Wave-particle duality
Light is not the only thing that exhibits both a wave nature and a particle nature – everything exhibits such wave-particle duality. The wavelength of an object is inversely proportional to its momentum.

$$\lambda = \frac{h}{p} = \frac{h}{mv}.$$ (Equation 27.8: **the de Broglie wavelength**)

To explain the results of a particular experiment, usually either the wave nature or the particle nature is used.

Heisenberg's uncertainty principle
Quantum physics actually puts a limit on how accurately we can know something. More specifically, the uncertainties in two related quantities, such as the position and momentum of an object, are related in such a way that the smaller the uncertainty in one of the quantities, the larger the uncertainty has to be in the other quantity.

$$\Delta x \, \Delta p \geq \frac{h}{4\pi} = 5.273 \times 10^{-35} \text{ J s}.$$ (Equation 27.9: **Heisenberg's uncertainty principle**)

End-of-Chapter Exercises

Exercises 1 – 12 are conceptual questions that are designed to see if you have understood the main concepts of the chapter.

1. Astronomers can determine the temperature at the surface of a star by looking at the star's color. Explain how the color of a star corresponds to its temperature, and comment on whether a blue star has a higher surface temperature than does a red star, or vice versa.

2. An incandescent light bulb gives off a bright yellow-white glow when it is connected to a wall socket. If the potential difference across the bulb is reduced, however, not only does the bulb get dimmer, the emitted light takes on a distinct orange hue. Explain this.

3. With a particular metal plate, shining a beam of blue light on the metal causes electrons to be emitted via the photoelectric effect. If we reduce the intensity of the light shining on the metal, without changing its wavelength, what happens? Explain your answer.

4. The work functions of gold, aluminum, and cesium are 5.1 eV, 4.1 eV, and 2.1 eV, respectively. If light of a particular frequency causes photoelectrons to be emitted when the light is incident on an aluminum surface, explain if we know whether this means that photoelectrons are emitted from a gold surface or a cesium surface when the light is incident on those surfaces.

5. With a particular metal plate, shining a beam of green light on the metal causes electrons to be emitted. (a) If we replace the green light by blue light, do we know that electrons will be emitted? (b) If the two beams have the same intensity and are incident on equal areas of the plate, do we get the same number of electrons emitted per second in the two cases? Assume that the probability that a photon will cause an electron to be emitted is the same in both cases (e.g., for every two photons incident on the plate, one electron is emitted).

6. The diagram in Figure 27.11 represents the Compton effect collision between a photon and an electron. The diagram shows the paths followed by the incident and outgoing photons, and the wavelengths of these photons. Which is the incident photon and which is the outgoing photon?

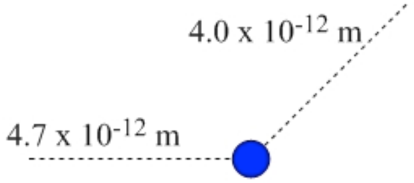


Figure 27.11: The diagram shows a representation of a particular Compton effect collision. The blue circle represents the initial position of the electron, which is at rest before the collision. The dashed lines represent the direction of the incident and outgoing photons – the lines are labeled with the photon wavelengths. For Exercise 6.

7. A photon is incident on an electron that is initially at rest. The photon experiences a Compton effect collision with the electron such that the photon, after the collision, is traveling in a direction exactly opposite to that of the incident photon. In what direction is the electron's velocity after the collision? Explain your answer.

8. One problem with solar sailboats is that the force they experience, because of photons from the Sun reflecting from the sails, cannot be directed toward the Sun. In an effort to overcome this problem, an inventor proposes a solar sailboat with a built-in light source. Instead of using sunlight to drive the sailboat, the inventor proposes attaching a high-power laser to the sailboat, and shining the light from the laser onto the sails. According to the inventor, by adjusting the angle of the sails, the sailboat can be made to turn in any direction, and, once pointed in the correct direction, the light from the laser can then propel the sailboat in the desired direction. What, if anything, is wrong with this idea?

9. Electrons are accelerated from rest and are then incident on a double slit. The pattern the electrons make on the screen is shown in Figure 27.12. If the potential difference through which the electrons are accelerated is reduced and a new pattern is observed on the screen, describe how the new pattern differs from that shown in Figure 27.12.

Figure 27.12: The pattern of dots created by electrons striking a screen a distance 2.40 m from a double slit, when an electron beam is incident on the slits. For Exercise 9.

Figure 27.13: A close up view of dots created by electrons striking a screen after passing through a double slit, for Exercise 10.

10. The interference pattern shown in Figure 27.12 for electrons that pass through a double slit is something of an idealization. If we take a close up view of the pattern on the screen, we see that it is really more like the pattern shown in Figure 27.13. How does this figure support the idea of wave-particle duality? In particular, comment on whether any part of the pattern is associated with the wave nature of electrons, and whether any part is associated with the particle nature of electrons.

11. Two experiments are carried out, using the same double slit. In the first experiment, a beam of electrons is incident on the double slit. In the second experiment, the electron beam is replaced by a beam of protons. In which experiment are the peaks in the interference patterns farther apart if the electrons and protons have equal (a) de Broglie wavelengths? (b) momenta? (c) speed? (d) kinetic energy?

12. To create a beam of fast-moving electrons, you accelerate electrons from rest by placing them between two metal plates that have a large potential difference across them. The electrons emerge from this system by means of a tiny hole you have drilled in the plate the electrons accelerate toward. You find, however, that instead of a narrow, well-defined beam, that the electrons are spread over a range of angles. (a) Come up with an explanation, based on the principles of physics covered in this chapter, to explain your observations. (b) Will the problem (the spread in the beam) get better or worse when you make the hole smaller? Explain. (c) Will the problem get better or worse when you increase the potential difference across the metal plates? Explain.

Exercises 13 – 16 involve the photoelectric effect.

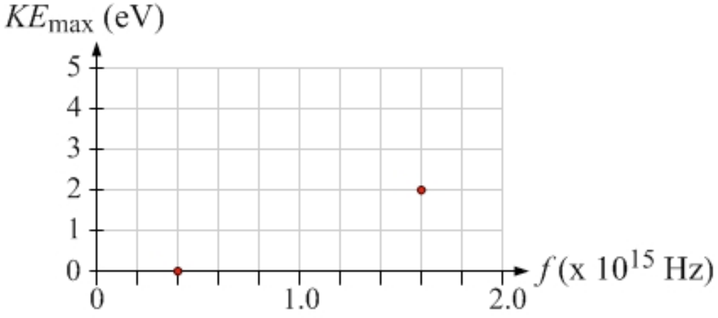
13. Gold has a work function of 5.1 eV. (a) What is the threshold (minimum) frequency of light needed to cause electrons to be emitted from a gold plate via the photoelectric effect? (b) If the frequency of the light incident on the gold plate is twice the threshold frequency, what is the maximum kinetic energy of the emitted electrons?

14. When ultraviolet light with a wavelength of 290 nm is incident on a particular metal surface, electrons are emitted via the photoelectric effect. The maximum kinetic energy of these electrons is 1.23 eV. (a) What is the work function of the metal? (b) What is the threshold frequency for this particular metal?

15. Iron has a work function of 4.5 eV. Plot a graph of the maximum kinetic energy (in eV) of photoelectrons emitted when light is incident on an iron plate as a function of the frequency of the light, which can vary from 0 to 2×10^{15} Hz.

16. An incomplete graph of the maximum kinetic energy (in eV) of photoelectrons emitted when incident light is incident on a particular metal is shown in Figure 27.14. Assume the two points shown are accurate. (a) What is the work function of the metal? (b) What is the threshold frequency in this case? (c) Complete the graph, to show the maximum kinetic energy at all frequencies up to 2×10^{15} Hz.

Figure 27.14: An incomplete graph of maximum kinetic energy of photoelectrons emitted from a particular metal plate, as a function of the frequency of the incident light. For Exercise 16.



Exercises 17 – 20 involve the Compton effect.

17. A photon with a wavelength of 5.0×10^{-12} m is incident on an electron that is initially at rest. If the photon that travels away from this collision is traveling in a direction that is at 120° to that of the incident photon, what is its wavelength?

18. A photon with a wavelength of 6.14×10^{-12} m is incident on an electron that is initially at rest. If the photon experiences a Compton effect collision with the electron, what is (a) the minimum possible, and (b) the maximum possible, wavelength of the photon after the collision? (c) In which direction does the electron travel after the collision in the situation described in (a) and (b)?

19. A photon is incident on an electron that is initially at rest. The photon experiences a Compton effect collision with the electron such that the photon, after the collision, is traveling in a direction perpendicular to that of the incident photon. (a) How does the wavelength of the photon after the collision compare to that of the photon before the collision? (b) Can the electron after the collision be traveling in a direction exactly opposite to that of the photon after the collision? Explain why or why not.

20. A photon collides with an electron that is initially at rest. After the collision, the photon has a wavelength of 4.0×10^{-12} m, and it is traveling in a direction that is at 45° to that of the incident photon. What is the wavelength of the incident photon?

Exercises 21 – 25 involve the de Broglie wavelength and wave-particle duality. Review Chapter 25 for some relevant concepts and equations.

21. When photons of a certain wavelength are incident on a particular double slit, the angle between the central maximum and one of the first-order maxima in the interference pattern is 10°. If the photons are replaced by electrons, and the electrons have a de Broglie wavelength less than that of the photon wavelength, what (if anything) will happen to the angle between the central maximum and one of the first-order maxima?

22. When light from a red laser, with a wavelength of 632 nm, is incident on a certain double slit, a particular interference pattern is observed. The laser light is replaced by a beam of electrons, all with the same energy, and exactly the same interference pattern is observed. What is (a) the wavelength, (b) frequency, and (c) energy of the electrons?

23. Electrons of a particular energy are incident on a double slit, producing an interference pattern with interference maxima and minima. When the electron energy is reduced, do the interference maxima get closer together, farther apart, or remain the same? Briefly justify your answer.

24. Figure 27.15 shows the $m = 0$ and $m = 1$ lines coming from an electron beam, in which the electrons have a de Broglie wavelength of 54 nm, that is incident on a double slit. The squares in the grid measure 20 cm × 20 cm. Determine the distance between the two slits in the double slit.

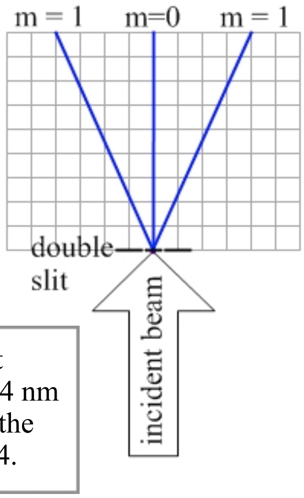


Figure 27.15: The $m = 0$ and $m = 1$ lines that result when electrons with a wavelength of 54 nm are incident on a double slit. The squares on the grid measure 20 cm × 20 cm. For Exercise 24.

25. Electrons of a particular energy are incident on a double slit. When the electrons are detected by a detector 3.0 m beyond the double slit, the distance between the central maximum and one of the first-order maxima in the interference pattern is found to be 8.0 mm. The distance between the slits in the double slit is 120 nm. What is the (a) wavelength, and (b) magnitude of the momentum of the electrons?

Exercises 26 – 30 involve the Heisenberg uncertainty principle. In all cases below, assume that the motion is one-dimensional.

26. What is the minimum uncertainty in an object's position if the object has a speed of 20 m/s and a mass of (a) 100 g? (b) 1×10^{-10} kg? (c) 1×10^{-25} kg?

27. According to the Heisenberg uncertainty principle, what is the minimum uncertainty in a proton's speed if the proton has an uncertainty in position of (a) 50 mm? (b) 500 nm? (c) 5.0×10^{-10} m?

28. Repeat Exercise 27, with an electron instead of a proton.

29. Imagine that you live in a strange world where Planck's constant *h* is 66.3 J s. You park your bike, which has a mass of 20 kg, in a location such that the uncertainty in the bike's location is 10 cm. What is the uncertainty in the bike's speed?

30. You place an electron on a wire that has a length of 6.0 nm, so that you know the electron is on the wire but you don't know exactly where it is. What is the uncertainty in the electron's momentum?

Exercises 31 – 34 involve applications of quantum physics.

31. In 2007, a team led by researchers at the University of Delaware announced a new world record for efficiency by a solar cell, at 42.8%. This means that the solar cell transformed 42.8% of the incident light energy into electrical energy. Estimate what area of these world-record solar cells would be required to supply the energy needs of a household of four people in the United States, based on an estimate of 2×10^{12} J of energy used annually by such a household. You can assume that the intensity of sunlight falling on the cells is 1000 W/m^2, and that, on average, the sun is shining on the solar cells for 6 hours per day.

32. Many luxury automobiles are now using xenon high-intensity discharge headlights. Compared to incandescent light bulbs, in which the filament is around 2800 K, the light given off by the high-intensity headlights has a spectrum similar to that of a black body at 4200 K. Does this help explain why the high-intensity discharge headlights have a blue tinge in comparison to the light from a standard incandescent bulb? Explain based on the principles of physics covered in this chapter.

33. One of the hot topics in research these days is the development of light-emitting diodes (LEDs) as a potential replacement for the incandescent bulbs that are commonly used for household lighting. The advantage of LEDs over incandescent bulbs is that LEDs are very efficient at transforming electrical energy into visible light, while it is commonly said that "incandescent bulbs give off 90% of their energy as heat." (a) Explain, using principles of physics, how an incandescent light bulb works. (b) Typically, the filament in an incandescent bulb is made from tungsten. Why is tungsten used instead of a cheaper, more readily available metal such as aluminum? (c) What is meant by the phrase "incandescent bulbs give off 90% of their energy as heat?"

34. Ernst Ruska was awarded a 50% share of the Nobel Prize in Physics in 1986. Write a couple of paragraphs regarding the work he did, mentioning in particular how the work is connected to the principles of physics discussed in this chapter.

35. An infrared thermometer is a thermometer that can determine an object's temperature without the thermometer needing to make contact with the object. For instance, you could aim the thermometer at the hot water flowing out of the faucet in your kitchen, and the thermometer would read the temperature of the hot water. (a) Briefly explain, using the principles of physics covered in this chapter, how such a thermometer works. (b) Explain why the thermometer is known as an infrared thermometer.

General problems and conceptual questions

36. One morning, as you wait for the toaster to finish toasting a couple of slices of bread, you measure the spectrum of light being emitted by the glowing toaster elements. The peak wavelength of this light is 600 nm. What is the temperature of the toaster elements?

37. You have probably heard the phrase "white hot" before. Approximately what temperature is a black body that is hot enough to look white?

38. A typical helium-neon laser pointer, emitting light with a wavelength of 632 nm, has a beam with an intensity of 800 W/m^2 and a diameter of 3.00 mm. How many photons are emitted by the laser pointer every second?

39. The work functions of gold, aluminum, and cesium are 5.1 eV, 4.1 eV, and 2.1 eV, respectively. When light of a particular frequency shines on a cesium surface, the maximum kinetic energy of the emitted photoelectrons is 2.5 eV. What is the maximum kinetic energy of the photoelectrons emitted when the same light shines on (a) an aluminum surface? (b) a gold surface?

40. We consider the visible spectrum to run from 400 nm to 700 nm. (a) What is the equivalent energy range, in eV, of photons for light in the visible spectrum? If white light, composed of wavelengths covering the full range of the visible spectrum but no more, is incident on a surface, will photoelectrons be emitted if the surface is (b) carbon (work function = 4.8 eV)? (c) potassium (work function = 2.3 eV)? Explain your answers to parts (b) and (c).

41. Zinc has a work function of 4.3 eV. A standard lecture demonstration involves attaching a zinc plate to an electroscope, and then charging the plate by rubbing it with a negatively charged rod. At that point, the electroscope indicates that it is charged, and we know that the charge is negative. (a) If light in the visible spectrum, from a bright incandescent light bulb, shines on the plate, do we expect photoelectrons to be emitted from the plate, causing the electroscope to discharge? Why or why not? (b) What is the maximum wavelength of light that would cause the electroscope to discharge? What part of the spectrum is this wavelength in?

42. Table 27.3 gives a set of data for a photo-electric effect experiment, showing the maximum kinetic energy of the emitted electrons at a number of different frequencies of the light incident on a particular metal plate. Plot a graph of the maximum kinetic energy as a function of frequency and determine, from your graph, (a) the threshold frequency, (b) the work function of the metal, and (c) the slope of that part of the graph above the threshold frequency.

Frequency ($\times 10^{15}$ Hz)	K_{max} (eV)
0.5	0
1.0	0
1.5	1.9
2.2	4.8
2.9	7.7
3.7	11.0

Table 27.3: A set of data from a photo-electric effect experiment, showing the maximum kinetic energy of the emitted electrons at a number of different photon frequencies, for Exercise 35.

43. In a particular photoelectric effect experiment, photons with an energy of 5.0 eV are incident on a surface, producing photoelectrons with a maximum kinetic energy of 2.2 eV. If the energy of the photons doubles, does the maximum kinetic energy of the photoelectrons also double? Explain your answer.

44. In a particular photoelectric effect experiment, photons with an energy of 5.6 eV are incident on a surface, producing photoelectrons with a maximum kinetic energy of 3.3 eV. (a) What is the work function of the metal? (b) What is the minimum photon energy necessary to produce a photoelectron in this situation? (c) What is the corresponding threshold frequency?

45. With a particular metal plate, shining a beam of green light on the metal causes electrons to be emitted. (a) If we replace the green light by red light, do we know that electrons will be emitted? (b) If the two beams have the same intensity and are incident on equal areas of the plate, do we get the same number of electrons emitted per second in the two cases, assuming that photons are emitted in both cases? Assume that the probability that a photon will cause an electron to be emitted is the same in both cases (e.g., for every two photons incident on the plate, one electron is emitted).

46. Figure 27.16 shows a graph of the maximum kinetic energy of emitted photoelectrons as a function of the energy of the photons that are incident on a particular surface. From this graph, determine (a) the work function of the surface, and (b) the threshold frequency.

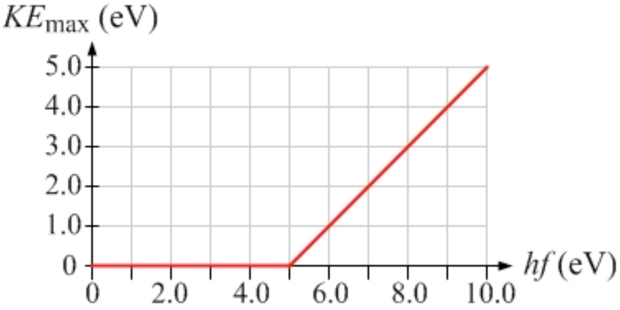


Figure 27.16: This graph shows the maximum kinetic energy of emitted electrons as a function of the energy of the incident photons, for Exercise 46.

47. A green helium-neon laser pointer emits light with a wavelength of 532 nm. The beam has an intensity of 900 W/m^2 and a diameter of 4.00 mm. (a) How many photons are emitted by the laser pointer every second? (b) What is the magnitude of the momentum of each of these photons? How much momentum is imparted to an object, in a 1.00-second interval, that (c) completely absorbs all these photons? (d) reflects all these photons straight back toward the laser pointer?

48. A laser beam that is completely absorbed by a black surface exerts a force of 2.5×10^{-8} N on the surface. (a) What is the net momentum transferred to the surface by the beam every second? (b) If the wavelength of light emitted by the laser is 632 nm, what is the magnitude of the momentum of each photon in the beam? (c) How many photons strike the surface every second?

49. A photon with a wavelength of 6.00×10^{-12} m is incident on an electron that is initially at rest. If the photon that travels away from this collision is traveling in a direction that is at 45° to that of the incident photon, what is its wavelength?

50. Return to the situation described in Exercise 49. Relative to the direction of the incident photon, in what direction does the electron travel after the collision?

51. Photons with a particular wavelength experience Compton-effect collisions with electrons that are at rest. Figure 27.17 shows a graph of the wavelength of the photons that travel away from the various collisions as a function of the cosine of the scattering angle (this is the angle between the direction of the incident photon and the direction of the photon after the collision). For the incident photons, determine the (a) wavelength, (b) frequency, and (c) energy, in eV.

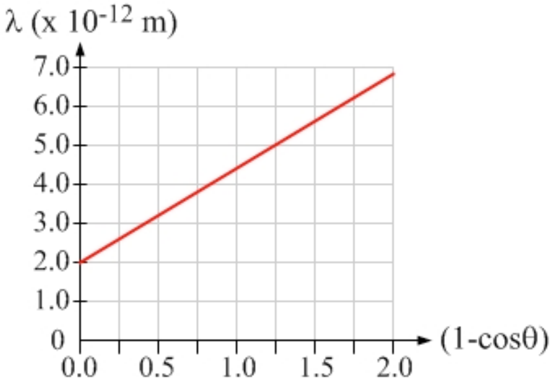


Figure 27.17: A graph of the wavelength of the photon after colliding with a stationary electron in a Compton effect collision, as a function of 1 minus the cosine of the scattering angle. The scattering angle is the angle between the direction of the incident photon and the direction of the photon after the collision. For Exercises 51 and 52.

52. Photons with a particular wavelength experience Compton-effect collisions with electrons that are at rest. Figure 27.17 shows a graph of the wavelength of the photons that travel away from the collision as a function of 1 minus the cosine of the scattering angle. We repeat the experiment with incident photons that have wavelengths twice as large as that of the original incident photons. If we plot a new graph, like that in Figure 27.17, how will the graphs compare in terms of their (a) slopes, and (b) *y*-intercepts?

53. A photon is incident on an electron that is initially at rest. The photon and electron experience a Compton-effect collision such that the electron, after the collision, is traveling in the same direction that the photon was before the collision. The electron's momentum after the collision is 5.0×10^{-22} kg m/s. (a) Write down an expression to show how the wavelength of the photon after the collision is related to that of the photon after the collision. (b) Write down an expression to show how the momentum of the photon before the collision is related to the momentum of the photon after the collision and the electron's momentum after the collision. Now solve your two equations to find the wavelength of the photon (c) before the collision, and (d) after the collision.

54. Electrons with an energy of 8.00 eV are incident on a double slit in which the two slits are separated by 400 nm. What is the angle between the two second-order maxima in the resulting interference pattern?

55. A beam of electrons, shining on a double slit, creates the pattern shown in Figure 27.18 at the center of a screen placed 2.40 m on the opposite side of the double slit from the electron source. If the de Broglie wavelength of the electrons is 135 nm, what is the distance between the two slits in the double slit?

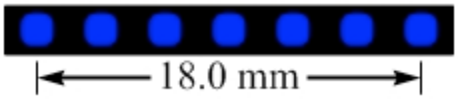


Figure 27.18: The pattern of dots created by electrons striking a screen a distance 2.40 m from a double slit, when an electron beam is incident on the slits. For Exercises 55 – 56.

56. A beam of electrons, shining on a double slit, creates the pattern shown in Figure 27.18 at the center of a screen placed 1.50 m on the opposite side of the double slit from the electron source. (a) If the distance between the two slits is 500 nm, what is the de Broglie wavelength of the electrons? (b) If the electrons achieved this wavelength by being accelerated from rest through a potential difference, what is the magnitude of the potential difference?

57. A baseball has a mass of 150 g. With what speed would a baseball have to be moving so that its de Broglie wavelength is 632 nm, the same as the wavelength of light from a red helium-neon laser?

58. Consider the opening image of this chapter, showing the head of an ant as imaged by an electron microscope. To be able to resolve tiny details, with a size much less than the wavelength of visible light, the de Broglie wavelength of the electrons also needs to be tiny. (a) What is the speed of an electron that has a de Broglie wavelength of 10 nm? (b) If the electron achieved this speed by being accelerated from rest through a potential difference, what was the magnitude of the potential difference?

59. To create a beam of fast-moving electrons, you accelerate electrons from rest by placing them between two metal plates that have a large potential difference across them. The electrons emerge from this system by means of a tiny hole you have drilled in the plate the electrons accelerate toward. You find, however, that instead of a narrow, well-defined beam, that the electrons are spread over a range of angles. To help solve this problem, make use of Equation 25.6, which states that the angle of the first minimum in the diffraction pattern from a circular opening of diameter D is $\theta_{\min} = 1.22\lambda / D$. (a) If the electrons are accelerated from rest through a potential difference of 500 V, what is their de Broglie wavelength? (b) What diameter hole is required for the beam to diffract through the hole so that $\theta_{\min} = 10°$? (c) If the hole diameter is reduced by a factor of 2, what is $\theta_{\min}$? (d) If, instead, the potential difference is doubled, what is $\theta_{\min}$?

60. Two students are having a conversation. Comment on the part of their conversation that is reported below. This one may require a little background reading.

Maggie: I really don't buy this wave-particle duality stuff. I mean, let's say we do this. We send an electron beam through a single slit. When the slit is wide, the electrons go straight through. The narrower we make the slit, the greater the probability that the electrons hit one of the edges of the slit, changing the direction. The narrower the slit, the greater the spread in the beam after passing through the slit – who needs waves, it's all particles.

Dipesh: That sounds sensible. Except, when you look at the pattern carefully, there are some angles where you don't get any electrons going – can you explain that just with particles?

Maggie: Oh, good point. OK, let's think about the double slit a little. What if, instead of firing a whole beam of electrons at the double slit, you just did one electron at a time. Wouldn't the electron have to choose one slit or the other? Plus, we shouldn't get interference any more - one electron wouldn't be able to interfere with itself, right?

Dipesh: What a cool idea. Let's google it and see if anyone has done an experiment like that. I'll type in "single electron interference double slit" and see what comes up.