

Answer to Essential Question 18.5: Each bulb has a potential difference less than the 120 V the bulb is designed for, so the bulbs are dimmer than usual. This reduces the filament temperature, lowering its resistance, which decreases the equivalent resistance of the circuit. This increases the current in the circuit, increasing the power dissipated in each bulb because $P = I^2R$. The decrease in R is offset by the increase in I , and the extra factor of I gives a net increase.

18-6 Resistors in Parallel

When charge has more than one path to choose from between two points in a circuit, we say that those paths are in parallel with one another. Interestingly, adding a resistor to a circuit can actually decrease the resistance of that circuit if the resistor is placed in parallel with another resistor in the circuit. Such is the case in Figure 18.12(b), where the circuit has a lower net resistance than the circuit in Figure 18.12(a).

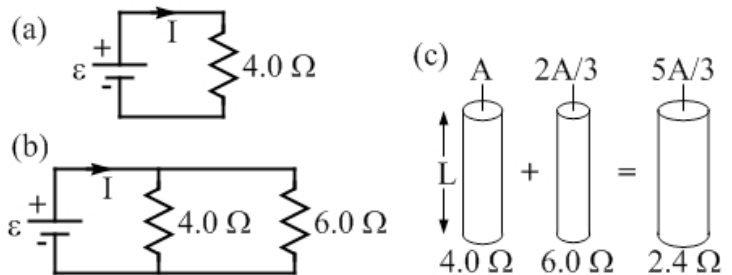


Figure 18.12: In (b), a $6.0\ \Omega$ resistor is placed in parallel with the $4.0\ \Omega$ resistor. (c) The charge now has a larger effective area to flow through, so the resistance of the circuit has been reduced.

If the resistors in Figure 18.12(b) have the same length and resistivity, then they must have different cross-sectional areas. The two resistors can be replaced by one equivalent resistor with the same length and resistivity, and with a cross-sectional area equal to the sum of the cross-sectional areas of the original resistors, as in Figure 18.12(c). Resistance is inversely proportional to area, so adding the second resistor in parallel actually decreases the resistance of the circuit.

If we have N resistors connected in parallel, their equivalent resistance is given by:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}. \quad (\text{Eq. 18.8: Equivalent resistance of resistors in parallel})$$

EXPLORATION 18.6 – Current in a parallel circuit

The emf of the battery in Figure 18.12(b) is 12 volts.

Step 1 – How does the current through the $4.0\ \Omega$ resistor in Figure 18.12(b) compare to that through the $6.0\ \Omega$ resistor? Explain why. When the charge has two paths to choose from, as it does in this circuit, more of the charge passes through the lower resistance path. Thus, the current through the $4.0\ \Omega$ resistor is larger than that through the $6.0\ \Omega$ resistor.

Step 2 – Find the equivalent resistance of the circuit in Figure 18.12(b) and use it to find the current through the battery in that circuit.

Applying equation 18.8 gives:
$$\frac{1}{R_{eq}} = \frac{1}{4.0\ \Omega} + \frac{1}{6.0\ \Omega} = \frac{3}{12\ \Omega} + \frac{2}{12\ \Omega} = \frac{5}{12\ \Omega}.$$

Inverting this to find the equivalent resistance gives $R_{eq} = (12/5)\ \Omega = 2.4\ \Omega$. In other words, the battery acts as if it is connected to a single $2.4\ \Omega$ resistor, and thus the current in the circuit, and through the battery, is $I = \varepsilon / R_{eq} = (12\ \text{V}) / 2.4\ \Omega = 5.0\ \text{A}$.

Step 3 – What is the current through each of the two resistors in Figure 18.12(b)? What is the sum of the currents? Why do we get this result? Each resistor in the circuit is directly connected to the battery, and thus each resistor has 12 V across it. Applying Ohm’s Law to each resistor:

$$I_{4\Omega} = \frac{\Delta V}{R} = \frac{12 \text{ V}}{4.0 \Omega} = 3.0 \text{ A} \quad \text{and} \quad I_{6\Omega} = \frac{\Delta V}{R} = \frac{12 \text{ V}}{6.0 \Omega} = 2.0 \text{ A} .$$

These sum to 5.0 A, the current through the battery. This is consistent with Kirchoff’s junction rule, which says the current entering a junction equals the current leaving the junction.

Key ideas for parallel circuits: Components in parallel with one another have the same potential difference across them. Current splits between parallel paths, with more current passing through the path with lower resistance. **Related End-of-Chapter Exercises: 21 and 24.**

In a calculation like that in Step 2, a common error is to forget to invert when applying Equation 18.8, stating the answer incorrectly as $R_{eq} = (5/12)\Omega = 0.42 \Omega$. Checking units can prevent this error. Also, a rule of thumb is that the equivalent resistance of two resistors in parallel is between half the smaller resistance and the smaller resistance. In Step 2 above, in which the smaller resistance is 4.0Ω , the equivalent resistance must be between 2.0Ω and 4.0Ω . The smaller the value of the larger resistance the smaller the equivalent resistance.

EXAMPLE 18.6 – Splitting the current

The current entering a particular section of a circuit is I . As shown in Figure 18.13, the current divides between two parallel paths, a current I_1 that passes through a resistor of resistance R_1 , and a current I_2 that passes through a resistor of resistance R_2 . What fraction of the current passes through each resistor? In other words, express I_1 and I_2 in terms of I , R_1 , and R_2 .

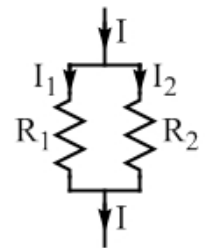


Figure 18.13: A section of a circuit in which two resistors are in parallel.

SOLUTION

We have two relationships we can use to find the answer. One is the junction rule, which tells us that $I = I_1 + I_2$, which we can rearrange to get $I - I_1 = I_2$. The second relationship is the idea that the potential difference across the two resistors is the same, because they are in parallel.

$$\Delta V = I_1 R_1 = I_2 R_2 . \text{ Using } I - I_1 = I_2 \text{ gives } I_1 R_1 = (I - I_1) R_2 = I R_2 - I_1 R_2 .$$

Solving for I_1 , and using that expression to solve for I_2 , gives:

$$I_1 = \frac{R_2}{R_1 + R_2} I \quad \text{and} \quad I_2 = \frac{R_1}{R_1 + R_2} I .$$

This is consistent with the idea that current prefers the path of least resistance, and tells us exactly how current splits between two branches that are in parallel with one another.

Essential Question 18.6: Two resistors, with resistances of 10Ω and 30Ω , are in parallel with one another (and only one another) in a circuit. What fraction of the current entering this part of the circuit passes through each resistor?