Answer to Essential Question 19.4: The velocity selector works for negatively charged particles without any changes. When the particle has a negative charge, the electric force and the magnetic force both reverse direction. For particles traveling at a speed of $v = E/B$, however, the two forces balance and the particles are undeflected. Faster particles would be deflected down, given the orientation of the fields in Figure 19.14, while slower particles would be deflected up.

19-5 The Magnetic Force on a Current-Carrying Wire

Starting with the magnetic force on a moving charge, $F_M = qvB \sin\theta$, let's derive the equation for the magnetic force on a current-carrying wire. In the equation below, we write the velocity as a length L divided by a time interval Δt. We also use the definition of current, as the amount of charge q passing a point in a certain time interval Δt. This gives:

$$F_M = qvB\sin\theta = q\frac{L}{\Delta t}B\sin\theta = \frac{q}{\Delta t}LB\sin\theta = ILB\sin\theta .$$

The magnitude of the magnetic force exerted on a wire of length L, carrying a current I, by a magnetic field of magnitude B is:

$F_M = ILB\sin\theta$,   (Equation 19.7: **The magnetic force on a current-carrying wire**)

where θ is the angle between the current direction and the magnetic field.

The direction of the force is given by a right-hand rule that follows the rule for charges. Point the fingers on your right hand in the direction of the current. Align your hand so that when you curl your fingers, they point in the direction of the magnetic field. Stick out your thumb, and it points in the direction of the magnetic force. The current direction is defined to be the direction of flow of positive charge, so the direction given by your hand never needs to be reversed, as it does when we find the direction of the magnetic force on a negatively charged particle.

EXPLORATION 19.5A – Three paths from a to b

As shown in Figure 19.16, three wires carry equal currents from point a to point b. The wires are in a uniform magnetic field directed to the right.

Step 1 – *Apply the right-hand rule to find the direction of the magnetic force exerted on each wire by the field.* For wire 1, pointing the fingers of the right hand in the direction of the current and then curling them into the direction of the magnetic field gives a force directed into the page. For wire 2, there is no force on the part that is parallel to the field. Apply the right-hand rule to the part of wire 2 that is perpendicular to the field – the force is directed into the page. For wire 3, we ignore the parts of the wire that are going left or right, parallel to the field, and focus on the sections that are going up or down, perpendicular to the field. Wire 3 goes up more than it goes down. Applying the right-hand rule with the current directed up gives a force into the page.
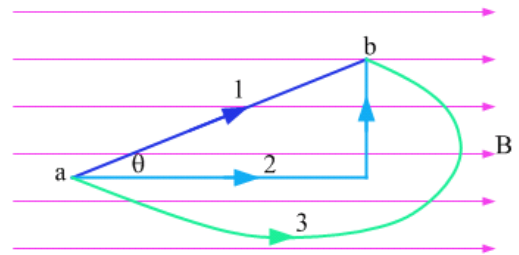

Figure 19.16: Three wires carry currents of the same magnitude from point a to point b in a uniform magnetic field.

Step 2 – *Rank the wires based on the magnitude of the force they experience. To help you, re-draw wire 3 as short segments that are either parallel or perpendicular to the magnetic field.*
Applying Equation 19.7, we find for wire 1 that $F_{M1} = ILB\sin\theta$, where wire 1 has a length L. For wire 2, only the part directed up, perpendicular to the field, experiences a force. This section of

wire 2 has a length of $L\sin\theta$, so applying equation 19.7 for wire 2 gives $F_{M2} = I(L\sin\theta)B\sin(90°)$. The factor of $\sin(90°)$ equals 1, giving: $F_{M1} = F_{M2} = ILB\sin\theta$.

Let's re-draw wire 3 using line segments that are either parallel to or perpendicular to the magnetic field, as in Figure 19.17. We could use shorter segments, so the wire would not look so jagged, but the argument that follows would still apply. The segments that are parallel to the field experience no force. For the segments perpendicular to the field, subtract the total length of segments that carry current down from the total length of segments that carry current up, because their forces are in opposite directions. The net displacement is $L\sin\theta$ up, the same as wire 2. Thus, all three wires experience the same force and the ranking is 1 = 2 = 3. The same argument applies for any wire: all wires carrying equal currents from a to b in a uniform magnetic field experience equal forces.
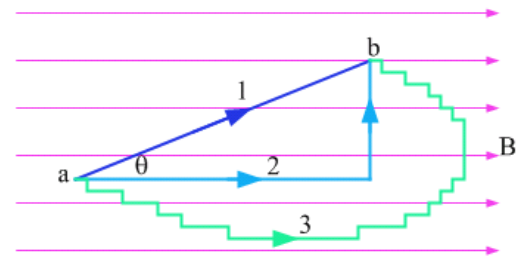

Figure 19.17: Wire 3 is replaced by a joined set of line segments, parallel and perpendicular to the magnetic field.

Key idea: All wires carrying equal currents from one point to another in a uniform magnetic field experience the same magnetic force. **Related End-of-Chapter Exercises: 51, 52.**

EXPLORATION 19.5B – The force on a current-carrying loop
Step 1 – *In what direction is the net magnetic force on a rectangular wire loop that carries a clockwise current I in a uniform magnetic field that is directed out of the page? The loop is in the plane of the page.* A diagram is shown in Figure 19.18 (a). To find the net magnetic force on the loop, we add, as vectors, the forces on each side of the loop. As shown in Figure 19.18 (b), the force on the left side cancels the force on the right side, and the force at the top cancels the force at the bottom. Thus, the net force on the loop is zero.
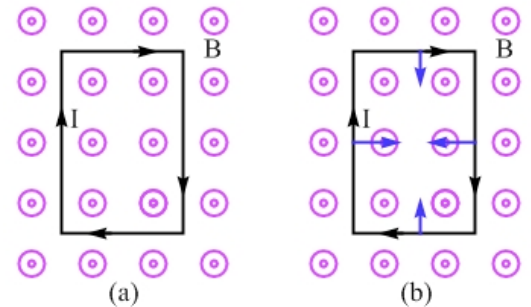

Figure 19.18: (a) A rectangular loop in a uniform magnetic field directed out of the page. (b) The magnetic force on each side of the loop is shown with an inward-directed arrow. The net force on the loop is zero.

Step 2 – *In what direction is the net force on the loop if the field is directed to the left?* In this special case, the forces on the top and the bottom sides are each zero. As Figure 19.19 (b) shows, the remaining forces cancel, and again the net force on the loop is zero. This is always true - as long as the magnetic field is uniform, the net magnetic force on any complete current-carrying loop is zero.
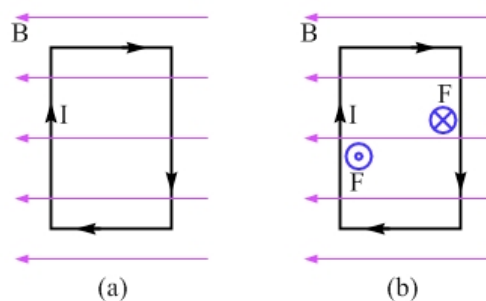

Figure 19.19: (a) A rectangular loop in a uniform magnetic field directed to the left. (b) In this case, the top and bottom sides experience no force because they are parallel to the magnetic field. The force on the left side is out of the page, while the force on the right side is into the page. These forces are of equal magnitude, so the net force is zero.

Key idea for current-carrying loops: In a uniform magnetic field, the net magnetic force acting on a current-carrying loop is zero. **Related End-of-Chapter Exercise: 53.**

Essential Question 19.5: Consider figures 19.18 and 19.19. The net magnetic force acting on a complete current-carrying loop is zero, but what about the net magnetic torque on the loop?