

PROBLEM 1 – 10 points

You have three polarizers. Polarizer A has its transmission axis at 0° relative to the vertical; polarizer B has its transmission axis at 30° to the vertical; and polarizer C has its transmission axis at 90° to the vertical.

[4 points] (a) You can arrange the three polarizers in any order you wish. If the incident light is **unpolarized**, in what order should you place the three polarizers so as to maximize the intensity of the light emerging from the last polarizer in the sequence?

- Polarizer A, then B, then C Polarizer C, then B, then A
 Either of the above, they'd give the same final intensity
 Polarizer B, then A, then C Polarizer B, then C, then A
 It doesn't matter what order they're in, none of the wave gets through

Briefly justify your answer: **B has to be between the other two. Because the transmission axes for A and C are perpendicular to one another they block all the light if light passes through them back-to-back. If the light goes through A first, then B, then C, the intensity is reduced by a factor of 2 passing through A, and then there is a 30° change from A to B and a 60° change from B to C. If you reverse the order you get the same factors, in a different order, which gives the same final intensity.**

[3 points] (b) If the incident light is **polarized at 30° to the vertical**, aligned with the transmission axis of polarizer B, in what order should you place the three polarizers so as to maximize the intensity of the light emerging from the last polarizer in the sequence?

- Polarizer A, then B, then C Polarizer C, then B, then A
 Either of the above, they'd give the same final intensity
 Polarizer B, then A, then C Polarizer B, then C, then A
 It doesn't matter what order they're in, none of the wave gets through

Again B must be between A and C. It's then best to maximize the intensity of the light emerging from the first polarizer, so the first polarizer should be the one out of A and C that is more closely aligned with the light. In this case that's A.

[3 points] (c) In part (b), let's say the polarizers are arranged so that the light passes first through C, then through B, then through A. If the intensity of the light incident on the first polarizer is 64 W/m^2 , find the intensity of the light emerging from:

- (i) polarizer C. $I_1 = I_0 \cos^2 \theta = (64 \text{ W/m}^2) \cos^2 60^\circ = (64 \text{ W/m}^2) \frac{1}{4} = 16 \text{ W/m}^2$
(ii) polarizer B. $I_2 = I_1 \cos^2 \theta = (16 \text{ W/m}^2) \cos^2 60^\circ = (16 \text{ W/m}^2) \frac{1}{4} = 4 \text{ W/m}^2$
(iii) polarizer A. $I_3 = I_2 \cos^2 \theta = (4 \text{ W/m}^2) \cos^2 30^\circ = (4 \text{ W/m}^2) \frac{3}{4} = 3 \text{ W/m}^2$

For part (b), the order A then B then C would result in a final intensity of 9 W/m^2

PROBLEM 2 – 10 points

A particular plane polarized electromagnetic wave, with a frequency of 100 MHz, is traveling through a vacuum in a direction we can call the x -axis. At $t = 0$, the electric field due to this wave at $x = 0$ has a magnitude of 300 V/m.

[2 points] (a) What is the wavelength of this wave?

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/s}}{1 \times 10^8 \text{ Hz}} = 3 \text{ m}$$

[2 points] (b) If this wave entered your eye, would you see anything? Explain why or why not.

No, you would not. Our eyes are sensitive to wavelengths in, approximately, the wavelength range from 400 nm to 700 nm. This wave is orders of magnitude out of that range. This wave is in the frequency range of FM radio signals, which are not visible to us.

[2 points] (c) At $t = 0$ and $x = 0$, what is the magnitude of the magnetic field due to this wave?

$$B = \frac{E}{c} = \frac{300 \text{ V/m}}{3 \times 10^8 \text{ m/s}} = 1 \times 10^{-6} \text{ T}$$

[2 points] (d) How much time passes, after $t = 0$, before the electric and magnetic fields at $x = 0$ are exactly the same as they are at $t = 0$? State the minimum non-zero time.

The fields should be exactly the same again after one period, which is the inverse of the frequency. That is a time of 10 ns.

[2 points] (e) If 300 V/m represents the amplitude of the electric field in this electromagnetic wave, what is the wave's average intensity?

$$I_{\text{average}} = \frac{E_{\text{max}} B_{\text{max}}}{2\mu_0} = \frac{(300 \text{ V/m}) \times (1 \times 10^{-6} \text{ T})}{2 \times 4\pi \times 10^{-7} \text{ Tm/A}} = \frac{300}{0.8\pi} \text{ W/m}^2 = 119 \text{ W/m}^2$$