Answer to Essential Question 18.10: Let's first determine the time constant, using Equation 18.11, and replacing the factor RC by τ. This gives:

$$(3.0 \text{ V}) = (12 \text{ V})\left(1 - e^{-(2.5 \text{ ms})/\tau}\right).$$

Divide both sides by 12 V: $0.25 = 1 - e^{-(2.5 \text{ ms})/\tau}$.

Re-arrange, then take the natural log of both sides: $\ln(0.75) = -\dfrac{2.5 \text{ ms}}{\tau}$.

Solving for the time constant gives: $\tau = -\dfrac{2.5 \text{ ms}}{\ln(0.75)} = 8.7 \text{ ms}$.

Multiplying ohms by farads gives seconds, so $C = \dfrac{\tau}{R} = \dfrac{8.69 \text{ ms}}{47\ \Omega} = 180\ \mu C$.

Chapter Summary

Essential Idea: Direct Current Circuits.

Electric circuits are essential to our daily lives, being part of electronic devices like cell phones and iPods. In this chapter we explored the principles of how basic circuits work.

Electric Current

Current is the rate at which charge flows. The symbol we use for current is I:

$$I = \frac{\Delta Q}{\Delta t}$$ (Equation 18.1: **Current, the rate of flow of charge**)

The unit for current is the ampere (A). 1 A = 1 C/s.

The direction of current is the direction positive charges flow.

Ohm's Law and Resistance

$$\Delta V = IR$$ (Equation 18.2: **Ohm's Law**)

The electrical resistance of a wire of length L and cross-sectional area A is:

$$R = \frac{\rho L}{A}.$$ (Equation 18.3: **Electrical resistance**)

The unit of resistance is the ohm (Ω). 1 Ω = 1 V/A.

The resistivity ρ depends on the material the resistor is made from.

Resistors in Series and Parallel

If N resistors are connected in series, their equivalent resistance is given by:

$$R_{eq} = R_1 + R_2 + \ldots + R_N .$$ (Eq. 18.7: **Equivalent resistance of resistors in series**)

If N resistors connected in parallel, their equivalent resistance is given by:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N} .$$ (Eq. 18.8: **Equivalent resistance of resistors in parallel**)

Electric Power, and the Cost of Electricity

$$P = I\,\Delta V = I^2 R = \frac{(\Delta V)^2}{R} \,.$$ (Equation 18.5: **Electrical power**)

The cost for operating a particular electrical device can be determined from:

$(\text{Number of hours it is on}) \times (\text{power rating in kW}) \times (\text{cost per kW-hr})$ (Eq. 18.6: **Electricity cost**)

The contraction/expansion method for analyzing series-parallel combination circuits

When a circuit has one battery with an emf ε , and resistors that have series and parallel connections, the current through, and potential difference across, each resistor can be found by:

1. Identifying two resistors that are either in series or in parallel, and replacing them by their equivalent resistance. Repeat this contraction until one resistor is left.
2. Determining the total current in the circuit by applying Ohm's Law: $I = \varepsilon / R_{eq}$.
3. Gradually reversing the steps in the contraction. When one resistor is expanded to two in series all three resistors have the same current, while when one resistor is expanded to two in parallel all three resistors have the same potential difference across them.

Applying Kirchoff's Rules to analyze Multi-loop Circuits

Kirchoff's Rules can also be applied to analyze a circuit. The loop rule is a statement of conservation of energy, while the junction rule is a statement of conservation of charge.

Loop rule: the sum of all the potential differences around a closed loop is zero.

Junction rule: the total current entering a junction equals the total current leaving the junction.

To analyze a multi-loop circuit (a circuit with multiple batteries connected in a way that they can not be replaced by a single equivalent battery) we use the following steps:

1. Label the currents in the various branches of the circuit.
2. Choose a loop and apply the loop rule to obtain an equation involving one or more of the unknowns (often, these are the currents). Repeat until each branch has been used at least once. Successive equations must involve a branch not involved in previous equations.
3. Apply the junction rule to obtain one or more equations relating the currents.
4. Solve the system of equations for the unknowns.

RC Circuits

In a series RC circuit, the current, and the potential differences across the resistor and the capacitor, change with time. When **the capacitor is being charged** by the battery:

$I = \frac{\varepsilon}{R} e^{-t/RC}$ (Eq. 18.10**: Current)** $\Delta V_C = \varepsilon \left(1 - e^{-t/RC}\right)$ (Eq. 18.11**: Capacitor voltage)**

$\Delta V_R = \varepsilon \; e^{-t/RC}$ (Eq. 18.12**: Resistor voltage)**

When the battery is removed and **the capacitor is discharging**:

$I = -\frac{\Delta V_{C,\max}}{R} e^{-t/RC}$ (Eq. 18.13) $\Delta V_C = \Delta V_{C,\max} \; e^{-t/RC}$ (Eq. 18.14**: Capacitor voltage)**

$\Delta V_R = -\Delta V_{C,\max} \; e^{-t/RC}$ (Eq. 18.15**: Resistor voltage)**