Answer to Essential Question 23.5: To add fractions you need to find a common denominator.
$\frac{1}{f} = \frac{1}{+12 \text{ cm}} + \frac{1}{+24 \text{ cm}} = \frac{2}{+24 \text{ cm}} + \frac{1}{+24 \text{ cm}} = \frac{3}{+24 \text{ cm}}$. This gives $f = \frac{+24 \text{ cm}}{3} = 8.0 \text{ cm}$.

23-6 Analyzing the Concave Mirror

In section 23-4, we drew one ray diagram for a concave mirror. Let's investigate the range of ray diagrams we can draw for such a mirror.

EXPLORATION 23.6 – Ray diagrams for a concave mirror

Step 1 – ***Draw a ray diagram for an object located 40 cm from a concave mirror that has a radius of curvature of 20 cm. Verify the image location on your diagram with the mirror equation.*** In drawing a ray diagram, it is helpful to know where the mirror's focal point is. For a spherical mirror, the focal point is halfway between the mirror's center of curvature and the point at which the principal axis intersects the mirror. Thus, the focal length in this case is +10 cm.

In Figure 23.29, two rays are shown. One is the parallel ray, which leaves the tip of the object, travels parallel to the principal axis, and reflects from the mirror so that it passes through the focal point. The second ray reflects off the mirror at the point at which the principal axis meets the mirror, reflecting as if the mirror was a vertical plane mirror.

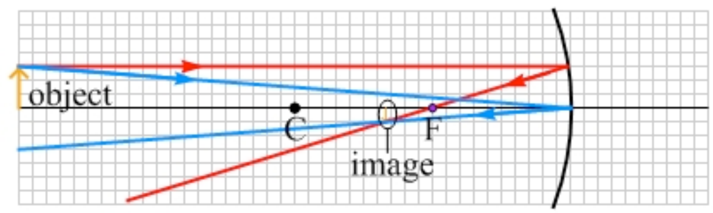

Figure 23.29: A ray diagram for the situation in which the object is far from the mirror. The squares in the grid measure 1.0 cm × 1.0 cm.

Applying the mirror equation, in the form of equation 23.5, to find the image distance:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(40 \text{ cm}) \times (+10 \text{ cm})}{(40 \text{ cm}) - (+10 \text{ cm})} = \frac{+400 \text{ cm}^2}{30 \text{ cm}} = +13.3 \text{ cm}.$$

This image distance is consistent with the ray diagram in Figure 23.29.

Step 2 – ***Repeat step 1, with the object now moved to the center of curvature.*** The parallel ray follows the same path as it did Figure 23.29. As shown in Figure 23.30, the ray that reflects from the center of the mirror follows a different path, because shifting the object changes the angle of incidence for that ray. This situation is a special case. When the object is located at the center of curvature, the image is inverted, also at the center of curvature, and the same size as the object because the object and image are the same distance from the mirror.

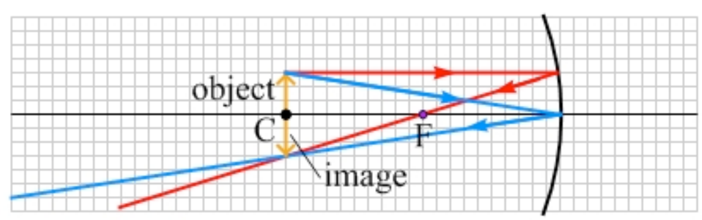

Figure 23.30: When the object is at the mirror's center of curvature, so is the image.

Applying the mirror equation to find the image distance, we get:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(20 \text{ cm}) \times (+10 \text{ cm})}{(20 \text{ cm}) - (+10 \text{ cm})} = \frac{+200 \text{ cm}^2}{10 \text{ cm}} = +20 \text{ cm}$$, matching the ray diagram.

Step 3 – *Repeat step 1, with the object 15 cm from the mirror.* No matter where the object is, the parallel ray follows the same path. The path of the second ray, in blue, depends on the object's position. The ray diagram (Figure 23.31) shows that the image is real, inverted, larger than the object, and about twice as far from the mirror as the object.

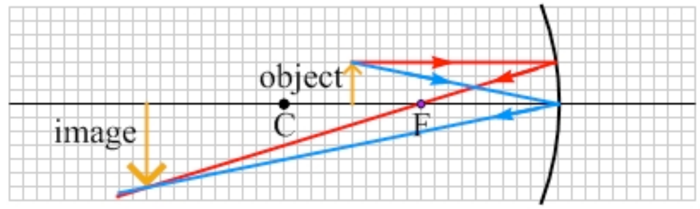

Figure 23.31: A ray diagram for a situation in which the object is between the mirror's center of curvature and its focal point.

Applying the mirror equation gives:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(15\text{ cm})\times(+10\text{ cm})}{(15\text{ cm})-(+10\text{ cm})} = \frac{+150\text{ cm}^2}{5.0\text{ cm}} = +30\text{ cm}$$, matching the ray diagram.

Step 4 – *Repeat step 1, with the object at the mirror's focal point.* As shown in Figure 23.32, the two reflected rays are parallel to one another, and never meet. In such a case the image is formed at infinity.

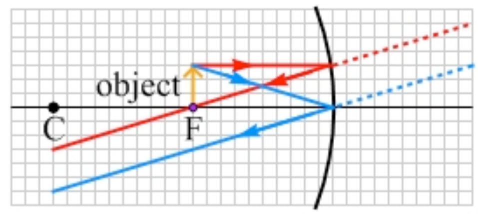

Figure 23.32: A ray diagram for a situation in which the object is at the focal point.

Applying the mirror equation gives:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(10\text{ cm})\times(+10\text{ cm})}{(10\text{ cm})-(+10\text{ cm})} = \frac{+100\text{ cm}^2}{0\text{ cm}} = +\infty,$$

which agrees with the ray diagram.

Step 5 – *Repeat step 1, with the object 5.0 cm from the mirror.* When the object is closer to the mirror than the focal point, the two reflected rays diverge to the left of the mirror, and they must be extended back to meet on the right of the mirror. The result is a virtual, upright image that is larger than the object, as shown in Figure 23.33.

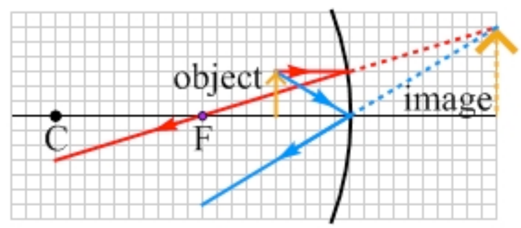

Figure 23.33: A ray diagram for a situation in when the object is between the mirror's surface and its focal point.

Applying the mirror equation to find the image distance gives:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(5.0\text{ cm})\times(+10\text{ cm})}{(5.0\text{ cm})-(+10\text{ cm})} = \frac{+50\text{ cm}^2}{-5.0\text{ cm}} = -10\text{ cm}.$$

Recalling the sign convention that a negative image distance is consistent with a virtual image, the result from the mirror equation is consistent with the ray diagram.

Key idea for concave mirrors: Depending on where the object is placed relative to a concave mirror's focal point, the mirror can form an image of the object that is real or virtual. If the image is real, it can be larger than, smaller than, or the same size as the object. If the image is virtual, the image is larger than the object. **Related End-of-Chapter Exercises: 27 and 47 – 49.**

Essential Question 23.6: When an object is placed 20 cm from a spherical mirror, the image formed by the mirror is larger than the object. What kind of mirror is it? What, if anything, can you say about the mirror's focal length?