Answer to Essential Question 24.7: The lens must be converging, because a diverging lens cannot produce an image that is larger than the object. A converging lens produces a real image only when the object distance is larger than the focal length, so the focal length in this case must be positive but less than 20 cm.

24-8 An Example Problem Involving a Lens

Let's begin by discussing a general approach we can use to solve problems involving a lens. We will then apply the method to a particular situation.

A general method for solving problems involving a lens

1. Sketch a ray diagram, showing rays leaving the tip of the object and being refracted by the lens. Where the refracted rays meet is where the tip of the image is located. The ray diagram gives us qualitative information about the location and size of the image and about the characteristics of the image.

2. Apply the thin-lens equation and/or the magnification equation. Make sure that the signs you use match those listed in the sign convention in section 24-6. The equations provide quantitative information about the location and size of the image and about the image characteristics.

3. Check the results of applying the equations with your ray diagram, to see if the equations and the ray diagram give consistent results.

Rays that are easy to draw

To locate an image on a ray diagram, you need a minimum of two rays. If you draw more than two rays, however, you can check the image location you find with the first two rays. You can draw any number of rays being refracted by the lens, but some are easier to draw than others because we know exactly where the refracted rays go for these rays. Such rays are shown on Figure 24.37, and include:

1. The ray that goes parallel to the principal axis, and refracts to pass through the focal point on the far side of the lens (converging lens), or away from the focal point on the near side of the lens (diverging lens).

2. The ray that passes straight through the center of the lens without changing direction.

3. The ray that travels along the straight line connecting the tip of the object and the focal point not associated with the first ray. This ray is refracted by the lens to go parallel to the principal axis.

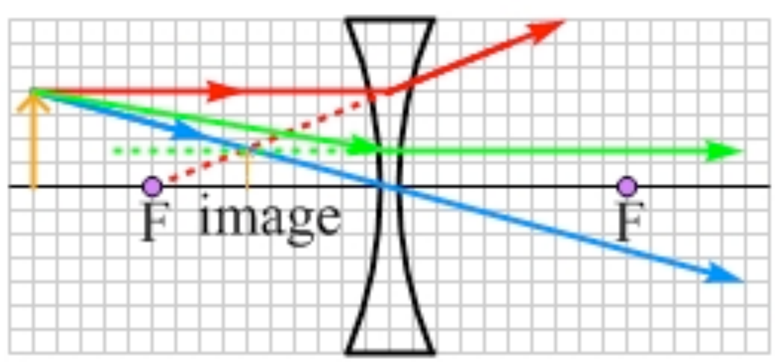

Figure 24.37: An example of the three rays that are easy to draw the refracted rays for. When you look at the object through the lens, your brain interprets the light as traveling in straight lines, so you see the image, and not the object.

EXAMPLE 24.8 – Applying the general method

When you look at a cat through a lens that has its focal points at distances of 24 cm on either side of the lens, you see an image of the cat that is 1.5 times as large as the cat. How far is the cat from the lens? Sketch a ray diagram to check your calculations.

SOLUTION

In this case, let's first apply the equations and then draw the ray diagram. The lens is clearly a converging lens, because only converging lenses produce images that are larger than the object. One possibility is that the lens produces a virtual, upright image, so the sign of the magnification is positive. Applying the magnification equation, we get:

$m = +1.5 = -\frac{d_i}{d_o}$, which tells us that $\frac{1}{d_i} = -\frac{1}{1.5 d_o}$.

Applying the thin-lens equation:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} - \frac{1}{1.5 d_o} = \frac{3}{3d_o} - \frac{2}{3d_o} = \frac{1}{3d_o}.$$

Thus, we find that $3d_o = f = +24 \text{ cm}$, so $d_o = +8.0 \text{ cm}$ and we can show that $d_i = -12 \text{ cm}$.

The ray diagram for this situation is shown in Figure 24.38, confirming the calculations.

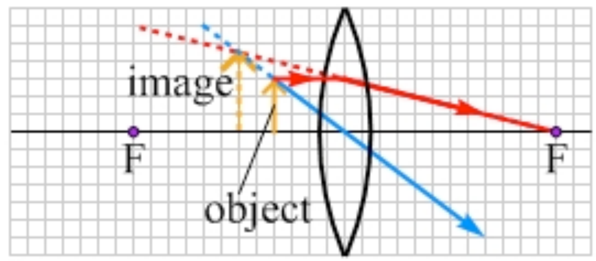

Figure 24.38: A ray diagram for the solution involving a virtual image. Each box on the grid measures 2 cm × 2 cm.

The solution above is only one of the possible answers. The image could also be real and inverted, so the sign of the magnification is negative. Applying the magnification equation, we get:

$m = -1.5 = -\frac{d_i}{d_o}$, which tells us that $\frac{1}{d_i} = +\frac{1}{1.5 d_o}$.

Applying the thin-lens equation:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{1.5 d_o} = \frac{3}{3d_o} + \frac{2}{3d_o} = \frac{5}{3d_o}.$$

Thus, we find that $d_o = \frac{5}{3} f = +40 \text{ cm}$, and we can show that $d_i = +60 \text{ cm}$.

The ray diagram for this situation is shown in Figure 24.39, again confirming the calculations above. These two rays, and all rays that travel from the tip of the object to the tip of the image, take the same time to get there.

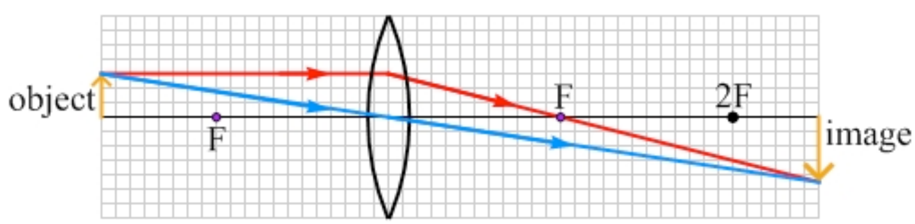

Figure 24.39: A ray diagram for the situation involving a real image. Each box on the grid measures 2 cm × 2 cm.

Related End-of-Chapter Exercises: 25 – 28.

Essential Question 24.8: Return to the situation described in Example 24.8. Would there still be two solutions if the image was smaller than the object? Explain.