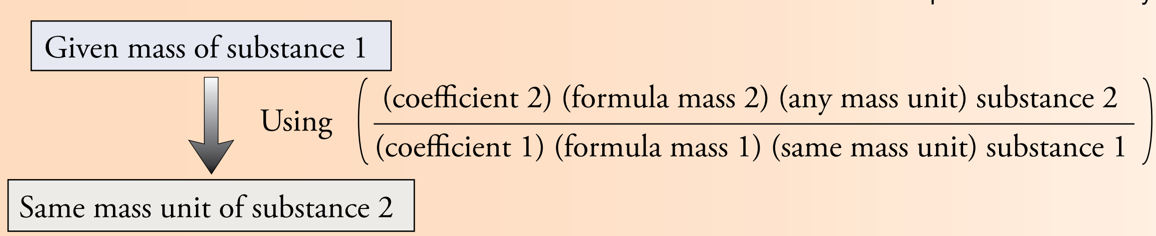
STEP 3 Convert directly from the mass unit of substance 1 that you have been given to the same mass unit of substance 2, using a conversion factor having the following general form.

$$? \text{ (unit) } 2 = (\text{given}) \text{ (unit) } 1 \left(\frac{(\text{coefficient } 2)\ (\text{formula mass } 2)\ (\text{any mass unit}) \text{ substance } 2}{(\text{coefficient } 1)\ (\text{formula mass } 1)\ (\text{same mass unit}) \text{ substance } 1}\right)$$

STEP 4 Calculate your answer and report it with the correct significant figures, in scientific notation if necessary, and with the correct unit.

EXAMPLE See Example 10.2.

Figure 10.2
Shortcut for Mass-Mass Equation Stoichiometry Problems.



Example 10.2 - Equation Stoichiometry

Objective 3
Objective 4

Aluminum sulfate is used in water purification as a coagulant that removes phosphate and bacteria and as a pH conditioner. It acts as a coagulant by reacting with hydroxide to form aluminum hydroxide, which precipitates from the solution and drags impurities down with it as it settles.

a. Write a complete, balanced equation for the reaction of water solutions of aluminum sulfate and sodium hydroxide to form solid aluminum hydroxide and aqueous sodium sulfate.

b. Write six different conversion factors that relate moles of one reactant or product to moles of another reactant or product.

c. If 0.655 Mg of $Al_2(SO_4)_3$ are added to water in a treatment plant, what is the maximum mass of $Al(OH)_3$ that can form?

Solution

a. The balanced equation is:

$$Al_2(SO_4)_3(aq) + 6NaOH(aq) \rightarrow 2Al(OH)_3(s) + 3Na_2SO_4(aq)$$

b. The stoichiometric relationships in the reaction lead to the following conversion factors.

$$\left(\frac{1 \text{ mol } Al_2(SO_4)_3}{6 \text{ mol NaOH}}\right) \quad \left(\frac{1 \text{ mol } Al_2(SO_4)_3}{2 \text{ mol } Al(OH)_3}\right) \quad \left(\frac{1 \text{ mol } Al_2(SO_4)_3}{3 \text{ mol } Na_2SO_4}\right)$$

$$\left(\frac{6 \text{ mol NaOH}}{2 \text{ mol } Al(OH)_3}\right) \quad \left(\frac{6 \text{ mol NaOH}}{3 \text{ mol } Na_2SO_4}\right) \quad \left(\frac{2 \text{ mol } Al(OH)_3}{3 \text{ mol } Na_2SO_4}\right)$$

c. We are asked to calculate the mass of $Al(OH)_3$, but we are not told what units to calculate. To choose an appropriate unit, keep the following criteria in mind.

- Choose a metric unit unless there is a good reason to do otherwise. For this problem, that could be grams, kilograms, milligrams, megagrams, etc.
- Choose a unit that corresponds to the size of the expected value. In this problem, for example, we expect the mass of $Al(OH)_3$ that forms from the large mass of 0.655 Mg of $Al_2(SO_4)_3$ to be large itself, so we might choose to calculate kilograms or megagrams instead of grams or milligrams.
- Choose a unit that keeps the calculation as easy as possible. This usually means picking a unit that is mentioned in the problem. In this example, megagrams are mentioned, so we will calculate megagrams.

We are asked to convert from amount of one compound in a reaction to amount of another compound in the reaction: an equation stoichiometry problem. Note that the setup below follows the general steps described in Sample Study Sheet 10.1 and Figure 10.1.

1	13	16	8
H	Al	S	O
1.00794	26.9815	32.066	15.9994

$$? \text{ Mg Al(OH)}_3 = 0.655 \cancel{\text{Mg Al}_2\text{(SO}_4)_3} \left(\frac{10^6 \cancel{\text{g}}}{1 \cancel{\text{Mg}}}\right) \left(\frac{1 \cancel{\text{mol Al}_2\text{(SO}_4)_3}}{342.154 \cancel{\text{g Al}_2\text{(SO}_4)_3}}\right)$$

$$\left(\frac{2 \cancel{\text{mol Al(OH)}_3}}{1 \cancel{\text{mol Al}_2\text{(SO}_4)_3}}\right) \left(\frac{78.0035 \cancel{\text{g}} \text{ Al(OH)}_3}{1 \cancel{\text{mol Al(OH)}_3}}\right) \left(\frac{1 \text{ Mg}}{10^6 \cancel{\text{g}}}\right)$$

$$= \mathbf{0.299 \text{ Mg Al(OH)}_3}$$

The setup for the shortcut is:

$$? \text{ Mg Al(OH)}_3 = 0.655 \cancel{\text{Mg Al}_2\text{(SO}_4)_3} \left(\frac{2 \times 78.0035 \text{ Mg Al(OH)}_3}{1 \times 342.154 \cancel{\text{Mg Al}_2\text{(SO}_4)_3}}\right)$$

$$= \mathbf{0.299 \text{ Mg Al(OH)}_3}$$

Note that this setup follows the steps described in Sample Study Sheet 10.1 and Figure 10.2.

Exercise 10.1 - Equation Stoichiometry

Objective 3
Objective 4

Tetrachloroethene, C_2Cl_4, often called perchloroethylene (perc), is a colorless liquid used in dry cleaning. It can be formed in several steps from the reaction of dichloroethane, chlorine gas, and oxygen gas. The equation for the net reaction is:

$$8C_2H_4Cl_2(l) + 6Cl_2(g) + 7O_2(g) \rightarrow 4C_2HCl_3(l) + 4C_2Cl_4(l) + 14H_2O(l)$$

a. Fifteen different conversion factors for relating moles of one reactant or product to moles of another reactant or product can be derived from this equation. Write five of them.

b. How many grams of water form when 362.47 grams of tetrachloroethene, C_2Cl_4, are made in the reaction above?

c. What is the maximum mass of perchloroethylene, C_2Cl_4, that can be formed from 23.75 kilograms of dichloroethane, $C_2H_4Cl_2$?

Because substances are often found in mixtures, equation stoichiometry problems often include conversions between masses of pure substances and masses of mixtures containing the pure substances, using percentages as conversion factors. See calculations like these at the textbook's Web site.