Answer to Essential Question 26.4: According to you, 50 years elapses on your clock during Isabelle's trip to Zorg. Using the time dilation equation with a relative speed of 0.8*c,* we can show that Isabelle's clock is running at 60% of the rate of yours, according to you. Thus, when 50 years passes by on your clock, only 30 years passes by on Isabelle's clock, according to you. This is consistent with the result of part (d) in Example 26-4.

As we showed in part (d), according to Isabelle, the time interval between Earth passing her and Zorg passing her to is 30 years. However, if you see her clock running at 60% of the rate of yours, Isabelle sees your clock running at 60% of the rate of hers. 60% of 30 years is 18 years, so, according to Isabelle, your clock only reads 18 years when Isabelle and Zorg meet.

26-5 The Breakdown of Simultaneity

Let's explore the behavior of time in more detail, beginning with extending our understanding of spacetime diagrams. From our work earlier in the book, we are used to being able to transform from one coordinate system to another by simply rotating the *x* and *y* axes through the same angle, as demonstrated in Figure 26.12. The two coordinate systems share the same origin, and a point such as $x = +3$ m, $y = +4$ m (a distance of 5 m from the origin) transforms into $x' = +5$ m, $y' = 0$ (not coincidentally also 5 m from the origin). For any point a distance *r* from the origin, we have the equivalent of the spacetime interval equation,

$$x^2 + y^2 = (x')^2 + (y')^2 = r^2 .$$

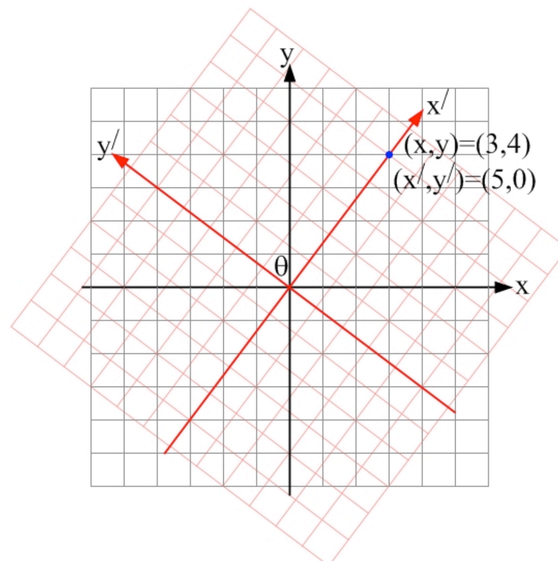

Figure 26.12: Rotating the *x* and *y* axes through the same angle transforms coordinate systems in the *x*-*y* plane.

We can something similar with the spacetime diagram, except that the space and time axes rotate in opposite directions. The angle of rotation depends on the relative velocity between one reference frame and the other. With the super-fast mosquito we looked at in Exploration 26-2, the mosquito's time axis, as viewed from your frame of reference, was rotated as it was because the mosquito was moving at half the speed of light with respect to you. Similarly, the mosquito's *x*-axis (as seen from your reference frame) is rotated through the same angle, but in the opposite direction, so that the slope of the mosquito's *x*-axis is equal to the mosquito's speed, with respect to you, expressed as a fraction of the speed of light. The mosquito's coordinate system, according to you, is shown in Figure 26.13.

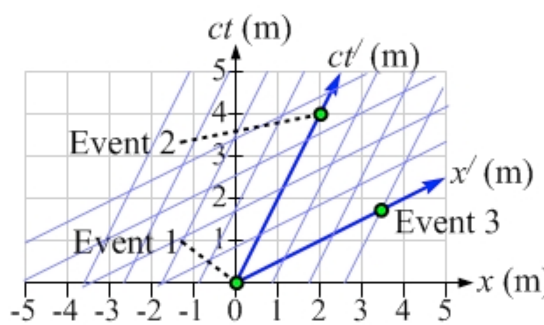

Figure 26.13: Transforming coordinate systems on a spacetime diagram is accomplished by rotating the time and space axes in opposite directions, by an angle determined by the relative velocity between the two coordinate systems.

Examining events 1 and 3 in Figure 26.13, we can see that events which are simultaneous in one reference frame may not be simultaneous in another. Events 1 and 3 are on the mosquito's *x*-axis, which means they take place at the same instant (at $t' = 0$, in this case) and are thus simultaneous in the mosquito's frame of reference. In your reference frame, however, Event 1 occurs before Event 3. Simultaneity is relative! In a situation like this, events are simultaneous for observers in different reference frames only when they occur at the same time and place.

Let's construct a spacetime diagram, from your frame of reference, for Isabelle's travels, from Example 26.4. Isabelle is traveling at 0.8c with respect to you so, in your reference frame, Isabelle's coordinate system is rather distorted. This is shown in Figure 26.14. If you and an observer called Yan, who is in your frame of reference but located on Zorg, synchronize your clocks, according to you and Yan, all four of the following events are simultaneous:

- Isabelle passes Earth
- Your clock reads zero
- Isabelle's clock reads zero
- Yan's clock reads zero

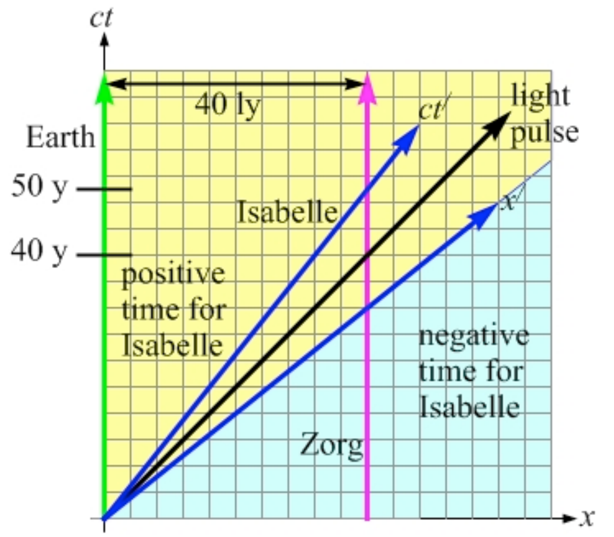

Figure 26.14: A spacetime diagram, from your frame of reference, for the situation of Isabelle's travels between Earth and Zorg. The boxes on the grid measure 4 ly × 4 ly.

Isabelle agrees that the first three events are simultaneous, but she disagrees that Yan's clock reads zero at that instant. From the spacetime diagram, we can divide spacetime into locations at which Isabelle records a positive time on her clock, and locations where Isabelle records a negative time. According to Isabelle, Yan's clock reads zero long before your clock reads zero. In fact, according to Isabelle, Yan's clock is set 32 years ahead of your clock! This helps explain a mystery regarding Essential Question 26.4. You think Isabelle takes 50 years to reach Zorg, and Yan sends a message to you that, when Isabelle passed Zorg, Yan's clock read 50 years and Isabelle's read 30 years. This makes complete sense to you. Isabelle's message to you, however, states that, when Zorg passed her, Yan's clock read 50 years, her clock read 30 years, but that your clock read only 18 years. This is consistent with Isabelle observing both your clock and Yan's clock to be running slow, but with Yan's clock 32 years ahead of your clock. During her trip, according to Isabelle, 18 years ticked by on both your clock and Yan's clock.

Differences in clock readings are NOT associated with the travel time of light
You should be clear that the different times and lengths recorded by observers in different reference frames are not caused by the observers being separated by some distance and light taking some time to travel from one observer to another. When we say things like "When Isabelle reaches Zorg, from her reference frame your clock reads 18 years," that is not the reading that Isabelle would see on your clock at that instant, looking back many light-years through a powerful telescope to Earth. Instead, we imagine multiple observers at different locations in Isabelle's reference frame, their clocks synchronized with Isabelle's. Each of these observers, when they pass Earth, can send a message to Isabelle to tell her what their clock reading was, and what your clock was reading, when they passed Earth. Many years after passing Zorg, Isabelle will finally get the message confirming what your clock was reading when she passed Zorg, according to an observer in her reference frame. Thus, the statement "When Isabelle reaches Zorg, from her reference frame your clock reads 18 years" is interpreted as "When Isabelle reaches Zorg, the observer in Isabelle's frame of reference who is next to your clock records that your clock reads 18 years." The observer is right there, so light travel time is not an issue.

Related End-of-Chapter Exercises: 44 – 46.

Essential Question 26.5: According to Isabelle, how many years after passing Zorg would she get the message stating what your clock read when she passed Earth, from the observer in Isabelle's frame of reference who sees Earth pass by when Isabelle sees Zorg pass by?