Answer to Essential Question 25.6: The shift for the wave reflecting from the top surface, Δ_t, depends on how n_2 compares to n_1. The shift for the wave reflecting from the bottom surface, Δ_b, depends on how n_3 compares to n_2.

	$n_2 > n_1$	$n_2 < n_1$
$n_3 > n_2$	$\Delta_t = \lambda_{film}/2$ $\Delta_b = 2t + (\lambda_{film}/2)$	$\Delta_t = 0$ $\Delta_b = 2t + (\lambda_{film}/2)$
$n_3 < n_2$	$\Delta_t = \lambda_{film}/2$ $\Delta_b = 2t$	$\Delta_t = 0$ $\Delta_b = 2t$

Table 25.3: Summarizing the various results for Δ_t and Δ_b.

25-7 Applying the Five-Step Method

EXPLORATION 25.7 – Designing a non-reflecting coating
High-quality lenses, such as those for binoculars or cameras, are often coated with a thin non-reflecting coating to maximize the amount of light getting through the lens. We can apply thin-film ideas to understand how such a lens works. Explaining why such lenses generally look purple will also be part of our analysis. In this example, we will assume light is traveling through air before it encounters the non-reflective coating (n = 1.32) that is on top of the glass (n = 1.52). Figure 25.26 shows the arrangement. The coating is completely non-reflective for just one wavelength, so we will design it to be non-reflective for light with a wavelength in vacuum of 528 nm, which is close to the middle of the visible spectrum.

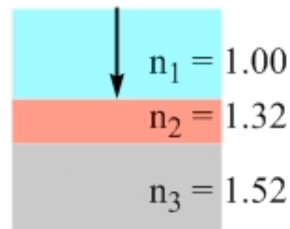

Figure 25.26: The arrangement of air (top), coating (middle), and glass (bottom) for a typical situation of a non-reflective coating on a glass lens.

Step 1 – *Determine Δ_t, the shift for the wave reflecting from the air-coating interface.* Because the coating has a higher index of refraction than the air, this wave is inverted upon reflection, giving $\Delta_t = \lambda_{film}/2$.

Step 2 – *Determine Δ_b, the shift for the wave reflecting from the coating-glass interface.* The glass has a higher index of refraction than the coating, so this wave is also inverted upon reflection. For a coating of thickness t, $\Delta_b = 2t + (\lambda_{film}/2)$.

Step 3 – *Determine Δ, the effective path-length difference.* $\Delta = \Delta_b - \Delta_t = 2t$.

Step 4 – *Bring in the appropriate interference condition.* In this situation, we do not want light to reflect from the coating. We can accomplish this by having the reflected waves interfere destructively. Setting the effective path-length difference equal to (m + 1/2) wavelengths gives:

$$2t = (m + 1/2)\lambda_{film} .$$

Step 5 – *Solve for the minimum possible coating thickness.* To solve for the smallest possible coating thickness, we choose the smallest value of m that makes sense, remembering that m is an integer. In this case, m = 0 gives the smallest coating thickness.

$$2t_{min} = (0 + 1/2)\lambda_{film} \qquad \Rightarrow \qquad t_{min} = \frac{\lambda_{film}}{4} = \frac{\lambda_{vacuum}}{4n_{film}} = \frac{528 \text{ nm}}{4 \times 1.32} = 100 \text{ nm} .$$

Step 6 – ***If a 100-nm-thick film produces completely destructive interference for 528 nm green light, what kind of interference will it produce for the violet end of the spectrum (400 nm) and the red end of the spectrum (700 nm)? Why does this make the lens look purple in reflected light?*** In the coating, 400 nm violet light has a wavelength of 400 nm / 1.32 = 303 nm. Thus, an effective path-length difference of $2t$ = 200 nm shifts one reflected violet wave relative to another by 200 nm / 303 nm, a shift of about 2/3 of a wavelength. The interference is partly destructive, so some violet light reflects from the coating. For red light of 700 nm, with a wavelength in the film of 700 nm / 1.32 = 530 nm, the relative shift is 200 nm / 530 nm = 0.38 wavelengths. Again, this produces partly destructive interference, so some red light reflects. When white light shines on the film, therefore, almost no green light is reflected, small amounts of yellow and blue are reflected, a little more orange and indigo are reflected, and even more red and violet are reflected. Thus, the reflected light is dominated by red and violet, which makes the film look purple.

Key ideas: The five-step method can be applied in all thin-film situations, to help us relate the film thickness to the wavelength of light. **Related End-of-Chapter Exercises: 31, 32, 54.**

EXAMPLE 25.7 – A soap film

A ring is dipped into a soap solution, creating a round soap film. (a) When the ring is held vertically, explain why horizontal bands of color are observed, as seen in Figure 25.27(a). (b) As time goes by, the film gets progressively thinner. Where the film is very thin, no light reflects from the film, so it looks like the film is not there anymore, as in the top right of Figure 25.27(b). Apply the first three steps of the five-step method to explain why, in the limit that the film thickness approaches zero, the two reflected waves interfere destructively.

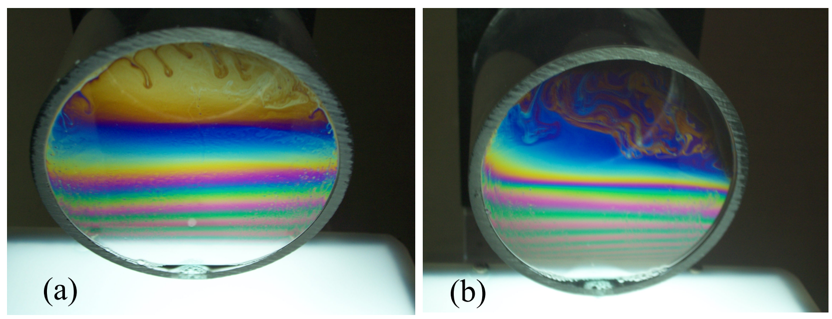

Figure 25.27: (a) A vertical soap film generally has horizontal bands. (b) When the film gets very thin, it does not reflect any light whatsoever, as is happening at the top right of the film in this case. Photo credit: A. Duffy.

SOLUTION

(a) The film thickness is approximately constant at a given height, with that thickness corresponding to constructive interference for a particular wavelength (color). Gravity pulls the fluid down toward the bottom of the film, so the film thickness decreases as the vertical position increases, changing the wavelength (color) associated with a particular height.

(b) The index of refraction of the soap film is essentially that of water (n = 1.33), with the film being surrounded by air (n = 1.00). The wave reflecting from the front surface of the film is in air, reflecting from the higher-n film, so it experiences a half-wavelength shift: $\Delta_t = \lambda_{film}/2$. The wave reflecting from the back surface of the film reflects from a lower-n medium, so the effective path-length is simply $\Delta_b = 2t$, where t is the film thickness. The effective path-length difference is therefore $\Delta = \Delta_b - \Delta_t = 2t - \lambda_{film}/2$. In the limit that the film thickness t approaches zero, the effective path-length difference has a magnitude of half a wavelength. Shifting one wave with respect to the other by half a wavelength produces destructive interference, and the interference is destructive for all wavelengths, so no light is reflected when the film is very thin.

Related End-of-Chapter Exercises: 10, 12.

Essential Question 25.7: For the situation shown in Exploration 25.7, the non-reflective coating on glass, what kind of interference results as the thickness of the coating approaches zero?