Answer to Essential Question 19.5: Figure 19.18, with the magnetic field perpendicular to the plane of the loop, is a special case, in which both the net force and the net torque are zero. In Figure 19.19, the forces acting on the loop would cause the loop to rotate, with the left side of the loop coming out of the page and the right side going into the page. In this case there is a net torque acting on the loop, which is generally the case.

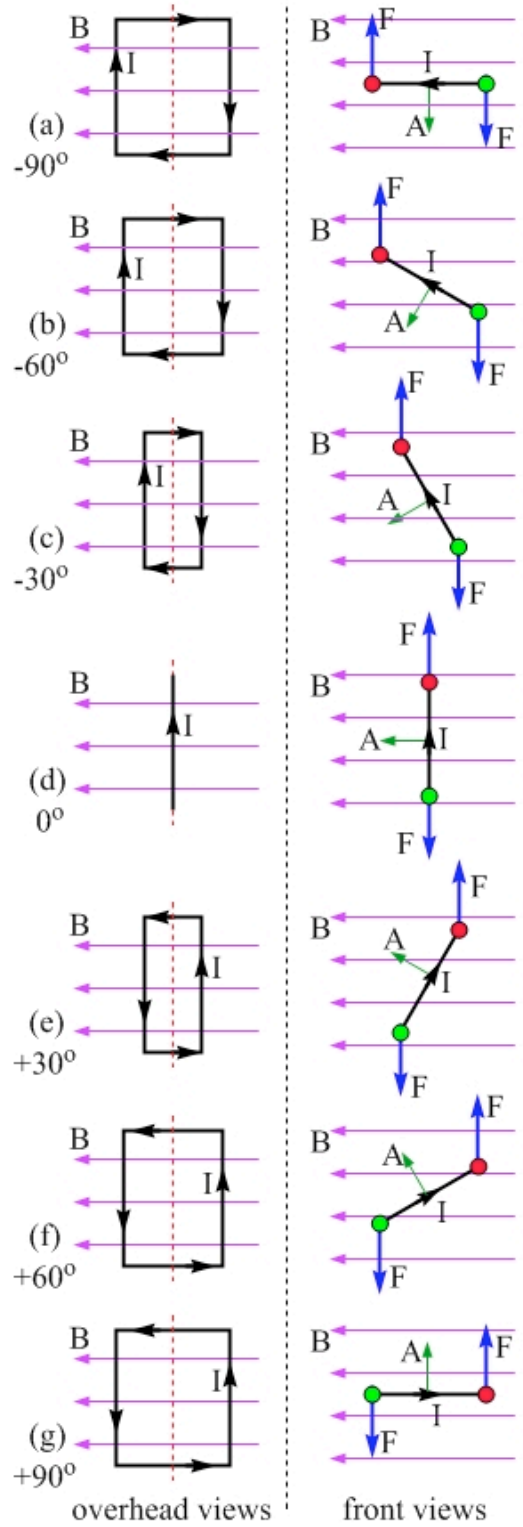## 19-6 The Magnetic Torque on a Current Loop

In a uniform magnetic field, a wire loop carrying current experiences no net force, but there is generally a net torque acting that tends to make the loop rotate. As we will see, we can exploit the interaction between the loop and the field to make a motor.

Let's begin by drawing a number of views of a rectangular current-carrying loop in a uniform magnetic field. The loop can rotate without friction about an axis parallel to its long sides, passing through the center of the loop. In the front view, imagine viewing the loop as if your eye is at the bottom of the page, looking along the page at the loop. The angles specified in Figure 19.20 are the angles between the loop's area vector, which is perpendicular to the plane of the loop, and the magnetic field.

Specifying area as a vector is something new. The magnitude of the area vector is the loop's length multiplied by its width. The direction of the area vector is perpendicular to the plane of the loop, but there are two directions that are perpendicular to this plane. The orientation of the area vector can be found by applying a right-hand rule. Curl the fingers on your right hand in the direction of the current flow. Your thumb, when you stick it out, gives the direction of the area vector. The angles specified in Figure 19.20 are the angles between the area vector and the magnetic field.

If the loop starts from rest in Figure 19.20(a), it will begin to rotate. The angular velocity increases in a clockwise direction, as observed from the front view. The clockwise torque continues until Figure 19.20(d), at which point there is no torque. In the absence of friction, the loop's angular momentum keeps it moving forward, with the counter-clockwise torque in Figures 19.20(e) – (g) slowing the loop down and bringing it instantaneously to rest in Figure 19.20(g). The loop then reverses direction, and goes through the pictures in reverse order.

> **Figure 19.20**: The arrow, labeled A in each front view, is the loop's area vector, which is directed perpendicular to the plane of the loop. When the area vector is perpendicular to the magnetic field, the loop experiences maximum torque. If the loop starts from rest in (a), the clockwise torque it experiences in orientations (a) – (d) cause the loop to rotate clockwise. After (d), the torque reverses direction, bringing the loop instantaneously to rest at (g), at which point the motion reverses.
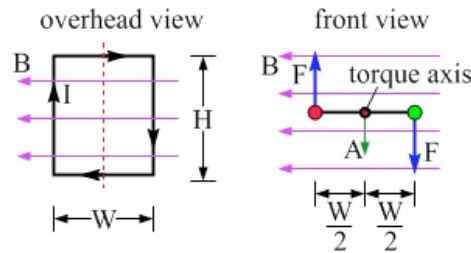
(a) -90°
(b) -60°
(c) -30°
(d) 0°
(e) +30°
(f) +60°
(g) +90°

overhead views   front views

The situation from Figure 19.20(a) is re-drawn in Figure 19.21, showing the height H and width W of the loop. Apply equation 10.9, $\tau = r\,F\sin\theta$, to find the torque applied by the field on the loop.

Figure 19.21: The situation from Figure 19.20(a), with dimensions given for the loop.

Take torques about the axis through the center of the loop, shown as a dashed red line in Figure 19.21 (overhead view). The distance from each force to this axis is $W/2$, each force has a magnitude of $F = IHB$ (from equation 19.7), and the angle between the line we're measuring the distance along and the line of the force is 90°. A factor of 2 accounts for the two identical torques, which are both clockwise. The magnitude of the net torque on the loop in this orientation is:

$$\tau = 2\frac{W}{2}IHB = IAB \text{ , where } A = HW \text{ is the area of the loop.}$$

As the loop rotates, the torque is reduced as the angle between each force and the plane of the loop changes from 90°. This angle is equal to the angle between the area vector and field, so we can write a general expression for the torque, and generalize to coils with more than one loop.

The magnitude of the torque applied by a uniform magnetic field on a coil of N loops carrying a current I, with an angle θ between the area vector $\bar{A}$ and the magnetic field $\bar{B}$, is:
$$\tau = NIAB\sin\theta \text{ .} \quad \text{(Equation 19.8: \textbf{Torque on a current-carrying coil in a magnetic field})}$$

The DC (Direct Current) Motor – an application of the torque on a current-carrying loop
  If the loop starts from rest in the orientation in Figure 19.20(a), then the loop will flop back and forth as the torque alternates between clockwise and counterclockwise. Friction will eventually bring the loop to rest in the zero-torque orientation of Figure 19.20(d). With one change, however, the loop becomes an electric motor – a device for transforming electrical energy into mechanical energy. We simply reverse the direction of the current each time the plane of the loop is perpendicular to the field (as in Figure 19.20(d)). With this change, the torque on the loop, and therefore the rotation of the loop, is always in the same direction.

  An easy way to accomplish the current reversal is to use a split-ring commutator, as shown in Figure 19.22. The split-ring commutator is a cylinder divided into two halves that are electrically insulated from one another. The cylinder rotates with the loop, and its left side rubs against a fixed wire connected to the positive terminal of a battery while its right side rubs against a fixed wire connected to the battery's negative terminal. As shown in Figure 19.22, the current reverses direction in the loop every half-rotation. From our perspective, however, the current always goes clockwise around the loop, which is why the torque is always in the same direction.
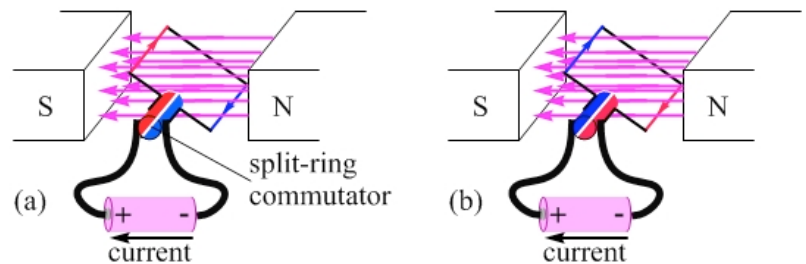

Figure 19.22: The split-ring commutator (the split cylinder that the dark wires from the battery are connect to) reverses the current in the loop every half rotation. From our perspective, the current is always clockwise, but from the loop's perspective the current has reversed between (a) and 180° later in (b).

Related End-of-Chapter Exercises: 54, 59.

Essential Question 19.6: In the situation shown in Figure 19.20, we ignored the magnetic forces acting on the shorter sides of the loop. Why can we do this?