Answer to Essential Question 21.10: The longer the pipe, the longer the wavelength of the fundamental. Wavelength is inversely proportional to frequency, so the longer the pipe, the smaller the frequency.

Chapter Summary

Essential Idea: Waves.

A wave is a way to transfer energy from one place to another without needing a net flow of material. Waves are in integral part of the way we communicate, whether it be the signals that are picked up by our cell phones, and turned into recognizable speech by the phone's circuitry and speaker, or the light that brings the world to our eyes.

Types of Waves

In this chapter, we dealt with **mechanical waves**, which need a medium through which to travel. Such waves can be **transverse**, in which the particles of the medium oscillate in a direction perpendicular to the direction the wave travels, or **longitudinal**, in which the particles of the medium oscillate along the same direction as the direction the wave travels. A wave on a string is generally transverse, while sound waves are longitudinal.

The wave equation

In general, the relationship between wave speed, *v*, frequency, *f*, and wavelength, *λ*, is:

$v = f\lambda$. (Equation 21.1: **Connecting speed, frequency, and wavelength**)

Equation of motion for a single-frequency transverse wave

In general, the displacement of any point in the medium, at any instant in time, when a single-frequency transverse wave is propagating through the medium in the *x*-direction, is given by an equation of the form:

$y = A\cos(\omega t \pm kx)$, (Equation 21.4: **Equation of motion for a transverse wave**)

where the plus sign is used when the wave is traveling in the negative *x*-direction, and the minus sign is used when the wave is traveling in the positive *x*-direction.

The wave number, *k*, is related to the wavelength, *λ*, in the same way that the angular frequency, *ω*, is related to the period, *T*:

$k = \frac{2\pi}{\lambda}$. (Equation 21.2: **the wave number**)

$\omega = \frac{2\pi}{T}$. (Equation 21.3: **the angular frequency**)

Wave speed

In general, the wave speed is determined not by the frequency and wavelength, but by properties of the medium itself. For example, the speed of a wave on a string is determined by the tension in the string, F_T, and the mass per unit length, *μ*:

$v = \sqrt{\frac{F_T}{m/L}} = \sqrt{\frac{F_T}{\mu}}$, (Eq. 21.5: **The speed of a wave on a string**)

Sound Intensity

Intensity is the power per unit area: $I = P/A$. Because of the way the human ear responds to sound, we generally use a logarithmic scale to measure the intensity level of a sound:

$$\beta = (10\text{ dB})\log\left(\frac{I}{I_0}\right),$$ (Equation 21.7: **Absolute sound intensity level, in decibels**)

where the reference intensity $I_0 = 1 \times 10^{-12}$ W/m^2 is known as the threshold of hearing, and the log is in base 10.

The Doppler Effect

The Doppler effect describes the shift in frequency of a wave that occurs when the source of the waves, and/or the observer of the waves, moves with respect to the medium the waves are traveling through. If the source emits a frequency f, the frequency f' received by the observer is:

$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right),$$ (Equation 21.11: **The general Doppler equation**)

where v is the speed of the wave through the medium, v_o is the speed of the observer, and v_s is the speed of the source. In the numerator, use the top (+) sign if the observer moves toward the source, and the bottom (–) sign if it moves away. In the denominator, use the top (–) sign if the source moves toward the observer, and the bottom (+) sign if it moves away.

Superposition and interference

When two or more waves overlap, we find the net effect by applying the **principle of superposition**: the net displacement of any point in a medium is the sum of the displacements at that point due to each of the individual waves. If the displacements of the individual waves are in the same direction at a point, we say that the waves experience **constructive interference**, leading to a large net displacement at that point. If the individual displacements are in opposite directions, **destructive interference** occurs, which means that the net displacement is small.

Beats

One example of superposition is when two waves of different frequencies interfere, leading to oscillations in the amplitude of the resultant wave. This is known as beats. The frequency at which the amplitude oscillates is the difference between the two frequencies.

$$f_{beat} = f_{high} - f_{low}.$$ (Equation 21.12: **the beat frequency**)

Standing waves

Standing waves are waves in which the **nodes** (points of zero displacement) and the **anti-nodes** (points of maximum displacement) remain at rest. Standing waves are generally produced by two identical waves traveling in opposite directions through a medium, and they describe the waves produced by string and wind instruments.

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L},$$ (**Standing-wave frequencies for strings and for pipes open at both ends**)

where L is the length of the string or pipe, v is the wave speed, and n is any integer. The lowest-frequency standing wave (for $n = 1$) is known as the **fundamental**, while the others are known as **harmonics**. Thus, harmonics are integer multiples of the fundamental.

$$f_n = \frac{nv}{4L},$$ (Eq. 21.15: **Standing-wave frequencies for a pipe open at one end only**)

where n is any odd integer.