

Answer to Essential Question 28.2: The area under the curve for the region beyond 1 Bohr radius from the nucleus is clearly larger than the area for the region less than 1 Bohr radius away from the nucleus – thus, the electron is more likely to be found farther than 1 Bohr radius away. A reasonable estimate for the ratio of the areas is 2:1. In other words, the electron, when in the ground state of hydrogen, is approximately twice as likely to be found at a radius farther than 1 Bohr radius from the nucleus than it is to be found at a distance less than one Bohr radius.

28-3 The Quantum Mechanical View of the Atom

In the Bohr model of the atom, only one quantum number, n , was used. With that single quantum number, Bohr came up with expressions for the energies of quantized energy levels, quantized angular momenta of the electrons, and quantized radii of the electron orbits. Our modern view of the atom, applying the equations of quantum mechanics, is a little more complicated, requiring four quantum numbers to completely specify the various electron energy states. These quantum numbers are:

1. n , the principal quantum number. To a first approximation, the energies of the quantized energy levels in an atom with an atomic number Z and just one electron are given by:

$$E_n = \frac{(-13.6 \text{ eV}) Z^2}{n^2}, \quad (\text{Eq. 28.3: Energies of the electron levels})$$

where $n = 1, 2, 3, \dots$

2. ℓ , the orbital quantum number. This quantum number quantizes the magnitude of the electron's orbital angular momentum (this is somewhat analogous to the angular momentum associated with a planet's orbit around the Sun). The magnitude of the orbital angular momentum is given by:

$$L = \sqrt{\ell(\ell+1)} \frac{h}{2\pi}, \quad (\text{Eq. 28.4: Orbital angular momentum})$$

where $\ell = 0, 1, 2, \dots, n-1$.

3. m_ℓ , the magnetic quantum number. This quantum number quantizes the direction of the electron's angular momentum (this is known as **space quantization**, and is illustrated in Figure 28.5). Conventionally, we say that this quantum number defines the z -component of the angular momentum. For a given value of the orbital quantum number, the different values of m_ℓ give electron states of the same energy unless a magnetic field is present, in which case the states have different energies (this is called the **Zeeman effect**).

$$L_z = \frac{m_\ell h}{2\pi}, \quad \text{where } m_\ell = -\ell, -\ell+1, \dots, \ell-1, \ell.$$

(Eq. 28.5: z -component of the orbital angular momentum)

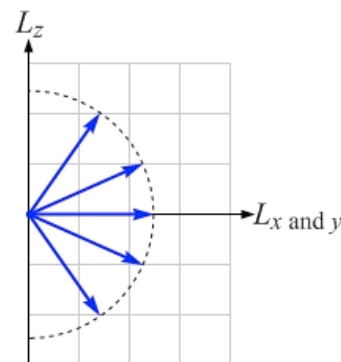


Figure 28.5: The allowed directions of the orbital angular momentum, when $\ell = 2$. The half-circle has a radius of $\sqrt{6}$ units. The units on both axes are angular momentum units of $h/(2\pi)$.

4. m_s , the spin quantum number. The concept of electron spin is somewhat analogous to the angular momentum associated with a planet's rotation about its own axis. The electron has two possible spin states, which we refer to as spin up and spin down. The two possible z -components of the spin angular momentum, S_z , are given by:

$$S_z = m_s \frac{h}{2\pi}, \quad (\text{Eq. 28.6: Spin angular momentum})$$

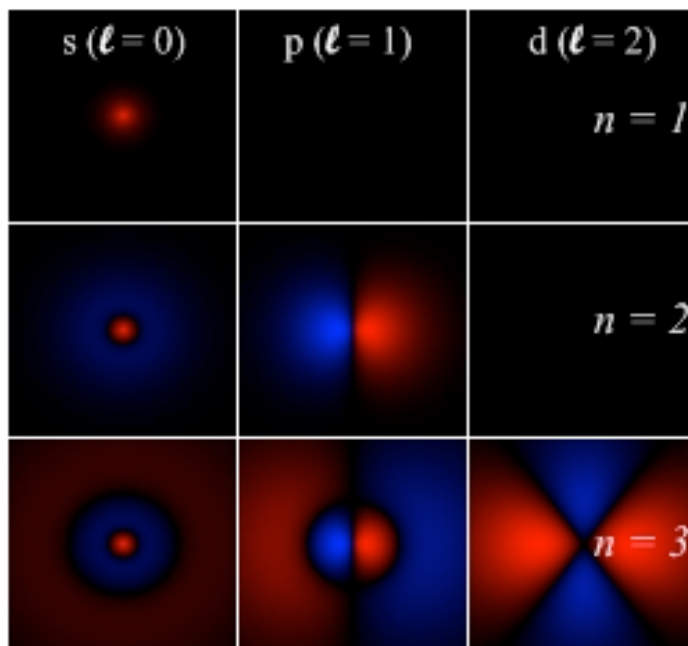
$$\text{where } m_s = \pm \frac{1}{2}.$$

Visualizing the wave functions

Figure 28.6 shows a common way to visualize the various electron wave functions for hydrogen. Remember that there is only one electron in the hydrogen atom, but the electron has an infinite number of states that it can choose between – some of the lower-energy states are shown below. When we look at pictures like this, we again interpret them in terms of probability. The brighter the picture is at a particular point, the more likely it is to find the electron at that point. Although the pictures are two-dimensional, the wave functions are three-dimensional.

The image in the top left of Figure 28.6 corresponds to the graph in Figure 28.4. At first glance, they appear to contradict one another. For instance, the graph in Figure 28.4 shows that the probability of finding the electron at $r = 0$ is zero, while the top left image in Figure 28.6 has the brightest (highest probability) spot right in the center, where $r = 0$. The probability of finding the electron at any particular point, not just the center point, is zero, because a point has no volume. It makes more sense to discuss the probability of finding the electron within a particular volume. Both the graph in Figure 28.4 and all six diagrams below are best interpreted that way. From that perspective, the diagram at the upper left and the graph in Figure 28.4 are consistent.

Figure 28.6: A visualization of the wave functions (or, atomic orbitals) for the $n = 1, 2,$ and 3 levels for hydrogen. The brighter it is in a particular region, the more likely it is to find the electron in that region. Conversely, there is no chance of finding the electron in a region that is black. The numbers on the right side of the diagram show values of n , the principal quantum number, for each row. The letters across the top represent the values of the orbital quantum number, ℓ , for a particular column. Only one wave function is shown in the first row because, when $n = 1$, the only possible value of the orbital quantum number is $\ell = 0$. Similarly, there are only two images in the second row because there are two allowed ℓ values when $n = 2$. The wave functions are positive in the red areas and negative in the blue areas. In all cases shown, $m_\ell = 0$.



Related End-of-Chapter Exercises: 8, 9, 19 – 23.

Essential Question 28.3: How many different electron states are there that have $n = 3$?