

The equation that describes the exponential decay in the number of nuclei of a particular radioactive isotope as a function of time t is

$$N = N_i e^{-\lambda t}, \quad (\text{Equation 29.12: The exponential decay of radioactive nuclei})$$

where N_i is a measure of the initial number of radioactive nuclei (the number at $t = 0$).

The decay constant, λ , is related to the half-life, $T_{1/2}$, (the time for half of the nuclei to decay) by

$$\lambda = \frac{\ln(2)}{T_{1/2}} = \frac{0.693}{T_{1/2}}. \quad (\text{Eq. 29.11: The connection between decay constant and half-life})$$

Nuclear fusion and nuclear fission

The most stable nuclei (those with the highest average binding energy per nuclei) are nickel-62, iron-58 and iron-56. Nuclei that are lighter than these most stable nuclei can generally become more stable (increasing the average binding energy per nuclei) by joining together with other light nuclei – this process is known as nuclear fusion.

Very heavy nuclei, in contrast, can generally become more stable by splitting apart, usually into two medium-sized nuclei and a few neutrons. This process is known as nuclear fission. The fission of uranium-235, driven by the bombardment of the uranium-235 atoms with neutrons, is exploited in a nuclear reactor to produce nuclear energy, while the fission of plutonium-239 is what drives the explosion of a nuclear bomb.

End-of-Chapter Exercises

Exercises 1 – 12 are mainly conceptual questions that are designed to see if you have understood the main concepts of the chapter.

- The symbol for the isotope iron-56 is ${}^{56}_{26}\text{Fe}$. A neutral iron-56 atom has how many (a) protons? (b) neutrons? (c) nucleons?
- Which of these numbers is larger, the mass of an iron-56 atom, or the total mass of the individual constituents (neutrons, protons, and electrons) of an iron-56 atom? Briefly explain your answer.
- If you could convert 1 kg of matter entirely to energy, how much energy would you get?
- (a) Fill in the blank to complete this decay process: ${}^{226}_{88}\text{Ra} \Rightarrow \text{---} + {}^4_2\text{He}$. (b) What kind of decay is this? (c) Based on the fact that this decay process happens spontaneously, which side of the equation do you expect to have more mass? Why?
- (a) What kind of radioactive decay process gives rise to a positron? (b) What is the electric charge of a positron? (c) Complete this sentence: The mass of a positron is the same as the mass of _____.
- Fill in the blanks to complete the following decay processes:
 (a) $\text{---} \Rightarrow {}^{15}_7\text{N} + {}^0_{+1}\text{e}^+ + \nu_e$. (b) ${}^{46}_{21}\text{Sc} \Rightarrow \text{---} + {}^0_{-1}\text{e}^- + \bar{\nu}_e$. (c) ${}^{60}_{28}\text{Ni}^* \Rightarrow \text{---} + \gamma$.

7. A particular sample contains a large number of atoms of a certain radioactive isotope, which has a half life of 1 hour. After 5 hours, approximately what percentage of the original radioactive nuclei remain?

8. You have two samples of radioactive material. At $t = 0$, sample A contains a large number of nuclei that have a half-life of 10 minutes, while sample B contains exactly the same number of a different kind of nuclei, which have a half-life of 40 minutes. (a) In which sample is the rate at which the nuclei decay larger, at $t = 0$? Briefly justify your answer. Find the ratio of the number of nuclei that have decayed in sample A to the number that have decayed in sample B after (b) $t = 40$ minutes, (c) $t = 1$ year (feel free to make a reasonable approximation in part (c)).

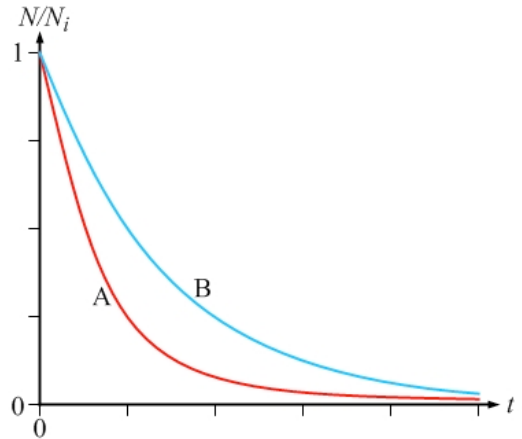
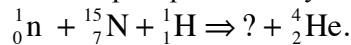


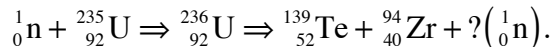
Figure 29.6: A graph of the ratio of the number of undecayed nuclei to the initial number of nuclei for samples of two different radioactive isotopes, labeled A and B, for Exercise 9.

9. Figure 29.6 shows a graph of the ratio of the number of undecayed nuclei to the initial number of nuclei for samples of two different radioactive isotopes, labeled A and B. Which isotope has the larger (a) decay constant? (b) half-life?

10. In stars that are more massive than the Sun, one of the primary methods of fusing hydrogen into helium is the CNO (carbon-nitrogen-oxygen) cycle, which is a sequence of fusion reactions. One of the reactions in the CNO cycle, which liberates about 5 MeV of energy, is shown below. Identify the isotope represented by the question mark.



11. How many neutrons are produced in the following fission reaction for a uranium-235 atom that combines with a neutron?



12. Modern human activities have altered the ratio of carbon-14 to carbon-12 in the atmosphere. A good example of this is the burning of fossil fuels, which contain carbon that is millions of years old. The half-life of carbon-14 is about 5700 years, while carbon-12 is stable. Based on this information, has all of our burning of fossil fuels increased or decreased the carbon-14 to carbon-12 ratio in the atmosphere? Explain your answer.

Exercises 13 – 18 involve applications of $E = mc^2$. For some of these exercises you will probably want to make use of the data in Table 29-3 in Section 29-8.

13. For an oxygen-16 atom, calculate (a) the mass defect, in atomic mass units, (b) the total binding energy, in MeV, and (c) the average binding energy per nucleon.

14. (a) What is the difference, in terms of constituents, between a carbon-12 atom and a carbon-13 atom? (b) Compare the mass of a carbon-13 atom to the mass of a carbon-12 atom plus the mass of the extra particle(s) that makes up a carbon-13 atom compared to a carbon-12 atom. Is there a difference? If so, why?

15. In almost all nuclei, there are more than 2 nucleons, and thus there is more than one pair of nucleons interacting via the nuclear force. The situation is much simpler in a deuterium (${}^2_1\text{H}$) atom, in which the binding energy is associated with the attractive nuclear force between the single neutron and the single proton. For the deuterium atom, calculate (a) the mass defect, in atomic mass units, (b) the binding energy, in MeV, which is almost entirely associated with the nuclear force between the proton and neutron.
16. In general, for a nucleus to be stable, the attractive forces between nuclei must balance the repulsive forces associated with the charged protons. A helium-4 atom is particularly stable. To help understand why this is, consider how many pairs of (a) interacting protons, and (b) interacting nucleons there are in a helium-4 atom. (c) Briefly explain why your answers to (a) and (b) support the idea that the helium-4 atom is particularly stable.
17. Lead-208 is stable, which is relatively rare for a high-mass nucleus. For ${}^{208}_{82}\text{Pb}$, calculate (a) the mass defect, in atomic mass units, (b) the total binding energy, in MeV, and (c) the average binding energy per nucleon.
18. Nickel-62 has the largest average binding energy per nucleon of any isotope. Calculate the average binding energy per nucleon for nickel-62.

Exercises 19 – 24 involve radioactive decay processes. Make use of the data in Table 29-3 in Section 29-8.

19. Plutonium-239 (${}^{239}_{94}\text{Pu}$) decays via alpha decay. (a) Write out the decay equation for plutonium-239. (b) Calculate the energy released in the decay of one Pu-239 atom.
20. Scandium-46 (${}^{46}_{21}\text{Sc}$) decays via the beta-minus process. (a) Write out the complete decay equation for scandium-46. (b) Calculate the energy released in the decay of one scandium-46 atom.
21. A certain isotope decays via the beta-plus process to neon-22 (${}^{22}_{10}\text{Ne}$). (a) Write out the complete decay equation for this situation. (b) The mass of a neon-22 atom is 21.99138511 u. Calculate the energy released in this beta-plus decay process.
22. (a) According to Table 29.3, what does krypton-85 decay into? (b) Write out the decay equation for krypton-85. (c) Calculate the energy released in this decay process.
23. Silver-111, ${}^{111}_{47}\text{Ag}$, which has an atomic mass of 110.905291 u, is radioactive, spontaneously decaying via either alpha decay, beta-plus decay, or beta-minus decay. The masses of the possible decay products are 110.904178 u for ${}^{111}_{48}\text{Cd}$, 110.907671 u for ${}^{111}_{46}\text{Pd}$, and 106.906748 u for ${}^{107}_{45}\text{Rh}$. (a) Explain how you can use these numbers to determine the spontaneous decay process for silver-111. (b) Write out the complete decay reaction for the spontaneous decay of silver-111.

Exercises 24 – 28 involve radioactivity.

24. Oxygen-15 has a half-life of 2 minutes. Nuclear activity is often measured in units of becquerels (Bq), which are the number of nuclear decays per second. If the activity of a sample of oxygen containing some oxygen-15 is 64×10^6 Bq at $t = 0$, what is the activity level of the sample at (a) $t = 4$ minutes, (b) $t = 16$ minutes, and (c) $t = 20$ minutes?
25. Fluorine-20 decays to neon-20 with a half-life of 11 seconds. At $t = 0$, a sample contains 120 grams of fluorine-20 atoms. How many grams of fluorine-20 atoms remain in the sample at (a) $t = 5.0$ seconds, (b) $t = 30$ seconds, and (c) $t = 1.0$ minutes?
26. A supply of fluorodeoxyglucose (FDG) arrives at a positron emission tomography clinic at 8 am. At 5 pm, at the end of the working day, the activity level of the FDG has dropped significantly because of the 110 minute half-life of the radioactive fluorine. Considering equal masses of FDG at 8 am and 5 pm, by what factor has the activity level been reduced by 5 pm?
27. Four hours after being injected with a radioactive isotope for a positron emission tomography scan, the activity level of the material injected into the patient has been reduced by a factor of 8 compared to its initial value (at the time of the injection). After another four hours elapses, will the activity level be reduced by another factor of 8? Briefly justify your answer.
28. After 3 hours, the activity level of a sample of a particular radioactive isotope, which decays into a stable isotope, has fallen to 20% of its initial value. Calculate the half-life of this isotope.

Exercises 29 – 33 involve nuclear fusion and nuclear fission. Make use of the data in Table 29-3 in Section 29-8.

29. (a) What is the second object produced in the following fusion reaction? ${}^2_1\text{H} + {}^6_3\text{Li} \Rightarrow {}^7_3\text{Li} + \text{---}$.
 (b) How much energy is produced in this reaction?
30. In Exercise 10, we examined one of the fusion reactions in the CNO (carbon-nitrogen-oxygen) cycle, which is a primary method of fusing hydrogen into helium in massive stars. The primary reactions in the CNO cycle are shown in Figure 29.7. Two more of the fusion reactions in the CNO cycle are ${}^1_1\text{H} + {}^{13}_6\text{C} \Rightarrow {}^{14}_7\text{N}$ and ${}^1_1\text{H} + {}^{14}_7\text{N} \Rightarrow {}^{15}_8\text{O}$. Calculate the energy released in each of these fusion reactions.

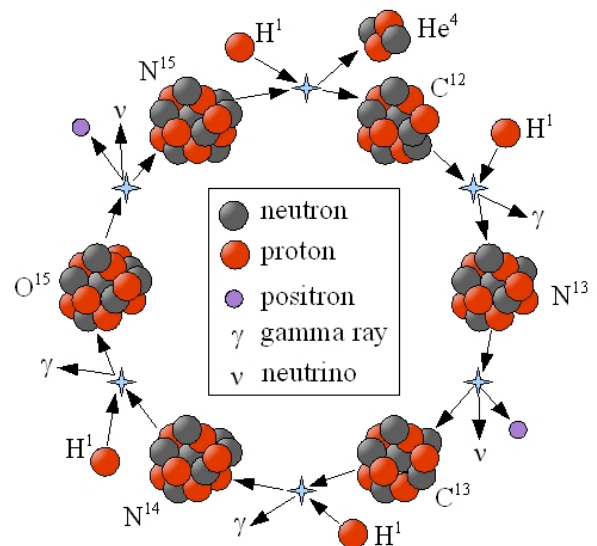


Figure 29.7: The CNO cycle of fusion reactions that produces much of the energy in massive stars. The CNO cycle has the net effect of fusing hydrogen into helium.

31. In relatively low-mass stars, such as our Sun, much of the energy generated by the star comes from the proton-proton chain, which has the net effect of fusing hydrogen into helium. The step that produces the helium-4 atom is: ${}^3_2\text{He} + {}^3_2\text{He} \Rightarrow {}^4_2\text{He} + {}^1_1\text{H} + {}^1_1\text{H}$. The helium-3 atoms are produced from the fusion of hydrogen isotopes in earlier steps in the process. How much energy is released in the step that produced helium-4? Note that the mass of ${}^3_2\text{He}$ is 3.01602932 u.
32. Complete the following fission reaction: ${}_0^1\text{n} + {}_{92}^{235}\text{U} \Rightarrow {}_{92}^{236}\text{U} \Rightarrow {}_{56}^{143}\text{Ba} + \text{---} + 3({}_0^1\text{n})$.
33. How much energy is released in the following fission reaction?
- $${}_0^1\text{n} + {}_{92}^{235}\text{U} \Rightarrow {}_{92}^{236}\text{U} \Rightarrow {}_{56}^{141}\text{Ba} + {}_{36}^{92}\text{Kr} + 3({}_0^1\text{n}).$$

Exercises 34 – 38 involve applications of nuclear physics.

34. A common application of nuclear radiation is **food irradiation**, which is the exposure of food products (such as meat, poultry, and fruit) to ionizing radiation to kill bacteria, prolong shelf life, and delay the ripening of fruit. One example of this treatment is the exposure of a meat product to a dose of 3 kilograys of gamma radiation from a cobalt-60 source. One gray (Gy) represents 1 joule of energy exposure per kilogram of food. Cobalt-60 decays by the beta-minus process to an excited form of nickel-60, which then decays to its ground state through the emission of two gamma rays, with energies of 1.17 MeV and 1.33 MeV. (a) How many nuclei are there in 1 gram of cobalt-60, which has an atomic mass of 59.9338171 u? (b) Cobalt-60 has a half-life of 5.27 years. How many nuclei, in a 1-gram sample of cobalt-60, would decay in 1 second? (c) If the two gamma rays associated with each decaying cobalt-60 nuclei deposit all their energy in a 1 kilogram package of meat, how long would the meat have to be exposed to receive a dose of 3 kGy?
35. When radiocarbon dating was carried out on the Shroud of Turin in 1988, the three labs doing the measurements agreed that the shroud dated to approximately the year 1300 AD, providing evidence against the idea that the shroud (a picture of which is shown in Figure 29.8) was the burial cloth of Jesus Christ. If the ratio of carbon-14 to carbon-12 in the atmosphere has remained constant over time at 1.2×10^{-12} , (a) approximately what ratio did the researchers find for the sample of shroud they measured?, and (b) what ratio would they have found if the cloth was 2000 years old?

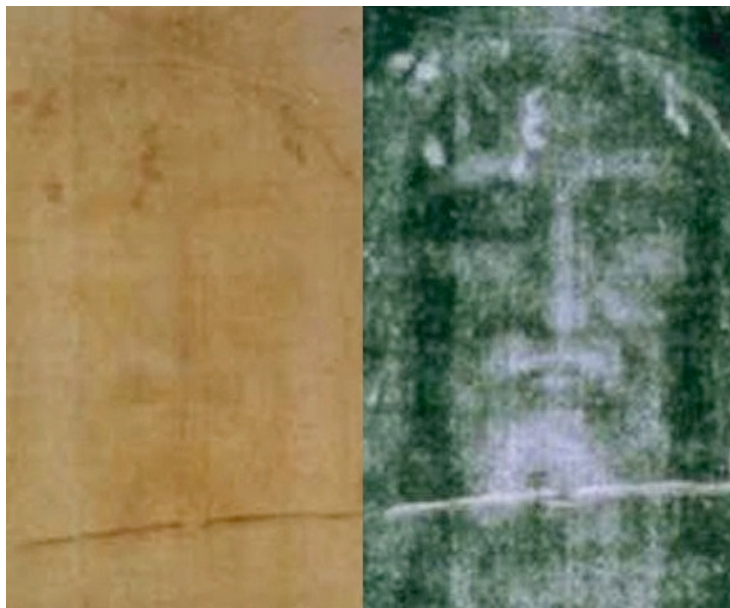


Figure 29.8: A photograph of part of the Shroud of Turin, on the left, and the corresponding negative image, on the right, for Exercise 35. Image credit: Wikimedia Commons.

36. Some smoke detectors have an alpha emitter, americium-241, in them. The alpha particles ionize air molecules in the space between two oppositely charged plates (a parallel-plate capacitor), and the current associated with these ions in the parallel-plate capacitor is measured by the smoke detector. When this current drops, because the ions are blocked by smoke particles, the detector's alarm sounds. (a) When an americium-241 atom decays via alpha decay, what does it decay into? (b) Using the data in Table 29.3 to help you, how much energy (in MeV) is released in the decay process of a single americium-241 atom? For reference, typical ionization energies are tens of electron volts.
37. Single photon emission computed tomography (SPECT) is a medical imaging procedure that makes use of technetium-99m, an excited form of technetium-99 that undergoes gamma decay with a half-life of 6 hours. The gamma rays are relatively low energy, around 140 keV, comparable to standard x-rays. SPECT is thus much like a CT scan, but with photons coming from inside the body rather than passing through the body. Technetium-99m is used in well over half of all medical imaging procedures that use radioactive nuclei, and is especially useful for bone scans and brain scans. By what factor has the activity level of the technetium-99m fallen to 48 hours after it was first administered to the patient?
38. Burning 20 tons of coal provides about 5×10^{11} J, which is approximately the annual energy requirement of an average person in the United States. If this amount of energy was provided by the electricity generated by the fission of uranium-235 in a nuclear reactor, instead, what mass of natural uranium would be required? About 0.7% of natural uranium is uranium-235 (almost all the rest is uranium-238, which does not fission like U-235 does). Assume that the fission of each U-235 atom provides 200 MeV of energy, and that the nuclear power plant has an efficiency of 33% in transforming the energy from the fission reactions into electricity.

General problems and conceptual questions

39. Marie Curie was a pioneer in the field of radioactivity. Do some research about Marie Curie and write a couple of paragraphs about her life and her contributions to nuclear physics.
40. Enrico Fermi made a number of important contributions to our understanding of nuclear physics. Do some research about Fermi and write a couple of paragraphs about his contributions to nuclear physics, describing how his work relates to the principles of physics discussed in this chapter.
41. Equation 29.1 gives the approximate radius of a nucleus, assuming it to be spherical. (a) What is the atomic mass number of a zinc-64 atom? (b) What is the cube root of zinc-64's atomic mass number? (c) Approximately what is the radius of a zinc-64 atom?
42. A particular nucleus has a radius of about 6×10^{-15} m. (a) Approximately what is the atomic mass number of the nucleus? (b) Approximately where would you find such a nucleus on the periodic table, assuming the nucleus is stable?
43. (a) If you convert the mass of an electron entirely to energy, how much energy, in keV, do you get? (b) If you convert the mass of a positron entirely to energy, how much energy, in keV, do you get? When an electron and positron encounter one another, they annihilate one another, and the particles are transformed entirely into two photons. Assume that the electron and positron are initially at rest before the annihilation process. After the annihilation process, what is the (c) energy, and (d) wavelength of each of the two photons?

44. The daily food intake of an average person consists of approximately 10 million joules of food energy. If we could generate this energy internally through the fusion of helium-3 and lithium-6 into two helium-4 atoms plus a proton, which releases about 17 MeV of energy, approximately what mass of helium-3 and lithium-6 would we need to take in every day?
45. Polonium-210 decays via alpha decay into lead-206 and an alpha particle. For the purposes of this exercise, use the approximation that the lead-206 atom has 50 times the mass of the alpha particle. We will also assume that the polonium-210 is at rest before the decay process. (a) Immediately after the decay, how does the momentum of the lead-206 atom compare to the momentum of the alpha particle? (b) Immediately after the decay, what is the ratio of the alpha particle's kinetic energy to the lead-206 atom's kinetic energy?
46. Try this at home. M&M's (the candy) are not radioactive, but a package of M&M's (or something equivalent, like coins) can be used as a model of a system with a half-life. Obtain a package of M&M's and, after first counting how many M&M's there are, do the following. Place all the M&M's in cup (starting with a number of M&M's that is a power of 2, like 64 M&M's, makes this exercise easier), and then shake them out onto a clean surface, like a plate. Remove all the M&M's that have their "m" down – those are the ones that have decayed. Count all the "m" up M&M's, representing undecayed nuclei, and place them back in the cup. Repeat the process until all the M&M's have decayed. (a) Make a table of your results, recording the number of "undecayed nuclei" as a function of the number of throws. (b) Use Equation 29.12 to predict the number of "undecayed nuclei" remaining – add this theoretical data to your table. (c) Plot a graph of your results, and draw the curve representing the theoretical results. (d) Account for any differences between your values and the theoretical values. (e) If we started with a very large number of radioactive nuclei, would we expect to see similar percentage deviations from the theoretical values? Explain. (f) If you repeated the experiment with the M&M's one thousand times, averaged your results, and plotted your averaged results against the theoretical values, would you expect more, less, or the same deviation from the theoretical values compared to the deviation obtained from a single trial? Explain. (g) Eat the M&M's.
47. Much like the M&M's in Exercise 46, a large number of six-sided dice can be used as a model of a system with a half-life. If you shake all the dice, how many throws represent one half-life if you define the "decayed" dice as (a) all those showing an even number, or (b) all those showing a 1?
48. Oxygen-15 has a half-life of 2 minutes. Fluorine-18 has a half-life of 110 minutes. For a sample of oxygen-15 to have the same decay rate as a sample of fluorine-18, how should (a) the number of radioactive nuclei in the two samples compare? (b) the masses of the two samples compare?
49. At $t = 0$, you have two samples of radioactive nuclei. Sample A contains N nuclei of a radioactive isotope that has a half-life of 3 hours. Sample B contains a to-be-determined number of nuclei of a different radioactive isotope, with a half-life of 4 hours. How many nuclei of the second radioactive isotope should sample B contain (at $t = 0$) if, at $t = 12$ hours, (a) the number of undecayed nuclei in the two samples is equal, or (b) the decay rate, measured in nuclei per second, of the two samples is equal.

50. You have two samples of radioactive nuclei. At $t = 0$, sample A contains N nuclei of a particular radioactive isotope, while sample B contains twice as many nuclei of a different radioactive isotope. Also at $t = 0$, the decay rate, measured in nuclei per second, of sample A is three times larger than the decay rate in sample B. How do the half-lives of the two different types of nuclei compare?

51. The graph in Figure 29.9 shows the ratio of the number of undecayed nuclei to the initial number of undecayed nuclei, as a function of time. Use the data in the graph to estimate the half-life of the nuclei.

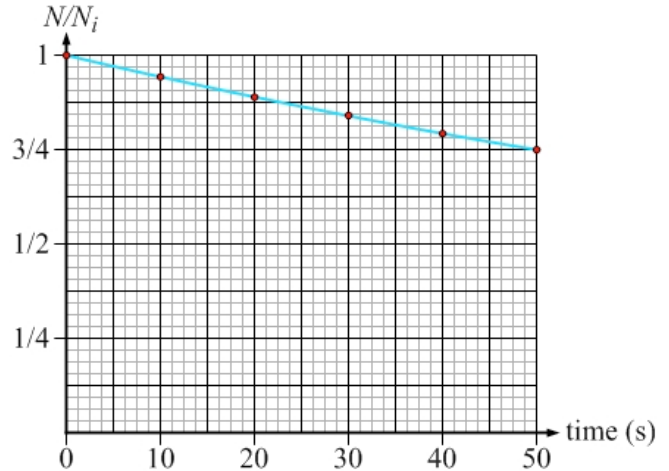


Figure 29.9: A graph of the number of undecayed nuclei to the initial number of undecayed nuclei, as a function of time, for Exercise 51.

52. Instead of defining the half-life for a particular isotope, we could define the “one-tenth-life” (or the life for any other fraction less than 1). After a time of a single one-tenth-life has passed, only $1/10^{\text{th}}$ of the initial number of radioactive nuclei remain. (a) Determine the ratio of a one-tenth-life to a half-life. (b) The half-life of cesium-137 is 30 years. What is the one-tenth-life for cesium-137? (c) After how many one-tenth-lives does the activity level of a sample of radioactive material drop to 1% of its initial value? (d) How much time elapses until the activity level of a sample of cesium-137 drops to 1% of its initial value?

53. After 6 days, the activity level of a sample of radioactive material has fallen to 40% of its initial value. After an additional 6 days elapses, what is the activity level of the sample, compared to its initial value? (Hint: there is a very easy way to do this calculation – it is possible to work out the answer without a calculator.)

54. Right now, a sample contains 1.0×10^{20} undecayed nuclei, and a much larger number of decayed nuclei. If the half-life of the undecayed nuclei is 5 months, how many undecayed nuclei did the sample contain a year ago?

55. At $t = 0$ s, a sample of radioactive nuclei contains 1.0×10^{20} undecayed nuclei, which have a half-life of 10 seconds. First, use equation 29.10, with $N = 1.0 \times 10^{20}$ nuclei, to estimate the number of undecayed nuclei remaining at (a) $t = 1$ s, (b) $t = 10$ s, and (c) $t = 20$ s. Second, use equation 29.12 to estimate the number of undecayed nuclei remaining at (d) $t = 1$ s, (e) $t = 10$ s, and (f) $t = 20$ s. (g) Explain which method is the correct method to use to accurately determine the number of undecayed nuclei remaining, and why the other method gives inaccurate results.

59. In 1991, a body was found by hikers in the Alps, just inside Italy along the border between Italy and Austria. After being removed from the ice that had preserved the body, the body was carefully studied. This individual is now known as Ötzi the Iceman, and one of the methods used to study Ötzi was radiocarbon dating. The radiocarbon dating process carried out at the University of Vienna resulted in a “radiocarbon age” of 4550 years BP. BP stands for “Before Present,” where present is defined to be the year 1950 AD, the year when the radiocarbon dating process was first done. In determining a radiocarbon age, the half-life of carbon-14 is taken to be 5568 years. Assuming the ratio of carbon-14 to carbon-12 has held constant in the atmosphere at a value of 1.2×10^{-12} , what was the ratio in Ötzi’s body in 1950? Note that correcting Ötzi’s age for known historical fluctuations in the carbon-14 to carbon-12 ratio, as well as the correct half-life of carbon-14, led to the conclusion that there was a more than 60% probability that Ötzi died between 3230 and 3100 BC.
60. Nickel has five different stable nuclides, but nickel-66 is not one of them. All of the stable nuclides of nickel have fewer neutrons than nickel-66 has, in fact. (a) What do you think the dominant radioactive decay process is for nickel-66? (b) What is the daughter nuclide produced by this decay process?
61. One of the largest stable nuclides is a nuclide of lead, lead-208, with 82 protons and 126 neutrons. This nuclide can be formed by various radioactive decay processes. (a) If lead-208 is produced by a single beta-plus decay, what was the original nuclide? (b) If lead-208 is produced by a single beta-minus decay, what was the original nuclide? (c) If lead-208 is produced by a single alpha decay, what was the original nuclide? (d) Calculate the amount of energy released in the alpha decay process.
62. Near the low atomic number end of the chart of the nuclides, nuclides with equal numbers of protons and neutrons tend to be stable. As the atomic number increases, however, nuclides need more neutrons than protons to be stable. Consider the nuclide with 50 protons and 50 neutrons. (a) What element is this nuclide? (b) What do you expect the dominant radioactive decay mode to be for this nuclide? Explain your answer.
63. Three students are having a conversation. Comment on how the answers obtained by each student compare to the correct answer to the question they are trying to solve. Explain what, if anything, is wrong with each of their methods.

Mike: OK, the question gives the half-life as 2 years, and it asks for the percentage remaining after just 1 year. Isn't that easy? If half of it decays in 2 years, then 25% decays in 1 year, so the fraction remaining after 1 year is 75%, right?

Jessica: I think you have to use one of the equations. I used Equation 29.12 ($N = N_i e^{-\lambda t}$), and got an answer around 70%.

Debbie: I think Jessica’s right, that you need to use an equation. I used Equation 29.10, though, after I re-arranged it to $\Delta N = -\lambda N(\Delta t)$. Then I subtracted what was lost from 100% to get the answer. I got more like 65% left, though.