Answer to Essential Question 21.5: The Doppler effect for sound (and for all mechanical waves) is not a relative velocity phenomenon. The relative velocity of the source and observer is the same in these two situations, but the observed frequency is different in the two situations. One interesting example is when $v_1 = v$, the wave speed. When the observer moves at speed v toward a stationary source, the observed frequency is twice the emitted frequency. When the source moves at a speed v toward a stationary observer, however, the observed frequency is infinite. We will investigate that situation further in the next section.

21-6 Sonic Booms, and the Doppler Effect in General

Essential Question 21.5 raises the question of what happens when a source of waves travels at the wave speed. We should also consider what happens when the source travels faster than the wave speed.

Let's begin by drawing a diagram like that in Figure 21.7, but with the source traveling to the right at the wave speed. In this special case, in Figure 21.8, because the source keeps up with the waves, the waves pile up at the source, leading to a large amplitude wave that moves with the source. This is known as a **sonic boom**, because a large amplitude corresponds to a loud sound. The observer at position A would hear the sonic boom when the source passed by.

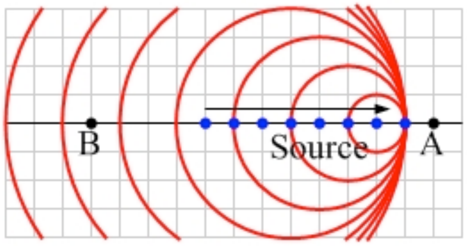

Figure 21.8: When the source moves at the wave speed, the waves pile up on one another at the source, creating a sonic boom.

Let's go further, and see what the picture looks like when the source travels faster than the waves. Figure 21.9 shows what happens when the source travels to the right at twice the wave speed. In this case, the waves pile up along lines that make an angle with the line of travel of the source. This pattern should look familiar to you, given that it looks like the waves left behind by a boat as it travels through water, as in the photograph in Figure 21.10. This tells us that the boat's speed is faster than the speed of the water waves.

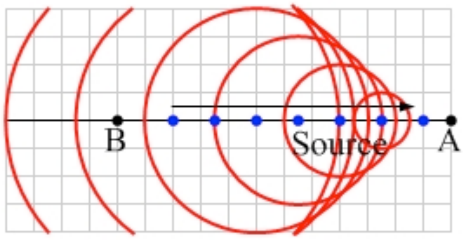

Figure 21.9: When the source moves faster than the waves, the waves create a wake pattern.

In section 21-5, we considered what happens when either the source moves or the observer moves, but not both. Let's now consider what happens in general, when both the source of a wave and the observer are moving with respect to the medium the waves are moving through. The general equation is simply a combination of the equations we derived in section 21-5 for the situations of a moving observer and a moving source.

Figure 21.10: A common example of the situation of a source of waves traveling faster through the medium than the waves themselves is in the wake created when a boat passes through water (here, the Avon Gorge in England). Photo credit: public-domain photo taken by Adrian Pingstone.

The Doppler effect: The Doppler effect describes the shift in frequency of a wave that occurs when the source of the waves, and/or the observer of the waves, moves with respect to the medium the waves are traveling through. The general equation for the observed frequency is:

$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right),$$ (Equation 21.11: **The general Doppler equation**)

where f' is the frequency observed by the observer, f is the frequency of the waves emitted by the source, v is the speed of the wave through the medium, v_o is the speed of the observer, and v_s is the speed of the source. In the numerator, use the top (+) sign if the observer moves toward the source, and the bottom (–) sign if the observer moves away from the source. In the denominator, use the top (–) sign if the source moves toward the observer, and the bottom (+) sign if the source moves away from the observer.

EXAMPLE 21.6 – Catching a moth

A particular bat emits ultrasonic waves with a frequency of 56.0 kHz. The bat is flying at 16.00 m/s toward a moth, which is moving at 2.00 m/s away from the bat. The speed of sound is 340.00 m/s. (a) Assuming the moth could detect the waves, what frequency waves would it observe? (b) The waves reflect off the moth and are detected by the bat. What frequency are the waves detected by the bat?

SOLUTION

(a) Here, we use the general Doppler equation, where f = 56.0 kHz and v = 340 m/s. The observer is the moth, so v_o is 2.00 m/s, and we use the bottom sign (the minus sign) in the numerator because the moth is traveling away from the bat. The bat is the source, so v_s = 16.00 m/s, and we use the top sign (the minus sign) in the denominator because the bat is traveling toward the moth. This gives:

$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right) = (56.0 \text{ kHz})\left(\frac{340.00 \text{ m/s} - 2.00 \text{ m/s}}{340.00 \text{ m/s} - 16.00 \text{ m/s}}\right) = (56.0 \text{ kHz}) \times 1.0432 = 58.42 \text{ kHz}$$

(b) Again, we use the general Doppler equation. This time, the moth acts as the source (because the moth reflects the waves back to the bat) and the bat is the observer. The frequency emitted by the moth is the frequency we found in part (a). Let's use f'' to denote the frequency of the waves detected by the bat, so f' = 58.42 kHz and v = 340 m/s. The observer is the bat, so v_o' is 16.00 m/s, and we use the top sign (the plus sign) in the numerator because the bat is traveling toward the moth. The moth is the source, so v_s' = 2.00 m/s, and we use the bottom sign (the plus sign) in the denominator because the moth is traveling away from the bat. This gives:

$$f'' = f'\left(\frac{v \pm v_o'}{v \mp v_s'}\right) = (58.42 \text{ kHz})\left(\frac{340.00 \text{ m/s} + 16.00 \text{ m/s}}{340.00 \text{ m/s} + 2.00 \text{ m/s}}\right) = (58.42 \text{ kHz}) \times 1.0409 = 60.8 \text{ kHz}$$

The bat can use the frequency of the detected wave to determine how fast, and in what direction, the moth is flying.

Related End-of-Chapter Exercises: 25 – 27, 46 – 48.

Essential Question 21.6: What happens when a source and observer have identical velocities? Is the observed frequency larger, smaller, or the same as the frequency emitted by the source?