Answer to Essential Question 24.6: To add fractions, you need to find a common denominator.
$\frac{1}{f} = \frac{1}{+12 \text{ cm}} + \frac{1}{+24 \text{ cm}} = \frac{2}{+24 \text{ cm}} + \frac{1}{+24 \text{ cm}} = \frac{3}{+24 \text{ cm}}$. This gives $f = \frac{+24 \text{ cm}}{3} = 8.0 \text{ cm}$.

24-7 Analyzing a Converging Lens

In section 24-4, we drew one ray diagram for a converging lens. Let's investigate the range of ray diagrams we can draw for such a lens. Note the similarities between a converging lens and a concave mirror. This section is very much a parallel of section 23-6, in which we analyzed the range of images formed by a concave mirror.

EXPLORATION 24.7 – Ray diagrams for a converging lens

Step 1 – *Draw a ray diagram for an object located 40 cm from a converging lens that has a focal length of +10 cm. Verify the image location on your diagram with the thin-lens equation.* Two rays are shown in Figure 24.32. One is the parallel ray, which leaves the tip of the object, travels parallel to the principal axis, and is refracted by the lens to pass through the focal point on the far side of the lens. The second ray passes straight through the center of the lens, undeflected.

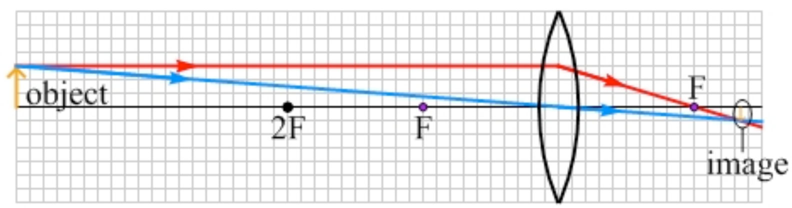

Figure 24.32: A ray diagram for the situation in which the object is far from the lens. The squares in the grid measure 1.0 cm × 1.0 cm.

Applying the thin-lens equation, in the form of equation 24.8, to find the image distance:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(40 \text{ cm}) \times (+10 \text{ cm})}{(40 \text{ cm}) - (+10 \text{ cm})} = \frac{+400 \text{ cm}^2}{30 \text{ cm}} = +13.3 \text{ cm}.$$

This image distance is consistent with the ray diagram in Figure 24.32.

Step 2 – *Repeat step 1, with the object now twice the focal length from the lens.* We draw the same two rays again, with the parallel ray (in red) being refracted so that it passes through the focal point on the far side of the lens, and the second ray (in blue) passing undeflected (approximately) through the center of the lens. As shown in Figure 24.33, this situation is a special case. When the object is located at twice the focal length from the lens, the image is inverted, also at twice the focal length from the lens (on the other side of the lens), and the same size as the object because the object and image are the same distance from the lens.

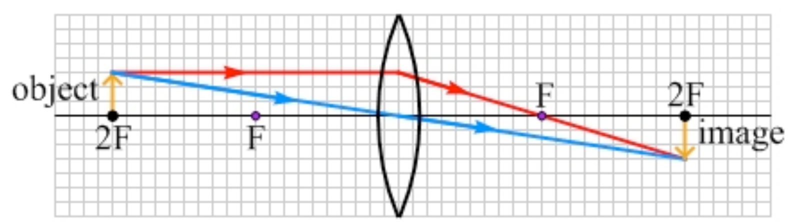

Figure 24.33: When the object is at twice the focal length from the lens, so is the image.

Applying the thin-lens equation to find the image distance, we get:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(20 \text{ cm}) \times (+10 \text{ cm})}{(20 \text{ cm}) - (+10 \text{ cm})} = \frac{+200 \text{ cm}^2}{10 \text{ cm}} = +20 \text{ cm},$$ matching the ray diagram.

Step 3 – *Repeat step 1, with the object 15 cm from the lens.* No matter what the object distance is, the parallel ray always does the same thing, being refracted by the lens to pass through the focal point on the far side. The path of the second ray, in blue, depends on the object's position. The ray diagram (Figure 24.34) shows that the image is real, inverted, larger than the object, and about twice as far from the lens as the object is.

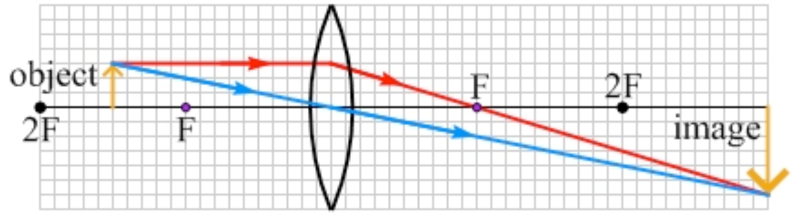

Figure 24.34: A ray diagram for a situation in which the object is between twice the focal length from the lens and the focal point.

Applying the thin-lens equation gives:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(15\text{ cm})\times(+10\text{ cm})}{(15\text{ cm})-(+10\text{ cm})} = \frac{+150\text{ cm}^2}{5.0\text{ cm}} = +30\text{ cm}$$, matching the ray diagram.

Step 4 – *Repeat step 1, with the object at a focal point.* As shown in Figure 24.35, the two refracted rays are parallel to one another, and never meet. In such a case the image is formed at infinity.

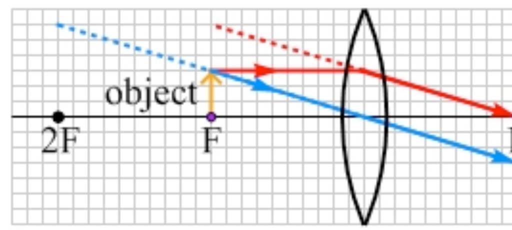

Figure 24.35: A ray diagram for a situation in which the object is at the focal point.

Applying the thin-lens equation gives:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(10\text{ cm})\times(+10\text{ cm})}{(10\text{ cm})-(+10\text{ cm})} = \frac{+100\text{ cm}^2}{0\text{ cm}} = +\infty ,$$

which agrees with the ray diagram.

Step 5 – *Repeat step 1, with the object 5.0 cm from the lens.* When the object is closer to the lens than the focal point, the refracted rays diverge to the right of the lens, and they must be extended back to meet on the left of the lens. The result is a virtual, upright image that is larger than the object, as shown in Figure 24.36. If you look at the object through the lens, your brain interprets the light as coming from the image.

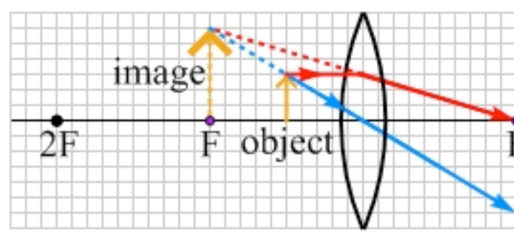

Figure 24.36: A ray diagram for a situation in which the object is between the lens and its focal point.

Applying the thin-lens equation gives:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(5.0\text{ cm})\times(+10\text{ cm})}{(5.0\text{ cm})-(+10\text{ cm})} = \frac{+50\text{ cm}^2}{-5.0\text{ cm}} = -10\text{ cm} .$$

Recalling the sign convention that a negative image distance is consistent with a virtual image, the result from the thin-lens equation is consistent with the ray diagram.

Key idea for converging lenses: Depending on where the object is relative to the focal point of a converging lens, the lens can form an image of the object that is real or virtual. If the image is real, it can be larger than, smaller than, or the same size as the object. If the image is virtual, the image is larger than the object. **Related End-of-Chapter Exercises: 44, 50, 53.**

Essential Question 24.7: When an object is placed 20 cm from a lens, the image formed by the lens is real. What kind of lens is it? What, if anything, can you say about the lens' focal length?