Answer to Essential Question 26.3: Who ages more slowly depends on who you ask. According to Jack, time passes more slowly for Jill, who is moving with respect to Jack, and thus Jack says that Jill is aging more slowly. According to Jill, time passes more slowly for Jack, who is moving with respect to Jill, and thus Jill says that Jack is aging more slowly. This brings us to the famous **twin paradox**, in which one twin goes off at high speed to explore a distant galaxy, and returns some years later to find that the twin who remained behind on Earth is considerably older than the traveling twin. The apparent paradox is why there is no symmetry in this situation. The twin who stayed behind should see the traveling twin to be aging more slowly, but the traveling twin should see the twin who stayed behind to be aging more slowly. The resolution of the paradox is that the situation is not symmetric – the traveling twin changed reference frames halfway through, from a frame of reference in which she travels away from the Earth to a new frame of reference in which she travels toward the Earth. In both reference frames, interestingly, she does observe the twin who remained on Earth to be aging more slowly than she is, but she also observes the age of the twin who remained behind to jump when the traveling twin switches reference frames.

26-4 Length Contraction

Time is not the only thing that behaves in an unusual way, space does too. Observers who view objects moving with respect to them, for instance, measure the length of the object to be contracted (shorter) along the direction of motion.

Let's now work through an example that ties together all of the ideas we have discussed thus far in the chapter

Length contraction: Assume that two points separated by some distance are in the same reference frame. An observer measures the proper length L_{proper} between these two points when the points are at rest with respect to that observer (that is, the observer is in the same frame of reference as the points). For an observer in a different reference frame, when the displacement between the points has a component parallel to the velocity of the points, the observer measures a contracted length between the points. This length contraction is most pronounced when the displacement from one point to the other is parallel to the velocity, in which case the length L measured by the observer is

$$L = \sqrt{1 - \frac{v^2}{c^2}}\, L_{proper} = \frac{L_{proper}}{\gamma} ,$$ (Equation 26.3: **Length contraction**)

where v is the relative speed between the two reference frames.

EXAMPLE 26.4 – Isabelle's travels

Let's say that Planet Zorg is at rest with respect to the Earth, and that in the reference frame of the Earth, the distance between Earth and Zorg is 40 light-years. Isabelle is passing by the Earth in her rocket ship traveling toward Zorg at a constant velocity of 0.8 c. At the instant she passes by, you, who are on Earth, send a light pulse toward Zorg.

(a) According to you, how long does the light pulse take to reach Zorg?
(b) According to you, how long does Isabelle take to reach Zorg?
(c) According to Isabelle, what is the spatial distance between the event of Isabelle passing Earth and the event of Isabelle arriving at Zorg?
(d) Use Equation 26.1 to find the time it takes Isabelle to reach Zorg, according to Isabelle.
(e) What is the distance between Earth and Zorg, according to Isabelle?
(f) How long does the light pulse take to reach Zorg, according to Isabelle?

SOLUTION

(a) According to you, the planets are separated by a distance of 40 light-years. Because light covers 1 light-year every year, it takes the light pulse 40 years to reach Zorg, according to you.

(b) According to you, Isabelle travels at 4/5 the speed that light does, so Isabelle should take 5/4 the time that light does.

$$t = \frac{40 \text{ light-years}}{0.8\,c} = 50\frac{\text{light-years}}{c} = 50 \text{ years}.$$

Note that light-years divided by the speed of light in vacuum, *c*, gives years. Also, note that we don't have to use a relativistic equation to get the answer. We are, instead, using the basic idea that for motion with constant speed, time is simply the distance divided by the speed.

(c) For Isabelle, these two events happen at the same location, right outside Isabelle's rocket. Thus, their spatial separation, according to Isabelle, is $\Delta x' = 0$.

(d) Equation 26.1 is the equation for the spacetime interval. According to you, the time between the events is 50 years which, when multiplied by *c*, gives $c\Delta t = 50$ light-years, and the events are separated spatially by $\Delta x = 40$ light-years. The spacetime interval is given by:

$$(\text{spacetime interval})^2 = (c\Delta t)^2 - (\Delta x)^2 = (50 \text{ light-years})^2 - (40 \text{ light-years})^2 = (30 \text{ light-years})^2.$$

Solving for the time interval from Isabelle's perspective, we get:

$$(c\Delta t')^2 = (\text{spacetime interval})^2 - (\Delta x')^2 = (30 \text{ light-years})^2 - 0 = (30 \text{ light-years})^2.$$

In Isabelle's reference frame, we have $c\Delta t' = 30$ light-years. Dividing by *c* gives a time interval of 30 years, so it only takes Isabelle 30 years to go from Earth to Zorg, according to Isabelle.

Note that 30 years is also the proper time between the two events, because Isabelle is present at both events. The proper length between Earth and Zorg, on the other hand, is the 40 light-years measured by you, because the planets are at rest in your reference frame. At this point, you may be concerned that it looks like Isabelle travels faster than light, since she travels to Zorg in 30 years while light travels there in 40 years, but we are about to resolve that.

(e) We could use the length contraction equation to determine the distance between the planets, according to Isabelle. A simpler method is that, according to Isabelle, Zorg travels toward her at a constant velocity of 0.8 *c*, and Zorg passes her 30 years after Earth passes her. Thus, Zorg must have covered a distance of 0.8 *c* × 30 years, which is 24 light-years, in that time, which represents the distance between the planets, according to Isabelle.

(f) According to Isabelle, the planets are separated by a distance of 24 light-years. Isabelle sees the light pulse traveling away from her at *c*, and Zorg coming toward her at 0.8*c*, for a relative speed between them of 1.8*c*, according to Isabelle. Thus, according to Isabelle, the pulse and Zorg would meet at a time of 24 light-years divided by 1.8*c*, or 13.3 years. Both you and Isabelle agree that nothing travels faster than light in this situation, by the way.

Related End-of-Chapter Exercises: 6, 23 – 25.

Essential Question 26.4: If your clock and Isabelle's clock both read zero when Isabelle passes Earth, what will the clocks read, according to both you and Isabelle, when Isabelle reaches Zorg?