

Answer to Essential Question 18.6: The fact that one resistance is three times the other means that three times as much current passes through the smaller resistor as through the larger resistor. Thus $\frac{3}{4}$ of the current passes through the $10\ \Omega$ resistor and $\frac{1}{4}$ passes through the $30\ \Omega$ resistor.

18-7 Series-Parallel Combination Circuits

In many circuits some resistors are in series while others are in parallel. In such series-parallel combination circuits we often want to know the current through, and/or the potential difference across, each resistor. Let's explore one method for doing this. This method can be used if the circuit has one battery, or when multiple batteries can be replaced by a single battery.

EXPLORATION 18.7 – The contraction/expansion method of circuit analysis

Four resistors are connected in a circuit with a battery with an emf of $18\ \text{V}$, as shown in Figure 18.14. The resistors have resistances $R_1 = 4.0\ \Omega$, $R_2 = 5.0\ \Omega$, $R_3 = 7.0\ \Omega$, and $R_4 = 6.0\ \Omega$. Our goal is to find the current through each resistor.

Step 1 – Label the currents at various points in the circuit. This can help determine which resistors are in series and which are in parallel. This is done in Figure 18.15(a). The current passing through the battery is labeled I . This current splits, with a current I_1 through resistor R_1 , and a current I_2 through resistor R_2 . The current I_2 goes on to pass through R_3 , so R_2 and R_3 are in series with one another. The two currents re-combine at the top right of the circuit, giving a net current of I directed from right to left through resistor R_4 and back to the battery.

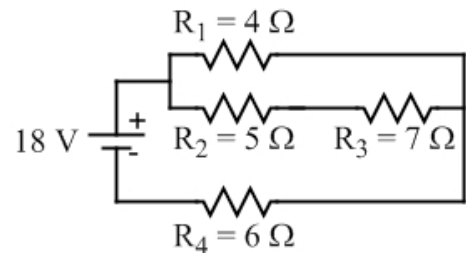


Figure 18.14: A series-parallel combination circuit with one battery and four resistors.

Step 2 – Identify two resistors that are either in series or in parallel with one another, and replace them by a resistor of equivalent resistance. Resistors R_2 and R_3 are in series, so they can be replaced by their equivalent resistance of $12.0\ \Omega$ (resistances add in series), as shown in Figure 18.15(b). Is this the only place we could start in this circuit? For instance, are resistors R_1 and R_2 in parallel? To be in parallel, both ends of the resistors must be directly connected by a wire, with nothing in between. The left ends of resistors R_1 and R_2 are directly connected, but the right ends are not, with resistor R_3 in between. In fact, the only place to start in this circuit is with R_2 and R_3 .

Step 3 – Continue the process of replacing two resistors by an equivalent resistor until the circuit is reduced to one equivalent resistor. In the next step, shown in Figure 18.15(c), the two resistors in parallel, R_1 and R_{23} , are replaced by their equivalent resistance of $3.0\ \Omega$. Finally, in Figure 18.15(d), the two resistors in series are replaced by their equivalent resistance, $9.0\ \Omega$.

Step 4 – Apply Ohm's Law to find the total current in the circuit. With only one resistor we know both its resistance and the potential difference across it, so we can apply Ohm's Law:

$$I = \frac{\varepsilon}{R_{eq}} = \frac{18\ \text{V}}{9.0\ \Omega} = 2.0\ \text{A}.$$

Step 5 – Label the potential at various points in the single-resistor circuit. Choose a point as a reference. Here we choose the negative terminal of the battery to be $V = 0$. The other side of the battery is therefore $+18$ volts. Wires have negligible resistance, so $\Delta V = IR = 0$ across each wire. Thus, all points along the wire leading from the negative terminal of the battery have $V = 0$, while all points along the wire leading from the positive terminal have $V = +18\ \text{V}$. See Figure 18.15(e).

Figure 18.15: The various steps in the contraction/expansion method.

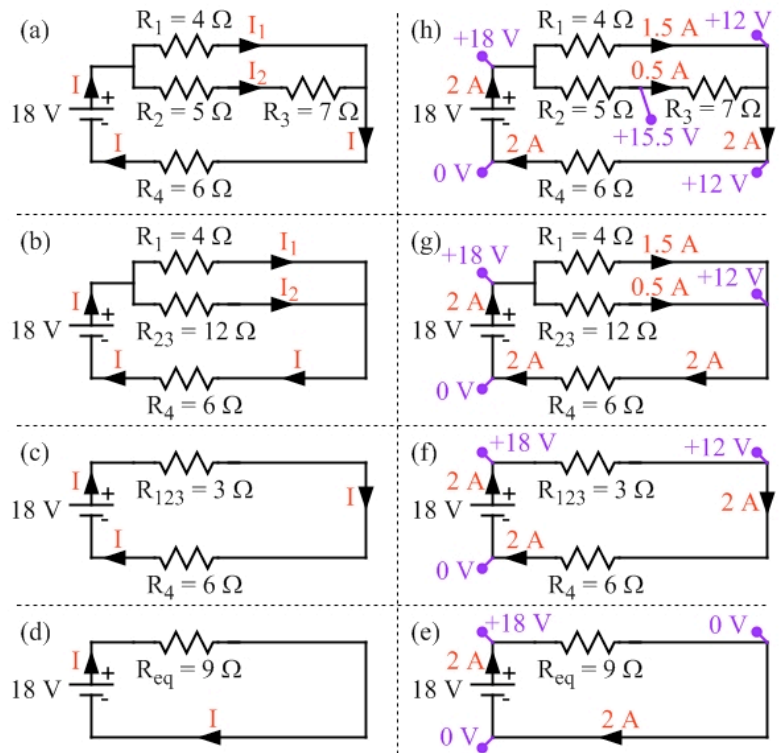
(a) Labeling the current at various points can help identify which resistors are in series and which are in parallel.

(b) R_2 and R_3 are in series, and can be replaced by one equivalent resistor, R_{23} .

(c) – (d) Resistors are replaced a pair at a time to find the circuit's equivalent resistance.

(e) Apply Ohm's Law to find the total current.

(f - h) Expansion reverses the steps. At each expansion step, we find the current and potential difference for each resistor.



Step 6 – Expand the circuit back from one resistor to two. Find the current through, and potential difference across, both resistors.

Expansion reverses the steps of the contraction. In Figure 18.15(f), we replace the $9.0\ \Omega$ resistor by the $3.0\ \Omega$ and $6.0\ \Omega$ resistors, in series, it came from. When a resistor is split into two in series, the current ($2.0\ \text{A}$ in this case) through all three resistors is the same. We can now use Ohm's law to find the potential difference. For the two resistors, we get $\Delta V_{3\Omega} = IR = 6.0\ \text{V}$ and $\Delta V_{6\Omega} = IR = 12\ \text{V}$. Thus, the wire connecting the $3.0\ \Omega$ and $6.0\ \Omega$ resistors is at a potential of $V = +12\ \text{V}$. This is consistent with the loop rule, and the fact that the direction of the current through a resistor is the direction of decreasing potential.

Step 7 – Expand the circuit back from two resistors to three. The $3.0\ \Omega$ resistor is replaced by the $4.0\ \Omega$ and $12.0\ \Omega$ resistors, in parallel, which it came from, as shown in Figure 18.15(g). When one resistor is split into two in parallel, the potential difference across all three resistors is the same. That is $6.0\ \text{V}$ in this case. We can then apply Ohm's Law to find the current in each resistor, giving $I_1 = \Delta V / R = 6\ \text{V} / 4\ \Omega = 1.5\ \text{A}$ and $I_2 = \Delta V / R = 6\ \text{V} / 12\ \Omega = 0.5\ \text{A}$. These add to the $2.0\ \text{A}$ through their equivalent resistor, as we expect from the junction rule.

Step 8 – Continue the expansion process, at each step finding the current through, and potential difference across, each resistor. In this circuit there is one more step. This is shown in Figure 18.15(h), where the $12.0\ \Omega$ resistor is split into the original $5.0\ \Omega$ and $7.0\ \Omega$ resistors. These resistors have the same current, $0.5\ \text{A}$, as the $12.0\ \Omega$ resistor, and their potential differences can be found from Ohm's Law and sum to the $6.0\ \text{V}$ across the $12.0\ \Omega$ resistor.

Key ideas for the contraction/expansion method: One way to analyze a circuit is to contract the circuit to one equivalent resistor and then expand it back. In each step in the contraction two resistors that are either in series or in parallel are replaced by one resistor of equivalent resistance. In the expansion when one resistor is expanded to two in series all three resistors have the same current, while when one resistor is expanded to two in parallel all three resistors have the same potential difference across them. **Related End-of-Chapter Exercises: 27, 32, 33, 36.**

Essential Question 18.7: Check the answer above by comparing the power associated with the battery to the total power dissipated in the resistors. Why should these values be the same?