***Answer to Essential Question 29.4***: Sodium-22 is to the left of sodium-23 on the chart of the nuclides, so we expect it to decay via beta-plus decay, taking the nucleus down and to the right on the chart. In fact, sodium-22 is a well-known positron emitter; it does decay via the beta-plus process. Through beta-plus decay, sodium-22, with 11 protons and 11 neutrons, produces the nuclide with 10 protons and 12 neutrons. That is neon-22, which happens to be stable.

## *29-5 Radioactivity*

A **radioactive nucleus** is a nucleus that will spontaneously undergo radioactive decay. For an individual radioactive nucleus, it is not possible to predict precisely when the nucleus will decay. However, the statistics of radioactive decay are well understood, so for a sample of radioactive material containing a large number of nuclei, we can accurately predict the fraction of radioactive nuclei that will decay in a particular time interval.

For a given isotope, we can generally look up the **half-life**. The half-life is the time it takes for half of a large number of nuclei of this particular isotope to decay. Half-lives can vary widely from isotope to isotope. For instance, the half-life of uranium-238 is 4.5 billion years; the half-life of carbon-14 is 5730 years; and the half-life of oxygen-15 is about 2 minutes.

For a particular sample of material, containing one isotope of radioactive nuclei, the number of radioactive decays that occur in a particular time interval is related to the half-life of that isotope, but it is also proportional to the number of radioactive nuclei in the sample. The time rate of decay is given by

$$\frac{\Delta N}{\Delta t} = -\lambda N \,,$$ (Equation 29.10: **Decay rate for radioactive nuclei**)

where $N$ is the current number of radioactive nuclei, and the **decay constant** $\lambda$ is related to the half-life, $T_{1/2}$, by the equation

$$\lambda = \frac{\ln(2)}{T_{1/2}} = \frac{0.693}{T_{1/2}} \,.$$ (Eq. 29.11: **The connection between decay constant and half-life**)

A process in which the rate that a quantity decreases is proportional to that quantity is characterized by exponential decay. The exponential equation that describes the number of a particular radioactive isotope that remain after a time interval $t$ is

$$N = N_i \, e^{-\lambda t} \,,$$ (Equation 29.12: **The exponential decay of radioactive nuclei**)

where $N_i$ is a measure of the initial number of radioactive nuclei (the number at $t = 0$).

**EXAMPLE 29.5 – Calculations for exponential decay**

A particular isotope has a half-life of 10 minutes. At $t = 0$, a sample of material contains a large number of nuclei of this isotope. How much time passes until (a) 80% of the original nuclei remain? (b) 1/8th of the original nuclei remain? See if you can answer part (b) in your head, without using Equation 29.12.

**SOLUTION**
(a) Let's first determine approximately what the answer is. We know that in 10 minutes (one half-life), 50% of the nuclei would decay. To get 20% of the nuclei to decay must take less than half of a half-life, so the answer should be something like 3 or 4 minutes. In Equation 29.12, we can use a number of different measures for $N$ and $N_i$, as long as we are consistent. For instance, we can measure them both in terms of mass (e.g., grams), or use the actual number of nuclei, or express them as a percentage or fraction. It makes sense to use fractions or percentage here, because the question was posed in terms of percentage. Thus, we're looking for the time when $N = 80\%$ of $N_i$, or when $N = 0.8\ N_i$. Note that the units on $\lambda$ and $t$ must match one another, but they can be completely different from the units on $N$ and $N_i$. Applying Equation 29.12, we get:

$$0.8 N_i = N_i\, e^{-\lambda t}, \quad \text{which becomes} \quad 0.8 = e^{-\lambda t}.$$

The inverse function of the exponential function is the natural log, so to bring down the $t$ term we can take the natural log of both sides. Thus, $\ln(0.8) = -\lambda t$.

Bringing in the half-life via Equation 29.11, we get:

$$t = \frac{-\ln(0.8)}{\lambda} = \frac{-\ln(0.8)}{\ln(2)} T_{1/2} = \frac{-\ln(0.8)}{\ln(2)} (10 \text{ min}) = 3.2 \text{ min}.$$

(b) If the fraction of nuclei remaining can be expressed as 1 divided by a power of 2, the time taken is an integer multiple of the half life (the integer being equal to the power to which 2 is raised). Table 29.2 illustrates the idea.

**Table 29.2**: The table shows the relationship between the number of half-lives that have passed and the fraction of radioactive nuclei remaining. In general, if the fraction remaining is expressed as $1/(2^x)$ (where $x$ does not have to represent an integer), the amount of time that has passed is $x$ multiplied by the half-life.

| **Number of half-lives** | **Fraction of radioactive nuclei remaining** |
|---|---|
| 0 | $1/(2^0) = 1$ |
| 1 | $1/(2^1) = 1/2$ |
| 2 | $1/(2^2) = 1/4$ |
| 3 | $1/(2^3) = 1/8$ |
| 4 | $1/(2^4) = 1/16$ |
| 5 | $1/(2^5) = 1/32$ |

In our example, if 1/8th of the nuclei remain, three half-lives must have passed. Three half-lives is 30 minutes, in this case.

A graph of Equation 29.11 is shown in Figure 29.3. This figure, as it must be, is also consistent with the data in Table 29.2.

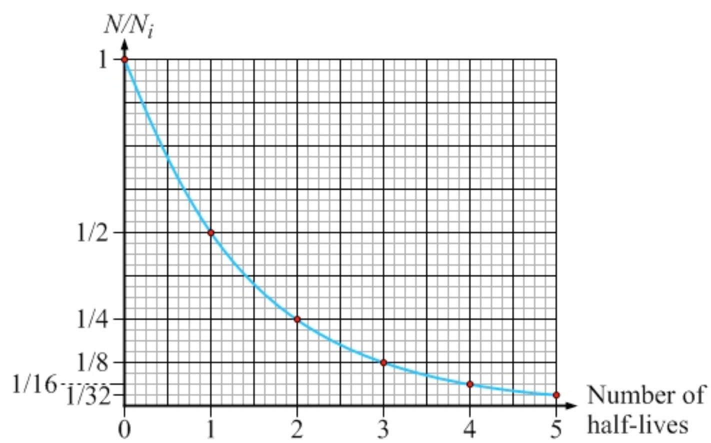


**Figure 29.3**: A graph of the exponential decay of radioactive nuclei, showing the fraction of nuclei remaining as a function of the number of half-lives that have elapsed.

**Related End-of-Chapter Exercises: 7–9, 48–54.**

***Essential Question 29.5***: Equation 29.10 can be re-arranged to $\Delta N = -\lambda N(\Delta t)$. This form of the equation gives a reasonable approximation of the number of decayed nuclei as long as $\Delta t$ is much smaller than what?