

Answer to Essential Question 25.1 (a) The only points for which changing the wavelength has no impact on the interference of the waves are points for which the path-length difference is zero. Thus, the point in question must lie on the perpendicular bisector of the line joining the sources. (b) If we re-arrange Equation 25.3 to solve for the sine of the angle, we get

$$\sin\theta = \frac{m\lambda}{d}$$

Thus, decreasing the wavelength decreases $\sin\theta$, so the pattern gets tighter. Decreasing d , the distance between the sources, has the opposite effect, with the pattern spreading out. We can understand the wavelength effect conceptually in that, when the wavelength decreases, we don't have to go as far from the perpendicular bisector to locate points that are half a wavelength (or a full wavelength) farther from one source than the other. Figure 25.3 shows the effect of decreasing the wavelength, or of decreasing the distance between the sources.

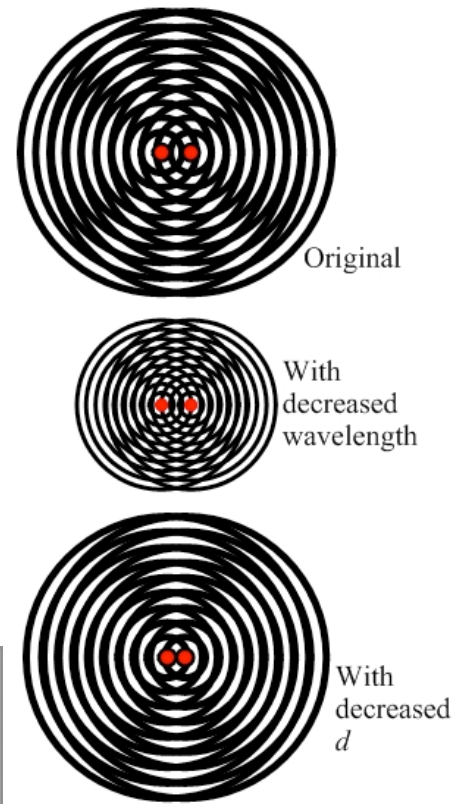


Figure 25.3: The top diagram shows the interference pattern produced by two sources. The middle diagram shows the effect of decreasing the wavelength of the waves produced by the sources, while the bottom diagram shows the effect of decreasing the distance between the sources.

25-2 The Diffraction Grating

Now that we understand what happens when we have two sources emitting waves that interfere, let's see if we can understand what happens when we add additional sources. The distance d between neighboring sources is the same as the distance between the original two sources, and the sources are arranged in a line. All the sources emit identical waves that are in phase.

EXPLORATION 25.2A – Adding sources

Step 1 – Consider a point a long way from two sources. The sources are a distance d apart. The point is one wavelength farther from one source than the other, so constructive interference occurs at the point. When we add a third source, so that we have three sources equally spaced in a line, separated by d , do we still get constructive interference taking place at the point?

Yes. As the diagram in Figure 25.4 shows, the path-length difference for the third source and the source it was placed closest to will also be one wavelength. Now we get constructive interference for three waves at once, not just two, so the amplitude of the resultant wave is larger than it was with only two sources.

Step 2 – If we consider a different point that is half a wavelength farther from one of two sources than the other, destructive interference occurs at the point. When we add a third source, so that we have three sources equally spaced in a line, separated by d , do we still get destructive interference taking place at the point?

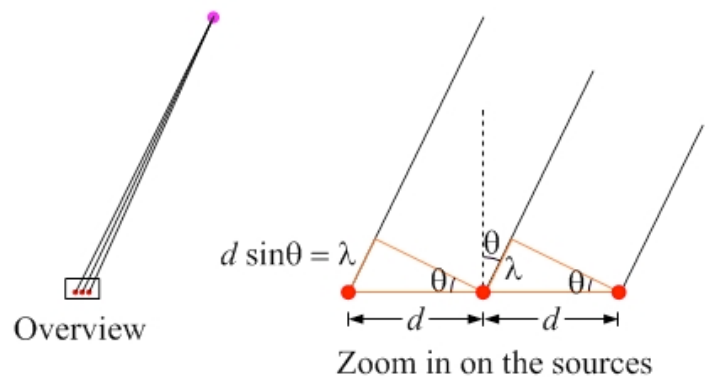


Figure 25.4: For a point that is one wavelength farther from one source than another, adding a third source results in even larger amplitude because of constructive interference.

No. The destructive interference at the point was caused by the cancellation between the waves from the first two sources. Adding a third source does not change the fact that the first two waves cancel one another, so there is nothing to cancel the third wave.

Step 3 – For three sources, what path-length difference (between zero and one wavelength) between neighboring sources results in completely destructive interference? With three sources, it turns out that there are two path-length differences between 0 and one wavelength that result in completely destructive interference, these being one-third and two-thirds of a wavelength.

Step 4 – What if we have N sources, where N is any integer greater than 1. Is there a general rule for predicting the angles at which constructive interference occurs? What about destructive interference? Constructive interference occurs at the same points for N sources that it does for 2 sources, so the equation $d \sin\theta = n\lambda$ still applies for situations with $N > 1$ sources. There are $N - 1$ places where destructive interference happens in between each interference maximum, so we generally dispense with an equation for destructive interference when $N > 2$.

Key idea: The equation $d \sin\theta = m\lambda$ applies to any number of sources > 1 , as long as the sources are equally spaced. With multiple sources, it is much easier to produce destructive interference than it is to produce completely constructive interference, so there is no simple equation for destructive interference.

Related End-of-Chapter Exercises: 7, 16 – 18, 38, 39, 48.

The Diffraction Grating

A diffraction grating is essentially a large number of equally spaced sources, and thus the $d \sin\theta = m\lambda$ equation applies.

One application of diffraction gratings is in spectroscopy, which involves separating light into its different wavelengths, a process that astronomers, or chemists, can use to determine the chemical makeup of the source producing the light. In actuality, a diffraction grating is typically a glass or plastic slide with a large number of slits (long thin openings between long thin lines). A diffraction grating (which should probably have been named an interference grating) offers two main advantages over a double slit. First, the more openings the light passes through, the brighter the interference maxima are. Second, the more openings there are, the narrower the bright lines are in the interference pattern, which is important when trying to resolve two similar wavelengths. Figure 25.5 shows the increased sharpness that results from adding slits.

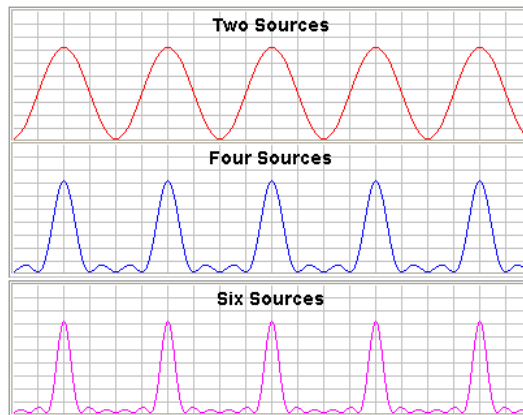


Figure 25.5: Adding sources (or slits that light goes through) results in sharper interference maxima. Each case shows the relative intensity at various points. The amplitude of the peaks also grows as sources are added.

EXPLORATION 25.2B – Double-slit geometry

When light of a single wavelength (say, from a laser) is incident on a double slit (or a diffraction grating, which gives a sharper pattern), we get a pattern of bright and dark fringes on a screen beyond the double slit. Such a pattern is shown in Figure 25.6. The bright fringes come from constructive interference, and the dark fringes come from destructive interference.

Step 1 – If we wanted to increase the distance between the bright spots on the screen, what would we change about d or λ ?

This should be something of a review. If we rearrange the equation for constructive interference, we get:

$$\sin\theta = \frac{m\lambda}{d}$$

Increasing the value of $\sin\theta$ will increase the value of θ , which means that the lines of constructive interference will be further apart, in terms of the angles between them. This will spread out the pattern on the screen. To increase $\sin\theta$, we can either replace the first laser with a laser that emits light of a longer wavelength (switch from blue to red, for instance), or we can switch to a double slit that has a smaller value of d . These two options are illustrated in Figure 25.7.

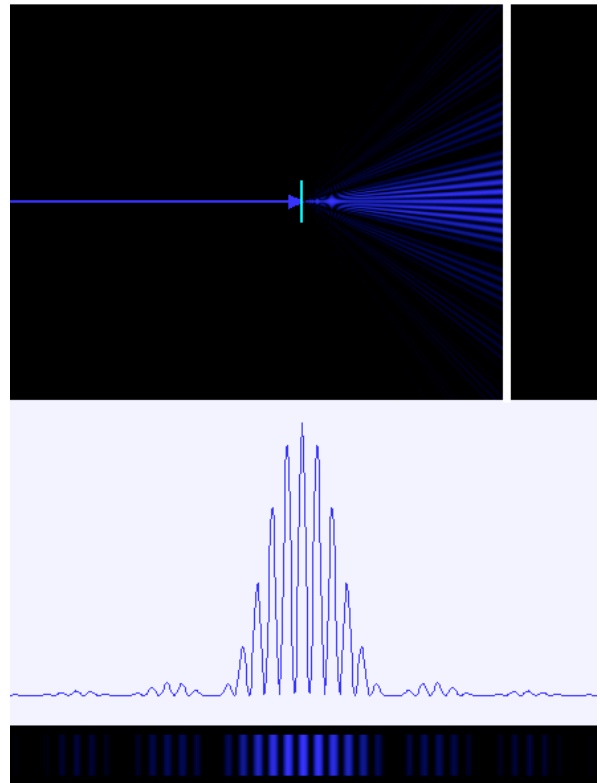


Figure 25.6: The geometry of the double-slit pattern, for blue light. The top shows an overview of the interference pattern. The bottom has the pattern of bright and dark fringes on the screen. In between is a graph of the intensity of the fringes in the pattern, as a function of position.

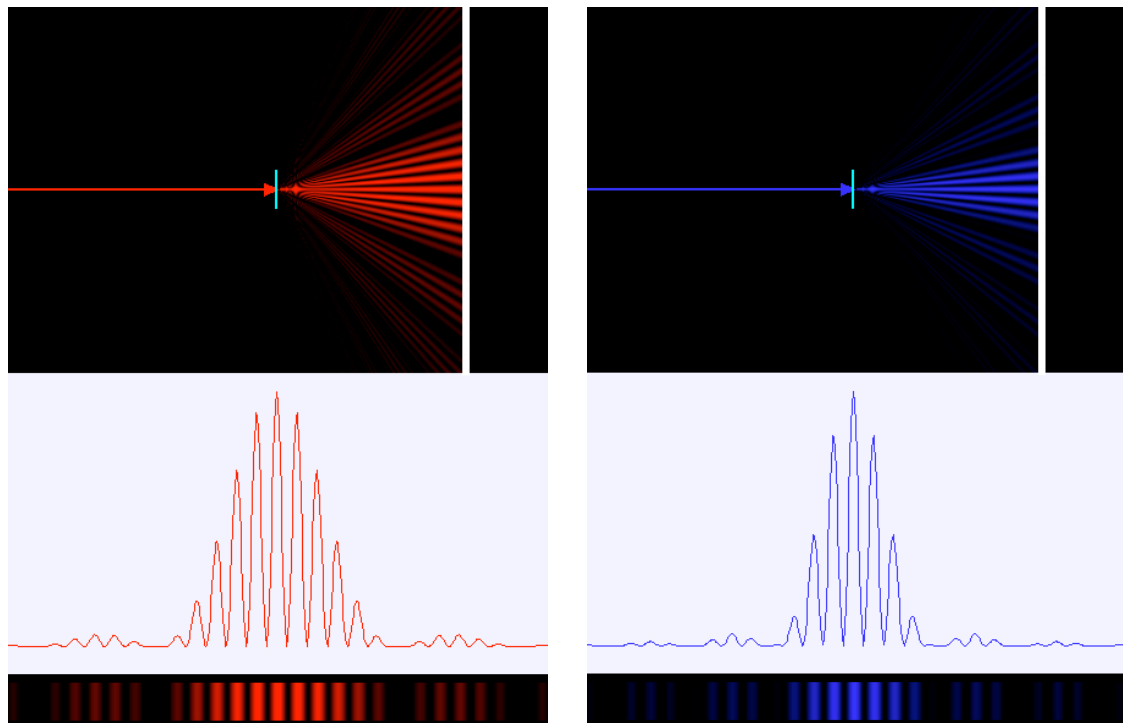


Figure 25.7: The pattern on the left shows how the situation of Figure 25.6 changes if we change to a larger wavelength, while the one on the right shows the change of switching to a smaller d .

Step 2 – What if we only have one laser and one double slit, so we can't change the wavelength or d . Is there a different way to increase the distance between the bright spots on the screen?

Yes. If we move the screen farther from the double slit, the screen will intercept the light from the grating after the bright lines in the pattern have been able to spread out farther, increasing the distance between the bright spots on the screen. This is illustrated in Figure 25.8.

Step 3 – As shown in Figure 25.9, let's use L to denote the distance from the double slit to the screen, and y_m to denote the distance from the central bright spot on the screen to the m^{th} bright spot. For instance, y_4 is the distance from the center of the pattern to one of the $m = 4$ bright spots on the screen. If the angle is small (say, 10° or less), we can use the approximation $\sin\theta \approx \tan\theta$. Using that assumption, derive an expression for y_m in terms of d , m , λ , and L .

We have two equations to work with here, one for $\sin\theta$, from above, and then one for $\tan\theta$, from the geometry of right-angled triangles.

$$\sin\theta = \frac{m\lambda}{d} \quad \text{and} \quad \tan\theta = \frac{y}{L}$$

The small-angle approximation enables us to set these two equations equal to one another. Doing that and solving for y gives:

$$y_m = \frac{m\lambda L}{d}$$

(Eq. 25.5: The distance from the center of the pattern to the m^{th} bright spot)

Step 4 – What would you do if you wanted to predict the position of a particular bright spot on the screen, but you could not use the small-angle approximation?

If we could not use the small-angle approximation, we could first use the $\sin\theta$ equation to find θ , and then take the tangent of that angle when we were using the $\tan\theta$ equation to find y .

Essential Question 25.2: A beam of light made up of three wavelengths, 660 nm (red light), 530 nm (green light), and 400 nm (violet light) is incident on a diffraction grating that has a spacing of $d = 1300$ nm. The first order spectrum, consisting of a violet line, a green line, and a red line, produced by the grating is shown in Figure 25.10. What are the colors of the other three beams (1 – 3) that come from the grating?

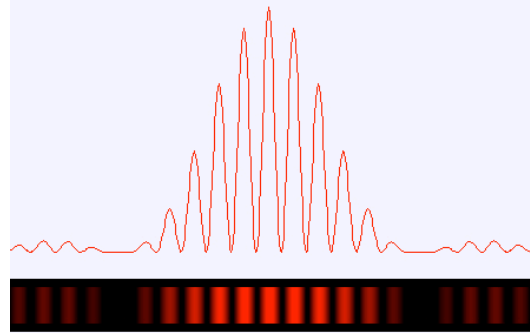
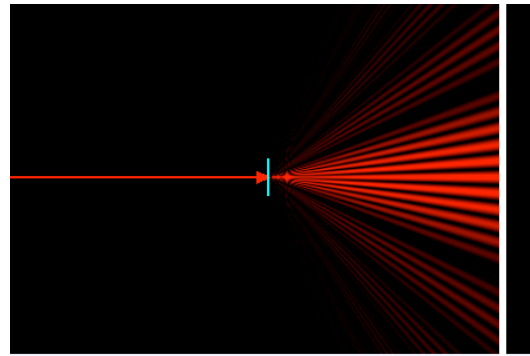


Figure 25.8: Spreading the dots out by moving the screen farther away, starting from the left-hand picture in Figure 25.7.

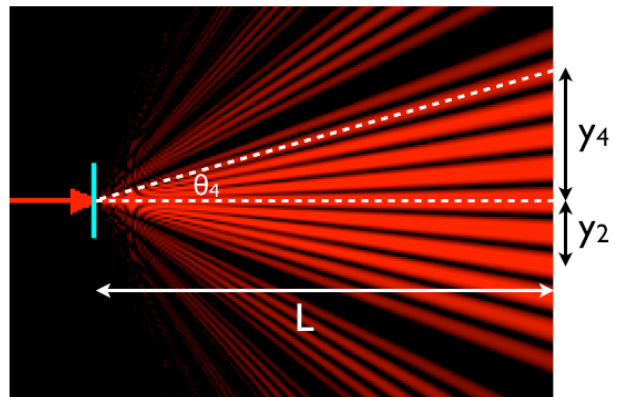


Figure 25.9: The geometry of the pattern, based on a right-angled triangle.

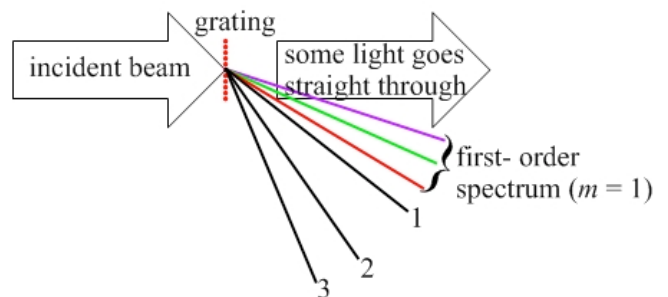


Figure 25.10: The first-order ($m = 1$) spectrum for the situation of Essential Question 25.2.