Answer to Essential Question 26.1: (a) The obvious answer is that you are at rest. However, the question really only makes sense when we ask what the speed is measured with respect to. Typically, we measure our speed with respect to the Earth's surface. If you answer this question while traveling on a plane, for instance, you might say that your speed is 500 km/h. Even then, however, you would be justified in saying that your speed is zero, because you are probably at rest with respect to the plane. (b) Your speed with respect to a point on the Earth's axis depends on your latitude. At the latitude of New York City (40.8° north) , for instance, you travel in a circular path of radius equal to the radius of the Earth (6380 km) multiplied by the cosine of the latitude, which is 4830 km. You travel once around this circle in 24 hours, for a speed of 350 m/s (at a latitude of 40.8° north, at least). (c) The radius of the Earth's orbit is 150 million km. The Earth travels once around this orbit in a year, corresponding to an orbital speed of 3×10^4 m/s. This sounds like a high speed, but it is too small to see an appreciable effect from relativity.

26-2 Spacetime and the Spacetime Interval

We usually think of time and space as being quite different from one another. In relativity, however, we link time and space by giving them the same units, drawing what are called spacetime diagrams, and plotting trajectories of objects through spacetime. A spacetime diagram is essentially a position versus time graph, with the position axes and time axes reversed.

EXPLORATION 26.2 – A spacetime diagram

We can convert time units to distance units by multiplying time by a constant that has units of velocity. The constant we use is *c*, the speed of light in vacuum.

Step 1 – ***Plot a graph with position on the x-axis and time, converted to distance units, on the y-axis. This graph is a spacetime diagram. We are at rest in this coordinate system, which means that the spacetime diagram is for our frame of reference. At t = 0 and x = 0, we send a pulse of light in the positive x-direction. On the graph, show the trajectory of this light pulse, which travels at the speed of light in vacuum. How far does the pulse travel in 5 meters of time?*** The spacetime diagram is shown in Figure 26.5. Because the pulse of light travels at the speed of light, the pulse's trajectory is a straight line with a slope of 1. The pulse travels 5 m of distance in 5 m of time, which makes things easy to plot.

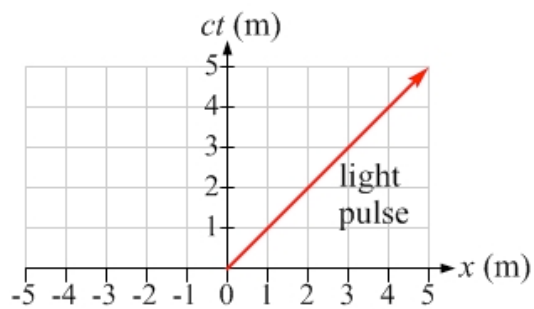

Figure 26.5: A spacetime diagram, showing the trajectory of a light pulse that travels in the +*x*-direction.

Step 2 – ***On the spacetime diagram, plot the trajectory (this is called a worldline) of a superfast mosquito that passes through x = 0 at t = 0. With respect to us, the mosquito moves in the positive x-direction at half the speed of light. What is the connection between the slope of the worldline and the velocity of the mosquito?*** This spacetime diagram is shown in Figure 26.6. Because the mosquito's velocity is constant, the slope of its worldline is constant. The mosquito is always half the distance from the origin that the light pulse is (at least according to us!).

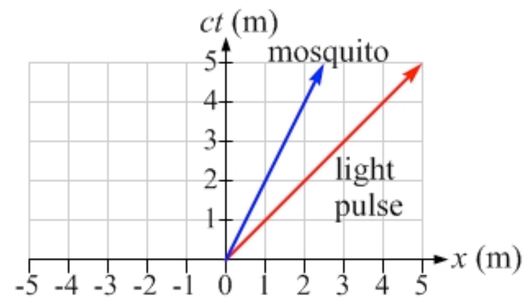

Figure 26.6: The spacetime diagram, showing the worldline of a mosquito traveling in the +*x*-direction at half the speed of light.

The slope of an object's worldline is the inverse of the velocity, if the velocity is expressed as a fraction of *c*. In this case, the mosquito has a velocity of +0.5, so the slope of its worldline is +2.

Step 3 – ***Add two events to the spacetime diagram. According to us, Event 1 occurs at x = 0 at t = 0, and Event 2 occurs at x = +2 m at t = +4 m of time. According to us, what is the time interval between the two events, and what is their spatial separation?***

Note that we define an event as something that takes place at a particular point in space and at a particular instant in time. The amended spacetime diagram with the events marked on it is shown in Figure 26.7. The time interval, converted to distance units, between the two events is $c\Delta t = 4\ \text{m}$. The spatial separation is $\Delta x = 2\ \text{m}$.

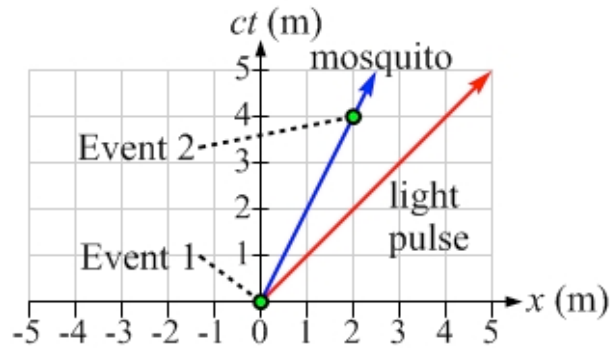

Figure 26.7: The spacetime diagram, now showing the space and time coordinates, as measured from our reference frame, of two events.

The spacetime interval: It turns out that observers in different constant-velocity reference frames always agree on the value of the spacetime interval between two events, as defined by

$$(c\Delta t)^2 - (\Delta x)^2 = (c\Delta t')^2 - (\Delta x')^2 = (\text{spacetime interval})^2 , \qquad \text{(Eq. 26.1: \textbf{The spacetime interval})}$$

where $c\,\Delta t$ and $c\,\Delta t'$ are the time intervals, converted to distance units, between the two events as measured from two different frames of reference, and Δx and $\Delta x'$ are the spatial separations between the two events, as measured in the same two reference frames. Note that, if the left-hand side of the equation gives a negative number, it is appropriate to reverse the order of the terms.

Step 4 – ***What is the spatial separation $\Delta x'$ between Event 1 and Event 2 as measured by the mosquito? Knowing this, use Equation 26.1 to find the time interval $c\Delta t'$ between the events, as measured by the mosquito.*** Both of the events lie on the worldline of the mosquito, so the mosquito thinks that they both take place at $x' = 0$. Thus, the spatial separation between them, as measured by the mosquito, is $\Delta x' = 0 - 0 = 0$. Let's now work out the value of the spacetime interval between these events, as measured by us.

$$(\text{spacetime interval})^2 = (c\Delta t)^2 - (\Delta x)^2 = (4\ \text{m})^2 - (2\ \text{m})^2 = 12\ \text{m}^2 .$$

In the mosquito's reference frame, then, the time difference is given by

$$(c\Delta t')^2 = (\text{spacetime interval})^2 - (\Delta x')^2 = 12\ \text{m}^2 - 0 = 12\ \text{m}^2 .$$

Thus, we observe that 4 m of time pass between the events, while the mosquito observes only $\sqrt{12}\ \text{m} = 3.5\ \text{m}$ of time passing between them. The main point here is that observers in different reference frames measure different amounts of time passing between the same two events.

Key ideas: In relativity, the emphasis is often on what is different in two different reference frames. However, as we have learned earlier (such as with energy conservation) the parameters that everyone agrees on are generally most important. In relativity, all observers agree on the value of the spacetime interval. In addition, in relativity we treat space and time as different components of a spacetime coordinate system, rather than treating them as completely different things, as we are more used to doing. **Related End-of-Chapter Exercises: 9 – 18.**

Essential Question 26.2: Return to Exploration 26.2. Add your worldline to the spacetime diagram shown in Figure 26.7.