

**Answer to Essential Question 19.3:** The direct way to solve this problem is to use Equation 19.2,  $F_M = qvB \sin\theta$ . The particles are in the same magnetic field, and  $\sin\theta = 1$  because each velocity is perpendicular to the magnetic field. Thus, in this situation the force magnitude is proportional to the magnitude of the charge multiplied by the speed. This combination of factors is 0 for particle 1,  $4qv$  for particle 2, and  $2qv$  for particles 3 and 4, so the ranking is  $2 > 3 = 4 > 1$ .

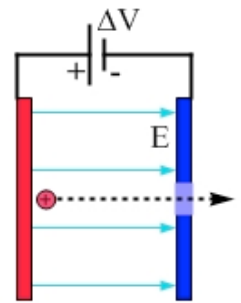
## 19-4 Mass Spectrometer: An Application of Force on a Charge

There are a number of practical devices that exploit the force that a magnetic field applies to a charged particle. Let's investigate one of these devices, the mass spectrometer.

### EXPLORATION 19.4 – How to make a mass spectrometer

Mass spectrometers, which separate ions based on mass, are often used by chemists to determine the composition of a sample. Let's explore one type of mass spectrometer, which uses electric and magnetic fields. For each step below, sketch a diagram to help you with the analysis.

**Step 1 – The accelerator.** *Release a charged particle from rest near one plate of a charged parallel-plate capacitor, so that the particle accelerates toward the other plate of the capacitor. Apply energy conservation to obtain a relation between  $\Delta V$ , the potential difference across the capacitor, and the speed of the particle when it emerges from a small hole in the second plate.* The particle shown in Figure 19.12 has a positive charge, but the accelerator can also work for negatively charged particles if we reverse the battery attached to the capacitor. Let's apply what we learned in Chapter 17. If we define the particle's electric potential energy to be zero at the negative plate ( $U_f = 0$ ), the potential energy is  $U_i = q(\Delta V)$  when the particle is next to



**Figure 19.12:** Charged particles are released from rest near the left-hand plate of a parallel-plate capacitor, accelerate across the gap, and emerge via a hole cut in the right-hand plate.

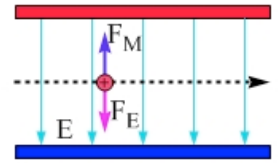
the positive plate. The particle has no initial kinetic energy ( $K_i = 0$ ), but as the particle accelerates across the gap between the plates the electric potential energy is transformed into kinetic energy. There is no work done by non-conservative forces ( $W_{nc} = 0$ ). With three terms being zero, our five-term conservation of energy equation,  $K_i + U_i + W_{nc} = K_f + U_f$ , becomes  $U_i = K_f$ . Using our expressions for potential and kinetic energy gives:

$$q(\Delta V) = \frac{1}{2}mv^2, \text{ so the particle's speed when it emerges is } v = \sqrt{\frac{2q(\Delta V)}{m}}.$$

**Step 2 – The velocity selector.** *The particle passes through a second parallel-plate capacitor in which the plates are parallel to the particle's velocity. In addition to the electric field inside the capacitor there is also a magnetic field, directed perpendicular to both the electric field and the velocity of the particle. The combined effect of the two fields is that a particle with just the right speed experiences no net force and passes undeflected through the velocity selector. Determine how this speed relates to the magnitudes of the fields.* In the situation shown in Figure 19.13, the top plate is positively charged, and the bottom plate is negatively charged, so the electric field in the capacitor is directed down. The electric force on the positive charge is also directed down, because  $\vec{F}_E = q\vec{E}$ . For the particle to experience no net force, the magnetic force must exactly balance the electric force. The magnitude of the magnetic force is given by  $F_M = qvB \sin\theta$ .

Because the velocity and magnetic field are at right angles,  $\theta = 90^\circ$  and  $\sin(90^\circ) = 1$ .

Setting the magnitudes of the two forces equal,  $F_M = F_E$ , gives  $qvB = qE$ . The factors of  $q$  cancel. Solving for the speed of the undeflected particles gives:

$$v = \frac{E}{B}. \quad (\text{Eq. 19.5: The speed of undeflected particles in a velocity selector})$$


There is a uniform magnetic field, directed perpendicular to the page, between the plates.

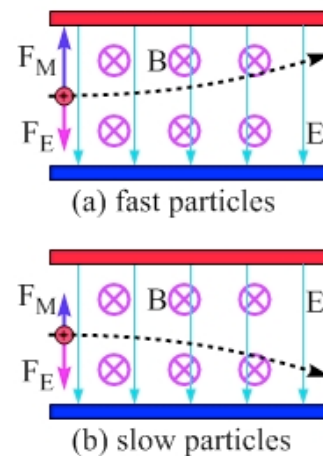
**Question:** Is the magnetic field in Figure 19.13 directed into or out of the page?

**Answer:** By the right-hand rule, to obtain a magnetic force directed up on a positive charge with a velocity to the right, the magnetic field is into the page.

**Question:** What happens to particles traveling faster than the undeflected particles? What happens to particles traveling slower than the undeflected particles?

**Answer:** The magnetic force depends on speed, while the electric force does not. For particles going faster than the selected speed, the magnetic force exceeds the electric force. The net upward force deflects the fast particles up out of the beam (see Figure 19.14). For relatively slow particles, the magnetic force is less than the electric force. The net downward force deflects the slow particles down out of the beam.

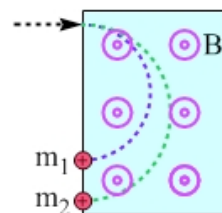
**Figure 19.13:** A charged particle with just the right velocity passes undeflected through the velocity selector because the magnetic force balances the electric force.



**Figure 19.14:** (a) A velocity selector deflects fast particles in one direction (up, in this case) and (b) slow particles in the opposite direction (down, here).

**Step 3 – The mass separator.** *Particles that pass undeflected through the velocity selector are sent into the mass separator, which consists of a uniform magnetic field that is perpendicular to the velocity of the particles. If the particles are collected after traveling through half-circles in the field, find an expression for the separation between particles of mass  $m_1$  and particles of mass  $m_2$ .* Consider equation 19.3 ( $r = mv/qB$ ), which gives the radius of the path followed by a charged particle in a uniform magnetic field. We give the particles the same charge in step 1, and they are in the same magnetic field, so the radius depends on mass and speed. The velocity selector ensures that all the particles reaching the mass separator have the same speed, so the radii differ only because the masses differ. As shown in Figure 19.15, the particles enter the field at the same point. After passing through a half-circle, the separation between the particles is the difference between the diameters of the circular paths. Thus, the separation is given by:

$$\Delta s = 2r_1 - 2r_2 = 2 \left( \frac{m_1 v}{qB} - \frac{m_2 v}{qB} \right) = \frac{2v}{qB} (m_1 - m_2) \quad (\text{Eq. 19.6: Separation in the mass spec.})$$



**Figure 19.15:** After traveling through a half-circle in the mass separator, particles of different mass are separated by a distance equal to the difference between the diameters of their paths.

**Key Ideas for the mass spectrometer:** The mass spectrometer is an excellent example of how we exploit uniform electric and magnetic fields, both individually and in combination, when working with charged particles.  
**Related End-of-Chapter Exercises:** 45 – 47.

**Essential Question 19.4:** If the particles in step 2 of Exploration 19.4 were negatively charged, would we need to reverse the direction of either field (or both fields) for the velocity selector to function properly, or would it work for negatively charged particles without any changes?