Answer to Essential Question 27.4: (a) To find the electron's speed, we can divide the magnitude of the electron's momentum by the electron's mass.

$$v_e = \frac{p_e}{m} = \frac{2.308 \times 10^{-23} \text{ kg m/s}}{9.11 \times 10^{-31} \text{ kg}} = 2.53 \times 10^7 \text{ m/s} .$$

This speed is less than 10% of the speed of light in vacuum, so we are safe applying the non-relativistic equations that we used in Example 27.4.

(b) We can find the direction of the electron's momentum from the components of the momentum.

$$\tan\phi = \frac{p_{e,y}}{p_{e,x}} = \frac{1.112 \times 10^{-23} \text{ kg m/s}}{2.022 \times 10^{-23} \text{ kg m/s}} \qquad \text{which gives } \phi = 28.8°.$$

27-5 Particles Act Like Waves

As we have learned in the previous two sections, light, which in many instances acts like a wave, also exhibits a particle nature. This was a rather surprising result, but the surprises kept coming. Recall that equation 27.6 relates the momentum of a photon to the wavelength of the light, $p = h/\lambda$. In 1923, Louis de Broglie (1892 – 1987) proposed turning the equation around and applying it to objects that we normally think of as particles.

The de Broglie wavelength: de Broglie proposed that Equation 27.6 could be applied to objects we think of as particles, such as electrons and neutrons, in the form:

$$\lambda = \frac{h}{p} = \frac{h}{mv} . \qquad \text{(Equation 27.8: \textbf{the de Broglie wavelength})}$$

In other words, de Broglie proposed that everything that moves has an associated wavelength. When de Broglie's idea was verified, de Broglie was awarded the Nobel Prize in Physics in 1929 for the idea.

For objects that we are used to dealing with in our daily lives, such as balls and cars and people, their de Broglie wavelength is so small that there is, effectively, no wave behavior. For instance, when you pass through a door you simply go through as a particle, rather than diffracting into the next room. For a person traveling at about 1 m/s, for instance, the de Broglie wavelength is about 1×10^{-35} m. This wavelength is so many orders of magnitude smaller than the objects and openings that we encounter everyday that our particle nature dominates.

In contrast, for tiny objects that we generally think of as particles, such as electrons and protons, their tiny mass produces a much larger wavelength. An electron with a kinetic energy of 10 eV, for instance, has a de Broglie wavelength of 3.9×10^{-10} m. That sounds small, but that wavelength is comparable in size to objects that an electron encounters. For instance, the spacing between atoms in a solid object is similar to the de Broglie wavelength of a 10 eV electron, and thus a crystal, with its regular array of atoms, can act as a diffraction grating for electrons. A sample diffraction pattern obtained by diffracting electrons from a crystal is shown in Figure 27.8. Similar patterns are obtained from diffraction with light.

Figure 27.8: An electron diffraction image obtained from electrons incident on a sample of anthopyllite asbestos. From this pattern, the crystal structure, including the spacing between the atoms, can be deduced. Image credit: California Department of Public Health.
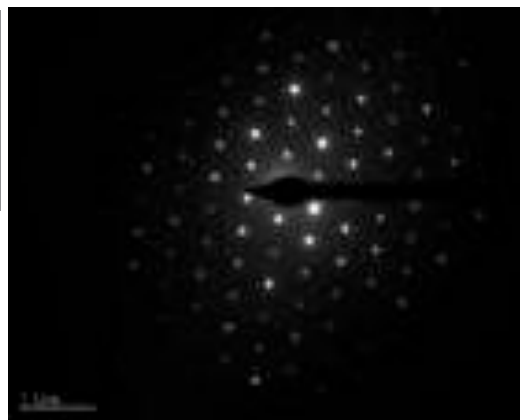
Experimental evidence for the de Broglie wavelength

In addition to the electron diffraction images that we just discussed, another persuasive piece of evidence supporting the idea of the de Broglie wavelength is the interference pattern obtained when electrons are incident on a double slit. Light of a particular wavelength produces a particular interference pattern when incident on a double slit. Replacing the light with electrons, which have a de Broglie wavelength equal to that of the wavelength of the light, results in the same interference pattern.

One application of the wave nature of electrons is in microscopy. One factor that limits the resolution of a light microscope is the wavelength of the light used. The same rules apply to an electron microscope, but the de Broglie wavelength of the electrons in an electron microscope can be 1000 times less than that of visible light. Such an electron microscope can resolve features 1000 times smaller than those resolved in a light microscope. An example of what an electron microscope can do is shown in Figure 27.9.
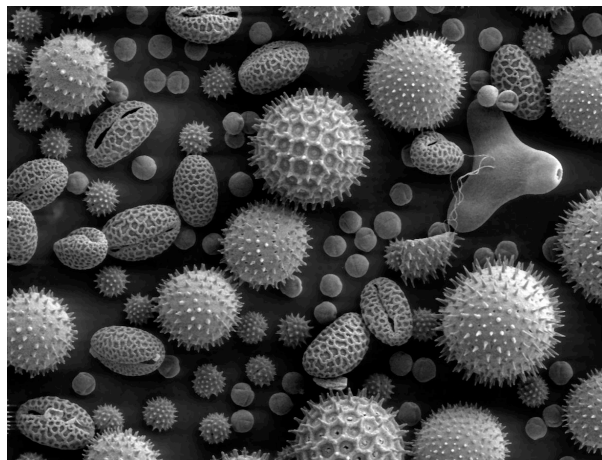
Experiments with other particles, including protons, neutrons, and hydrogen and helium atoms, have also been carried out, all of which have verified that such objects exhibit both a particle nature and a wave nature, with a wavelength given by the de Broglie equation.

Figure 27.9: An electron microscope image showing various types of pollen. Image credit: Dartmouth Electron Microscope Facility.

Wave-particle duality

The fact that everything, including ourselves, exhibits both a particle nature and a wave nature is known as wave-particle duality. Typically, to explain the result of a particular experiment, we use either the wave model or the particle model. However, recent experiments have shown interesting mixes of both. When electrons are incident on a double slit, for example, an interference pattern is produced on a screen beyond the slits – this shows the wave nature of electrons. If you add a detector that tells you which slit the electron passed through in every case, however, the pattern changes to that expected for particles. If your detector is faulty, however, and only tells you 30% of the time which slit an electron passed through, the resulting pattern is a mix of that expected from the wave nature (contributing 70% to the pattern, in this case) and that expected from the particle nature (the other 30%). This experiment also demonstrates how the act of making an observation can change the results of an experiment.

Related End-of-Chapter Exercises: 10, 21 – 25, 57.

Essential Question 27.5: Protons have a mass approximately 1800 times larger than the mass of the electron. If electrons traveling at a speed *v* produce a particular interference pattern when they encounter a particular double slit, with what speed would the protons have to travel to produce exactly the same interference pattern using the same double slit?