

Answer to Essential Question 18.4: The bulb flickers at 120 Hz. It is dim at the start of the cycle in Figure 18.9, bright when the voltage goes positive, dim when the voltage passes through zero, and bright again when the voltage goes negative. Thus, for each cycle of the voltage, the bulb goes through two cycles, so its frequency is twice that of the voltage. Note that the bulb does not go completely off when the voltage passes through zero because the filament takes time to cool.

18-5 Resistors in Series

In many circuits, resistors are placed in series, as in Figure 18.10(b), so that the charge flows through the resistors in sequence. Let's start with a battery connected to a single $10\ \Omega$ resistor, which is made from a wire of a particular resistivity and cross-sectional area. If we break the resistor into two pieces, one 40% as long as the original and one 60% as long, the pieces have resistances of $4\ \Omega$ and $6\ \Omega$, respectively, because resistance is directly proportional to length.

If the $4\ \Omega$ and $6\ \Omega$ resistors are connected by a wire of negligible resistance, the battery sees no difference between the single $10\ \Omega$ resistor and the $4\ \Omega$ and $6\ \Omega$ resistors that are connected in series – the battery is still trying to force charge to flow through a total resistance of $10\ \Omega$. We can generalize by splitting the original resistor into more than two pieces, but we always end up with the same total resistance.

Thus, we can say that when we have N resistors connected in series, the equivalent resistance of the set of resistors is:

$$R_{eq} = R_1 + R_2 + \dots + R_N \quad (\text{Eq. 18.7: Equivalent resistance of resistors in series})$$

EXPLORATION 18.5A – Finding the current through one resistor

The emf of the battery in Figure 18.10(b) is 20 volts.

Step 1 – How does the current through the $4\ \Omega$ resistor in Figure 18.10(b) compare to that through the $6\ \Omega$ resistor? Explain why. The current is the same at all points in a series circuit. All the charge that flows through the $4\ \Omega$ resistor keeps going to flow through the $6\ \Omega$ resistor. The rate of flow is also the same, because charge cannot pile up anywhere in the circuit. Because current is the rate of flow of charge this means the current is the same through both resistors.

Step 2 – What is the current through the $4\ \Omega$ resistor in Figure 18.10(b)? It is tempting to apply Ohm's Law directly to the $4\ \Omega$ resistor, with a potential difference of 20 volts. Resist this temptation, because the potential difference across that resistor is something less than 20 volts! The most straightforward way to proceed is to first find the equivalent resistance of the circuit, which is $4\ \Omega + 6\ \Omega = 10\ \Omega$ in this case, and apply Ohm's Law to find the current in the circuit:

$$I_{\text{circuit}} = \frac{\Delta V}{R_{eq}} = \frac{20\ \text{volts}}{10\ \Omega} = 2.0\ \text{A}.$$

The current is the same at all points in a series circuit, so this is the current through each resistor.

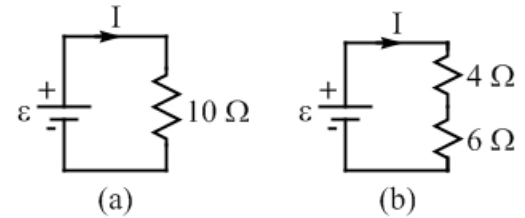


Figure 18.10: A single $10\ \Omega$ resistor can be thought of as two resistors, whose resistances add to $10\ \Omega$, placed in series with one another.

Step 3 – What is the potential difference across each of the two resistors in Figure 18.10(b)? What is the sum of the potential differences? Why do we get this result? Now we can apply Ohm’s Law to the individual resistors, to find that:

$$\Delta V_{4\Omega} = IR = (2.0 \text{ A})(4.0 \Omega) = 8.0 \text{ V} \quad \text{and} \quad \Delta V_{6\Omega} = IR = (2.0 \text{ A})(6.0 \Omega) = 12 \text{ V}.$$

These add together to equal 20 volts, the battery voltage. This is to be expected from Kirchoff’s loop rule. In a series circuit, the loop rule tells us that the sum of the potential differences across the resistors equals the battery voltage.

Key ideas for series circuits: Components in a series circuit have equal currents passing through them. The sum of the potential differences across the resistors in a series circuit is equal to the battery voltage. **Related End-of-Chapter Exercises: 25 and 46.**

EXAMPLE 18.5B – Which bulb is brighter?

A standard 40 W light bulb is connected in series with a standard 100 W light bulb and an electrical outlet, which we can treat as a 120 V battery, as shown in Figure 18.11. (a) Provide a qualitative analysis to show which bulb is brighter. (b) Analyze the circuit quantitatively to estimate the power dissipated in each bulb. Hint: we calculated the resistance of these bulbs in Section 18.4.

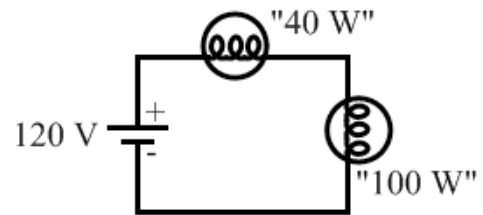


Figure 18.11: A circuit with a 100 W light bulb and a 40 W light bulb connected in series.

SOLUTION

(a) It’s easy to think of justifications for all three possible answers. An argument in favor of the 100 W bulb being brighter is that such a bulb is brighter than a 40 W bulb when used at home. We could also argue that the bulbs are equally bright because they are in series, and therefore have the same current. The correct answer, however, is that the bulb marked as 40 W is brighter in this case. The bulbs do have equal currents, but the brightness is determined by the power. In this case let’s use the equation $P = I^2R$. Because the current is the same, the bulb with higher resistance has more power dissipated in it and is brighter. We showed in Section 18.4 that the 40 W bulb has a larger resistance. Note that bulbs are designed to be placed in parallel, so it should not be too surprising that they behave in unexpected ways when they are in series.

(b) Let’s use the resistances we calculated in Section 18.4, 144 Ω for the 100 W bulb and 360 Ω for the 40 W bulb. The equivalent resistance of the two bulbs in series is the sum of these values, about 500 Ω . Knowing the equivalent resistance enables us to find the current in the circuit, from Ohm’s Law: $I = \Delta V / R_{eq} = (120 \text{ V}) / (500 \Omega) = 0.24 \text{ A}$. The power dissipated in each bulb is:

$$P_{40W} = I^2R = (0.24 \text{ A})^2 (360\Omega) = 21 \text{ W} \quad \text{and} \quad P_{100W} = I^2R = (0.24 \text{ A})^2 (144\Omega) = 8.3 \text{ W}.$$

The bulb marked “40 W” dissipates more power, and is thus brighter.

Essential Question 18.5: In the situation above, the bulbs will not actually have the resistances we used in the calculation. Why not? Will the bulbs actually dissipate more power or less power?