Answer to Essential Question 23.4: The image would be located where the object in Figure 23.23 is located. This demonstrates an important fact about light rays – they are reversible. As Figure 23.25 shows, we can simply reverse the direction of the rays from Figure 23.23 to obtain the appropriate ray diagram. Note that we can do this only when the image is a real image.

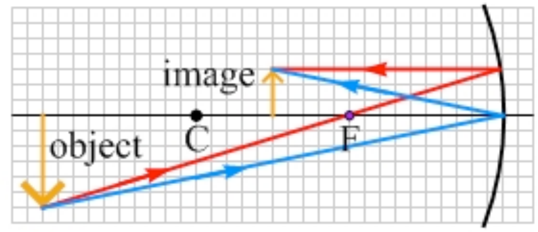

Figure 23.25: When the image is a real image, the ray diagram is reversible.

23-5 A Quantitative Approach: The Mirror Equation

The branch of optics that involves mirrors and lenses is generally called geometrical optics, because it is based on the geometry of similar triangles. Let's investigate this geometry, and use it to derive an important relationship between the image distance, object distance, and the focal length.

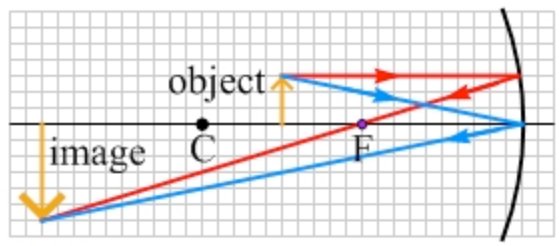

Figure 23.26: The ray diagram we constructed in section 23-4, for an object in front of a concave mirror.

Let's look again at the ray diagram we drew in Figure 23.23 of section 23-4, shown again here in Figure 23.26.

Remove the parallel ray (and its reflection), and examine the two shaded triangles in Figure 23.27, bounded by the other ray and its reflection, the principal axis, and the object and image. The two triangles are similar, because the three angles in one triangle are the same as the three angles in the other triangle. We can now define the following variables: d_o is the object distance, the distance of the object from the center of the mirror; d_i is the image distance, the distance of the image from the center of the mirror; h_o is the height of the object; h_i is the height of the image.

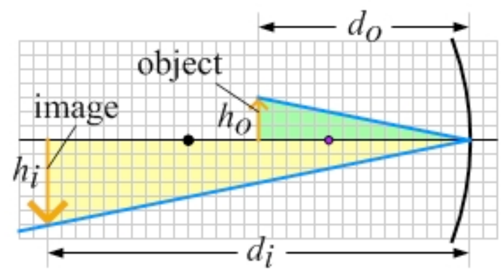

Figure 23.27: Similar triangles, bounded by the principal axis, the object and image, and the ray that reflects from the center of the mirror.

Using the fact that the ratios of the lengths of corresponding sides in similar triangles are equal, we find that:

$$-\frac{h_i}{h_o} = \frac{d_i}{d_o} . \qquad \text{(Equation 23.3)}$$

The image height is negative because the image is inverted, which is why we need the minus sign in the equation. Let's now return to Figure 23.26, and use the parallel ray instead. This gives us the shaded similar triangles shown in Figure 23.28.

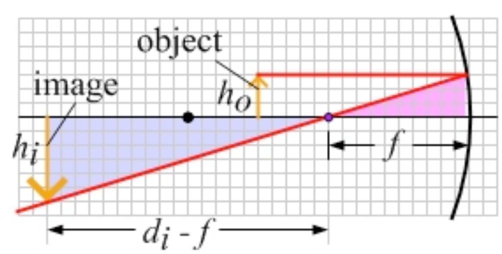

Figure 23.28: Similar triangles, with two sides bounded by the principal axis and the reflection of the parallel ray, and a third side that is equal to the object height (small triangle) or the image height (large triangle).

We use an approximation, which is valid as long as the object height is relatively small, that the length of the smaller triangle is f, the focal length. Again, using the fact that the ratios of the lengths of corresponding sides in similar triangles are equal, we find that:

$$\frac{d_i - f}{f} = -\frac{h_i}{h_o} .$$

Simplifying the left side, and bringing in equation 23.3, we get: $\frac{d_i}{f} - 1 = \frac{d_i}{d_o}$.

Dividing both sides by d_i gives: $\frac{1}{f} - \frac{1}{d_i} = \frac{1}{d_o}$, which is generally written as:

$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$. (Equation 23.4: **The mirror equation**)

The mnemonic "If I do I di" can help you to remember the mirror equation.

Often, we know the focal length f and the object distance d_o, so equation 23.4 can be solved for d_i, the image distance:

$d_i = \frac{d_o \times f}{d_o - f}$ (Equation 23.5: **The mirror equation, solved for the image distance**)

Sign conventions

We derived the mirror equation above by using a specific case involving a concave mirror. The equation can also be applied to a plane mirror, a convex mirror, and all situations involving a concave mirror if we use the following sign conventions.

The focal length is positive for a concave mirror, and negative for a convex mirror.

The image distance is positive if the image is on the reflective side of the mirror (a real image), and negative if the image is behind the mirror (a virtual image).

The image height is positive when the image is above the principal axis, and negative when the image is below the principal axis. A similar rule applies to the object height.

Magnification

The magnification, m, is defined as the ratio of the height of the image (h_i) to the height of the object (h_o). Making use of Equation 23.3, we can write the magnification as:

$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$. (Equation 23.6: **Magnification**)

The relative sizes of the image and object are as follows:

- The image is larger than the object if $|m| > 1$.
- The image and object have the same size if $|m| = 1$.
- The image is smaller than the object if $|m| < 1$.

The sign of the magnification tells us whether the image is upright (+) or inverted (–) compared to the object.

Related End-of-Chapter Exercises: 15 – 19.

Essential Question 23.5: As you are analyzing a spherical mirror situation, you write an equation that states: $\frac{1}{f} = \frac{1}{+12 \text{ cm}} + \frac{1}{+24 \text{ cm}}$. What is the value of $1/f$ in this situation? What is f?