Answer to Essential Question 24.5: To draw the refracted rays properly, we know that when we extend the refracted rays back, they will pass through the tip of the image, which we located in Figure 24.27. Three additional rays are shown in Figure 24.28.

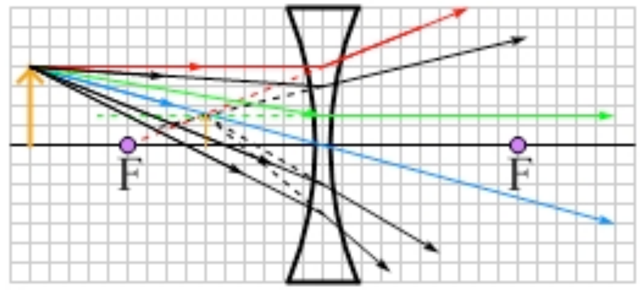

Figure 24.28: For all rays of light that leave the tip of the object and reflect from the mirror, the refracted rays can be extended back to pass through the tip of the image.

24-6 A Quantitative Approach: The Thin-Lens Equation

Even though mirrors and lenses form images using completely different principles (the law of reflection versus Snell's law), we use the same equation to relate focal length, object distance, and image distance, for both mirrors and lenses. This surprising result comes from the fact that the formation of images with both mirrors and lenses can be understood using the geometry of similar triangles. Let's look at how that works for lenses.

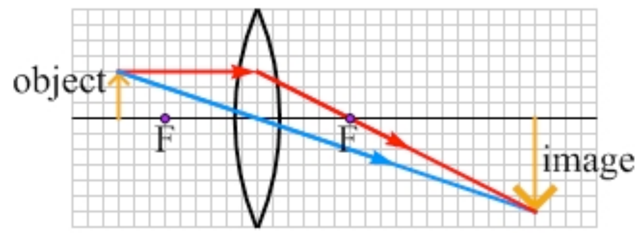

Figure 24.29: The ray diagram we constructed in section 24-4, for an object in front of a converging lens.

Let's look at the ray diagram we drew in Figure 24.22 of section 24-4, shown again here in Figure 24.29.

Remove the red rays, and examine the two triangles in Figure 24.30, one shaded green and one shaded yellow, bounded by the blue rays, the principal axis, and the object and image. The two triangles are similar, because the three angles in one triangle are the same as the three angles in the other triangle. We can now define the following variables: d_o is the object distance, the distance of the object from the center of the lens; d_i is the image distance, the distance of the image from the center of the lens; h_o is the height of the object; h_i is the height of the image.

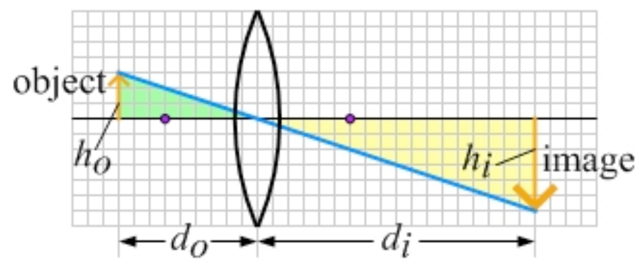

Figure 24.30: Similar triangles, bounded by the principal axis, the object and image, and the ray that passes through the center of the lens.

Using the fact that the ratios of the lengths of corresponding sides in similar triangles are equal, we find that:

$$-\frac{h_i}{h_o} = \frac{d_i}{d_o} . \qquad \text{(Equation 24.6)}$$

The image height is negative because the image is inverted, which is why we need the minus sign in the equation. Let's now return to Figure 24.29, and remove the ray that passes through the center. This gives us the shaded similar triangles shown in Figure 24.31.

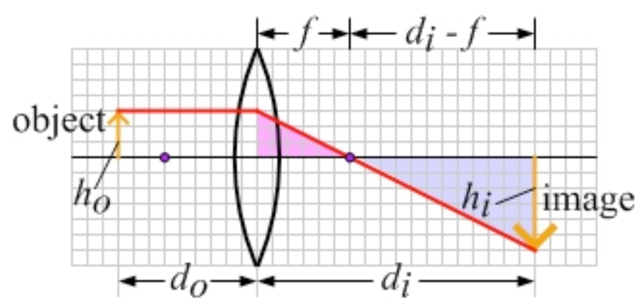

Figure 24.31: Similar triangles, with two sides bounded by the principal axis and the parallel ray, and a third side that is equal to the object height (smaller triangle) or the image height (larger triangle).

Again, using the fact that the ratios of the lengths of corresponding sides in similar triangles are equal, we find that: $\frac{d_i - f}{f} = -\frac{h_i}{h_o}$.

Simplifying the left side, and bringing in equation 24.6, we get: $\frac{d_i}{f} - 1 = \frac{d_i}{d_o}$.

Dividing both sides by d_i gives: $\frac{1}{f} - \frac{1}{d_i} = \frac{1}{d_o}$, which is generally written as:

$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$. (Equation 24.7: **The thin-lens equation**)

The mnemonic "If I do I di" can help you to remember the thin-lens equation.

Often, we know the focal length f and the object distance d_o, so equation 24.7 can be solved for d_i, the image distance:

$d_i = \frac{d_o \times f}{d_o - f}$ (Equation 24.8: **The thin-lens equation, solved for the image distance**)

Sign conventions

We derived the lens equation above by using a specific case involving a convex lens. The equation can be applied to all situations involving a convex lens or a concave lens if we use the following sign conventions.

The focal length is positive for a converging lens, and negative for a diverging lens.

The image distance is positive, and the image is real, if the image is on the side of the lens the light passes through to, and negative, and the image is virtual, if the image is on the side the light comes from.

The image height is positive when the image is above the principal axis, and negative when the image is below the principal axis. A similar rule applies to the object height.

Magnification

The magnification, m, is defined as the ratio of the height of the image (h_i) to the height of the object (h_o). Making use of Equation 24.6, we can write the magnification as:

$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$. (Equation 24.9: **Magnification**)

The relative sizes of the image and object are as follows:

- The image is larger than the object if $|m| > 1$.
- The image and object have the same size if $|m| = 1$.
- The image is smaller than the object if $|m| < 1$.

The sign of the magnification tells us whether the image is upright (+) or inverted (–) compared to the object.

Related End-of-Chapter Exercises: 21 – 24.

Essential Question 24.6: As you are analyzing a thin-lens situation, you write an equation that states: $\frac{1}{f} = \frac{1}{+12 \text{ cm}} + \frac{1}{+24 \text{ cm}}$. What is the value of $1/f$ in this situation? What is f?