Figure 18.21: Connecting the new resistor across the battery does not change the current elsewhere in the circuit.

Answer to Essential Question 18.8: Yes, it can be done by placing the 5.0 Ω resistor in parallel with the original set of resistors. This is how circuits work in your house; turning on a lamp, for instance, does not affect a TV, even when the lamp and the TV are plugged into the same outlet.
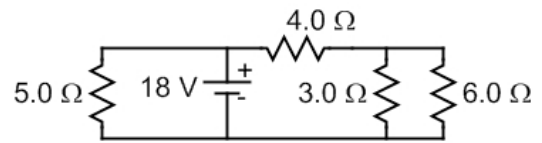
18-9 Multi-loop Circuits

In many circuits with more than one battery the batteries cannot be replaced by a single battery. In such cases we rely on Kirchoff's Rules, the loop rule and the junction rule.

Figure 18.22: A multi-loop circuit with three batteries and four resistors.

EXPLORATION 18.9 – Analyzing a multi-loop circuit

Solve for the current through each of the resistors in the circuit shown in Figure 18.22. The emf's of the three batteries are: $\varepsilon_1 = 2.0$ V; $\varepsilon_2 = 15$ V; and $\varepsilon_3 = 12$ V . The resistances of the four resistors are $R_1 = 1.0\ \Omega$; $R_2 = 5.0\ \Omega$; $R_3 = 2.0\ \Omega$; $R_4 = 3.0\ \Omega$.
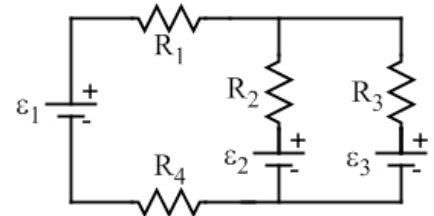
Step 1 – *Choose a direction for the current, and label the current, in each branch of the circuit.* A branch is a path from one junction to another in a circuit. Figure 18.23 shows the two junctions (points at which more than two current paths come together) as circles, with the three different branches running from one junction to the other. Within each branch, everything is in series, so each branch has its own current, labeled $I_1, I_2,$ and I_3 . In many cases, such as this one, we do not know for certain which way the current goes in a particular branch. We simply choose a direction for each current, and if we are incorrect we will get a minus sign for that current when we solve for it.

Step 2 – *Based on the directions chosen for each current, put a + sign at the higher-potential end of each resistor and a – sign at the lower-potential end.* This step is optional, but can make it easier to set up the loop-rule equations correctly in step 3. Current is directed from the high-potential end of a resistor to the low-potential end, giving the signs in Figure 18.23.
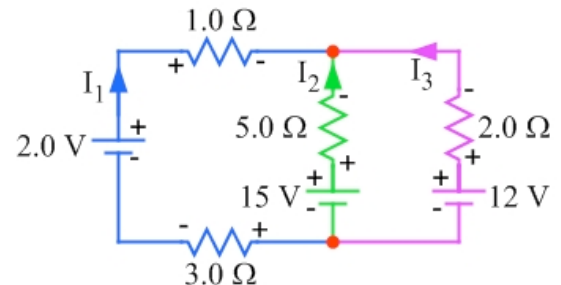

Figure 18.23: The three currents in the circuit are labeled, and directions are chosen for them. The resistors are labeled with high-potential (+) and low-potential (−) ends based on the direction chosen for current; the current direction through a resistor is in the direction of decreasing potential.

Step 3 – *Apply the loop rule to one complete loop in the circuit.* This means applying the equation $\sum \Delta V = 0$. ΔV for a battery is the battery emf, which is positive if we go from the – terminal to the + terminal and negative if we go the other way. ΔV for a resistor is IR for that resistor, positive if we go from – to + (with the current) and negative if we go the other way (against the current). Let's start with the inside loop on the left, going clockwise around the loop starting from the lower-left corner. Keeping track of the signs carefully, we get:

$$+2.0\ \text{V} - (1.0\ \Omega)I_1 + (5.0\ \Omega)I_2 - 15\ \text{V} - (3.0\ \Omega)I_1 = 0$$

This simplifies to $-13\ \text{V} - (4.0\ \Omega)I_1 + (5.0\ \Omega)I_2 = 0$.   [Equation 1]

Step 4 – *Keep applying the loop rule to complete loops. Each new equation should involve a branch not involved in any previous equations. Stop writing down loop equations when you have involved every branch at least once.* For this particular circuit, we only have one more

branch (the one on the right) to involve, so we just need one more loop equation. We can either write a loop equation for the inside loop on the right or for all the way around the outside. Let's try the inside loop on the right. Starting at the lower right and going counter-clockwise, we get:

$$+12 \text{ V} - (2.0 \text{ }\Omega)I_3 + (5.0 \text{ }\Omega)I_2 - 15 \text{ V} = 0 \,.$$

This simplifies to $-3 \text{ V} + (5.0 \text{ }\Omega)I_2 - (2.0 \text{ }\Omega)I_3 = 0$.   [Equation 2]

We could still go around the outside to obtain a third loop equation; however, because that third equation can be obtained from Equations 1 and 2 above it does not give us any new information, and thus does not give us the third independent equation we need.

Step 5 – *Apply the junction rule to come up with additional equations relating the variables.*
The junction rule says that the sum of the current entering a junction must equal the sum of the current leaving the junction. In this circuit both junctions give the same equation, so no matter which junction we choose we get:

$$I_1 + I_2 + I_3 = 0 \,.$$ [Equation 3]

This actually tells us that at least one of the currents must be directed opposite to the way we guessed, because we cannot have three currents coming into a junction and no current directed away (or vice versa), so we expect at least one current to have a minus sign in the end.

Step 6 – *Solve the equations to find the currents in the circuit.* This is now an exercise in algebra, solving three equations in three unknowns. Here is one way to do it. Noting that I_2 appears in both Equation 1 and Equation 2, solve Equation 3 for I_2 and substitute it into both Equations 1 and 2. From Equation 3 we get $I_2 = -I_1 - I_3$.

Equation 1 becomes: $-13 \text{ V} - (9.0 \text{ }\Omega)I_1 - (5.0 \text{ }\Omega)I_3 = 0$.   [Equation 4]

Equation 2 becomes: $-3 \text{ V} - (5.0 \text{ }\Omega)I_1 - (7.0 \text{ }\Omega)I_3 = 0$.   [Equation 5]

Multiply Equation 4 by a factor of +7: $-91 \text{ V} - (63 \text{ }\Omega)I_1 - (35 \text{ }\Omega)I_3 = 0$. [Equation 6]

Multiply Equation 5 by a factor of -5: $+15 \text{ V} + (25 \text{ }\Omega)I_1 + (35 \text{ }\Omega)I_3 = 0$. [Equation 7]

Add Equations 6 and 7: $-76 \text{ V} - (38 \text{ }\Omega)I_1 = 0$, which gives $I_1 = -2.0 \text{ A}$. Substituting this into Equation 4 (or Equation 5) gives $I_3 = +1.0 \text{ A}$. Using Equation 3 then gives $I_2 = +1.0 \text{ A}$.

Key idea for a multi-loop circuit: In a circuit with multiple batteries we can use Kirchoff's loop rule and Kirchoff's junction rule to solve for any unknown parameters. The loop rule is actually a statement of conservation of energy applied to circuits, while the junction rule is a statement of conservation of charge.   **Related End-of-Chapter Exercises: 37 – 41, 61, 62.**

Essential Question 18.9: Re-draw Figure 18.23 with the current in each branch labeled correctly. Also, label the potential at all points in the circuit, if the lower junction is at a potential of $V = 0$. What is the potential difference between the two junctions?