Answer to Essential Question 25.5: In this situation, the condition for constructive interference is that the path-length difference is half a wavelength. Using an integer number of wavelengths plus a half-wavelength would also produce constructive interference, but it would also decrease the wavelength. To get the lowest frequency, we need the longest wavelength. The wave that reflects from the floor is inverted upon reflection, and an inversion is equivalent to traveling an additional half wavelength, so the net shift is a full wavelength. This gives a path-length difference of 3.76 m – 3.42 m = 0.34 m. The path-length difference is half a wavelength, so a full wavelength is 0.68 m, corresponding to a frequency of v/λ = 340 m/s / 0.68 m = 500 Hz.

25-6 Thin-Film Interference: The Five-Step Method

The photograph in Figure 25.24 shows some colorful soap bubbles. The beautiful colors of the bubbles are caused by thin-film interference, interference between light reflecting from the outer surface of a soap bubble and light reflecting from the inner surface of the bubble. The colors we see are directly related to the thickness of the bubble wall. The basic process of thin-film interference is illustrated in Figure 25.25.
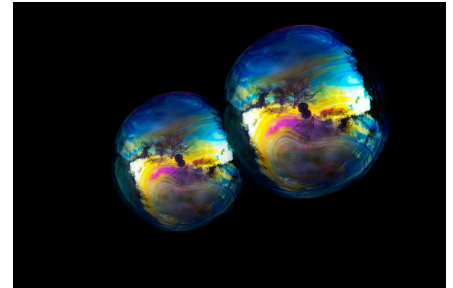
Figure 25.24: The colors in these soap bubbles are produced by thin-film interference - interference between light reflecting from the outer and inner surfaces of a bubble. Photo credit: George Horan, from publicdomainpictures.net
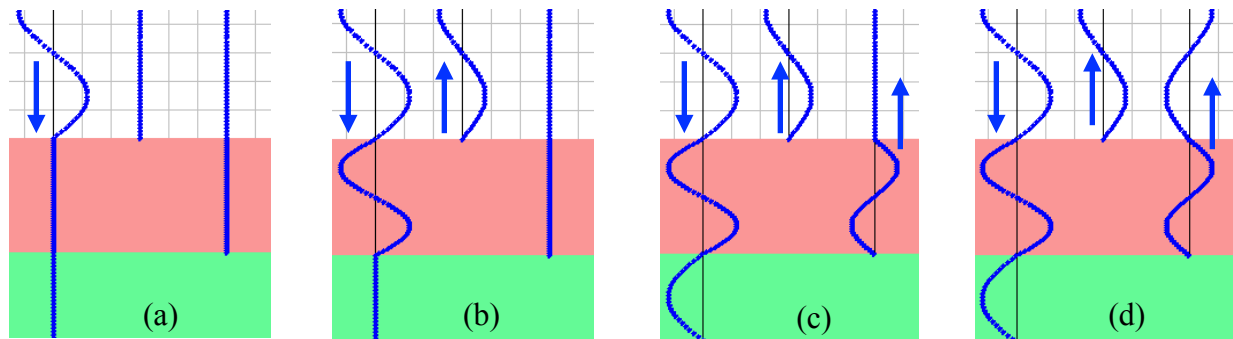

(a)   (b)   (c)   (d)

Figure 25.25: These successive images are separated in time by one period of the wave. The thin film, shown in pink, is characterized by an index of refraction n_2. (a) A wave traveling in medium 1 is incident on the interface separating media 1 and 2 along the normal to the interface. (b) Part of the wave is transmitted into medium 2, while part reflects back into medium 1. The reflected wave is shifted to the right for clarity. The thin film happens to be exactly one wavelength thick. In general, the wave reflecting from the top surface of the film can be inverted upon reflection, or reflect without being inverted. In this case, the reflected wave is inverted because $n_2 > n_1$. (c) At the interface separating media 2 and 3, the wave is also partly reflected and partly transmitted. This reflected wave is shown on the far right of the diagram. In this case, the reflected wave is not inverted upon reflection because $n_3 < n_2$. (d) The two reflected waves interfere with one another in medium 1. By adjusting the thickness of the thin film, this interference can be completely constructive, completely destructive, or something in between.

Note that, in Figure 25.25, the wave that reflects off the bottom surface of the film travels a total extra distance of 2 wavelengths, compared to the wave that reflects off the film's top surface. What kind of interference occurs between the two reflected waves? As we can see from Figure 25.25(d), the waves interfere destructively. The extra path length is an integer number of wavelengths, but the inversion upon reflection at the top surface introduces a half wavelength shift that causes peaks in one reflected wave to align with troughs in the other, and vice versa.

Let's take a systematic approach to analyzing a thin-film situation. The basic idea is to determine the effective path-length between the wave reflecting from the top surface of the film and the wave reflecting from the bottom surface. For a film of thickness t, and with a wave incident along the normal, the effective path-length difference accounts for the extra distance of $2t$ traveled by the wave that reflects from the bottom surface of the film, as well as for any inversions that occur when a wave reflects from a higher-n medium.

For a wave that does get inverted by reflecting from a higher-n medium, we will treat the inversion as an extra half-wavelength contribution to the wave's path-length. We do this because for a sine wave, inverting the wave is equivalent to shifting the wave by half a wavelength. However, a general thin-film situation involves three different media, and hence three different wavelengths! Which wavelength is it that matters? The wave that reflects from the bottom surface of the film is the one that travels the extra distance. Because the extra distance traveled is in the thin film, the wavelength that matters is the wavelength in the thin film. It is helpful to remember the relationship between the wavelength in the film and the wavelength in vacuum:

$$\lambda_{film} = \frac{\lambda_{vacuum}}{n_{film}}. \quad \text{(Eq. 25.8: } \textbf{An expression for the wavelength of light in the thin film)}$$

The Five-Step Method for analyzing thin films
Our approach will assume that the wave starts in medium 1, and is normally incident on a thin film (medium 2) of thickness t that is on a third medium (medium 3), as shown in Figure 25.25.

Step 1 – Determine Δ_t, the shift for the wave reflecting from the top surface of the film. This contribution to the path length is non-zero only if the wave is inverted upon reflection. If $n_2 > n_1$, $\Delta_t = \lambda_{film} / 2$. If $n_2 < n_1$, $\Delta_t = 0$.

Step 2 – Determine Δ_b, the shift for the wave reflecting from the bottom surface of the film. This contribution to the path length is at least $2t$, because that wave travels an extra distance t down through the film, and t back up through the film. There is an extra half-wavelength contribution if the wave is inverted upon reflection. If $n_3 > n_2$, $\Delta_b = 2t + \lambda_{film} / 2$. If $n_3 < n_2$, $\Delta_b = 2t$.

Step 3 – Determine Δ, the effective path-length difference. Simply subtract the results from the previous steps to find the relative shift between the two waves. $\Delta = \Delta_b - \Delta_t$.

Step 4 – Bring in the interference condition appropriate to the situation. If the interference is constructive, such as when we see a particular color reflecting from the thin film, we set the effective path-length difference equal to an integer number of wavelengths ($\Delta = m\lambda_{film}$, where m is an integer). If the interference is destructive, $\Delta = (m+0.5) \lambda_{film}$.

Step 5 – Solve the resulting equation. In general, the equation relates the thickness of the thin film to the wavelength of the light.

Related End-of-Chapter Exercises: 24 – 28.

Essential Question 25.6: Fill in Table 25.2, which summarizes the various possibilities for what Δ_t and Δ_b can be.

	$n_2 > n_1$	$n_2 < n_1$
$n_3 > n_2$	$\Delta_t =$ $\Delta_b =$	$\Delta_t =$ $\Delta_b =$
$n_3 < n_2$	$\Delta_t =$ $\Delta_b =$	$\Delta_t =$ $\Delta_b =$

Table 25.2: A table for summarizing the various results for Δ_t and Δ_b.