

**Answer to Essential Question 29.1:** The isotope  $^{12}_6\text{C}$  is certainly correct – it is the isotope of carbon that makes up 98.9% of naturally occurring carbon on Earth. On the other hand,  $^{14}_8\text{C}$  can not be correct. The lower number represents the number of protons in the nucleus, and carbon atoms all have 6 protons. The correct notation for carbon-14 is  $^{14}_6\text{C}$ .

## 29-2 $E = mc^2$

As we discussed in Section 29-1, the atomic mass unit is defined as  $1/12^{\text{th}}$  the mass of a neutral carbon-12 atom. Table 29.1, copied from Section 29-1, shows the mass of a neutron, proton, and electron in both kilograms and atomic mass units. The data seems to be contradictory. We will explore, and resolve, that apparent contradiction in Exploration 29.2.

**Table 29.1:** The charge and mass of the neutron, proton, and electron.

Particle	Charge	Mass (kg)	Mass (u)
neutron	0	$1.674929 \times 10^{-27}$ kg	1.008664
proton	+e	$1.672623 \times 10^{-27}$ kg	1.007276
electron	-e	$9.109390 \times 10^{-31}$ kg	0.00054858

### EXPLORATION 29.2 – Nuclear binding energy

**Step 1 – How many neutrons, protons, and electrons are in a neutral carbon-12 atom?** A neutral carbon-12 atom has six neutrons, six protons, and six electrons.

**Step 2 – Find the total mass of the individual constituents of a neutral carbon-12 atom. Compare this total mass to the mass of a neutral carbon-12 atom.**

The total mass of six neutrons, six protons, and six electrons is:

$$\begin{aligned}
 &6 \times 1.008664 \text{ u} \\
 &+ 6 \times 1.007276 \text{ u} \\
 &+ 6 \times 0.00054858 \text{ u} \\
 \hline
 &= 12.098931 \text{ u}
 \end{aligned}$$

The total mass of a carbon-12 atom is, by definition, exactly 12 u. This seems to be a contradiction – how can the masses of the individual constituents of the atom add up to more than the mass of the atom itself?

To resolve the contradiction, we apply the most famous equation in physics,  $E = mc^2$ . The idea is that the missing mass, amounting to 0.098931 u, is converted to energy, in the form of the binding energy that holds the atom together.

The difference in mass between the individual constituents of an atom and the atom itself is known as the **mass defect**. The mass that is “missing” from the atom is the atom’s binding energy (almost of this binding energy is associated with the nucleus, rather than the electrons). The conversion from mass to energy is done using Einstein’s famous equation:

$$E = mc^2, \quad (\text{Equation 29.2: The equivalence of mass and energy})$$

where  $c = 2.998 \times 10^8$  m/s is the speed of light in vacuum.

**Step 3 – Use Equation 29.2 to determine the energy, in MeV (mega electron volts) associated with a mass of 1 u.** Let's first convert 1 u, expressed in units of kilograms, to joules.

$$E = mc^2 = (1.660539 \times 10^{-27} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 = 1.492493 \times 10^{-10} \text{ J}.$$

We can now convert to MeV using the conversion factor  $1 \text{ MeV} = 1.602176 \times 10^{-13} \text{ J}$ .

$$1.492493 \times 10^{-10} \text{ J} \times \frac{1 \text{ MeV}}{1.602176 \times 10^{-13} \text{ J}} = 931.5 \text{ MeV}.$$

**Step 4 – Use the conversion factor derived in step 3 to find the binding energy, in MeV, of a carbon-12 atom, as well as the binding energy per nucleon.** The conversion factor we just derived is that 1 u is equivalent to 931.5 MeV of energy. Using this conversion factor to convert our value for the mass defect of carbon-12 from part (b), 0.098931 u, we get

$$0.098931 \text{ u} \times \frac{931.5 \text{ MeV}}{1 \text{ u}} = 92.15 \text{ MeV}.$$

To more easily compare different atoms, the binding energy for an atom is often expressed in terms of the binding energy per nucleon. Carbon-12 has 12 nucleons, so dividing the 92.15 MeV binding energy by 12 nucleons gives 7.68 MeV per nucleon.

**Key idea:** The binding energy for an atom is its mass defect (the difference between the total mass of the atom's neutrons, protons, and electrons and the mass of the atom), converted to energy. Electron binding energies are measured in electron volts, whereas atomic binding energies are measured in millions of electron volts – almost all the binding energy for an atom is associated with the energy holding the nucleus together.

**Related End-of-Chapter Exercises:** 2, 3, 13 – 15, 17, 18, 43.

### Comparing chemical energy to nuclear energy

Much of the energy we use on a daily basis comes from chemical energy, such as through the burning of fossil fuels. Burning a gallon of gasoline in a car engine, for instance, typically produces about  $1.3 \times 10^8 \text{ J}$  of energy. However, the energy obtained comes from changes in bonds between atoms, and this is associated with the electron energy levels. Breaking and forming bonds (generally associated with carbon atoms, when we're talking about fossil fuels) typically frees up a few electron volts at a time.

The relatively modest energy output, per unit mass, available from chemical energy contrasts with the much larger energy output, per unit mass, available from nuclear energy sources. Converting mass directly to energy gives enormous amounts of energy, but it is not feasible to convert all mass to energy. It is possible to convert a fraction of the mass associated with a nucleus into energy - as we will discuss in more detail later in this chapter, this can be done through the processes of nuclear fusion or nuclear fission (nuclear power plants make use of nuclear fission). Changes in the nucleus through these processes generally produce millions of electron volts per nucleus, several orders of magnitude more than is obtained from sources of chemical energy. Tapping into the vast sources of energy associated with the nucleus is why nuclear power is so attractive.

**Essential Question 29.2:** The power output of the Sun is about  $4 \times 10^{23} \text{ J/s}$ . This energy comes from converting mass into energy. How much mass is the Sun losing every second?