Answer to Essential Question 17.7: The net field inside a conductor is zero. Using $E_{net} = 0$ in equation 17.9 gives κ = infinity. Thus, conductors have infinite dielectric constants.

17-7 Energy in a Capacitor, and Capacitor Examples

When a capacitor stores charge, creating an electric field between the plates, it also stores energy. The energy is stored in the electric field itself. The energy density (the energy per unit volume in the field) is proportional to the square of the magnitude of the field:

$$\text{Energy density} = \frac{1}{2}\kappa\varepsilon_0 E^2 . \qquad \text{(Eq. 17.10: **Energy per unit volume in an electric field**)}$$

U, the total potential energy stored in the capacitor is thus the energy density multiplied by the volume of the region between the capacitor plates, $A \times d$. The energy can be written as:

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}Q\Delta V = \frac{Q^2}{2C} . \qquad \text{(Equation 17.11: **Energy stored in a capacitor**)}$$

Let's now explore the two main ways we set up capacitor problems.

EXPLORATION 17.7A – The capacitor stays connected to the battery

A parallel-plate capacitor is connected to a battery that has a voltage of V_0. The capacitor has an initial capacitance of C_0, with air filling the space between the plates. The capacitor stores a charge Q_0, has an electric field of magnitude of E_0, and stores an energy U_0. With the battery connected to the capacitor, the spacing between the plates is doubled, and then a dielectric with a dielectric constant 5 times that of air is inserted, completely filling the space between the plates.

Step 1 – *Do any of the parameters given above stay constant during this process?* Yes, the potential difference is constant. Because the capacitor is connected to the battery at all times, the potential difference across the capacitor matches the battery voltage.

Step 2 – *Complete Table 17.2 to show the values of the various parameters after each step.*

Situation	Potential difference	Capacitance	Charge	Electric field	Energy
Initially	V_0	C_0	Q_0	E_0	U_0
Spacing is doubled					
Dielectric inserted					

Table 17.2: Fill in the table to show what happens to the various parameters in this situation.

The completed table is Table 17.3. The potential difference is constant, so we start with that column. We then use Equation 17.7 to find the capacitance. Doubling the distance between the plates reduces the capacitance by a factor of 2; increasing the dielectric constant by a factor of 5 then increases *C* by a factor of 5, for an overall increase by a factor of 5/2 = 2.5.

Knowing ΔV and *C*, we can use Equation 17.6, $Q = C\Delta V$, to find the charge on the capacitor. Because the potential difference is constant the charge changes in proportion to the capacitance. Similarly, Equation 17.8, $E = |\Delta V| / d$, tells us what happens to the magnitude of the electric field, and the form of equation 17.11 that says the energy is proportional to $C(\Delta V)^2$ tells us what happens to the energy. It is interesting that inserting the dielectric has no effect on the electric field – this is because the battery increases the charge by a factor of 5 to keep the potential difference, and therefore the field, the same when the dielectric is inserted.

Situation	Potential difference	Capacitance	Charge	Electric field	Energy
Initially	V_0	C_0	Q_0	E_0	U_0
Spacing is doubled	V_0	$0.5\,C_0$	$0.5\,Q_0$	$0.5\,E_0$	$0.5\,U_0$
Dielectric inserted	V_0	$2.5\,C_0$	$2.5\,Q_0$	$0.5\,E_0$	$2.5\,U_0$

Table 17.3: Keeping track of the various parameters as changes are made to the capacitor.

Key idea: When a capacitor remains connected to a battery the capacitor voltage is constant – it equals the battery voltage. If changes are made we first determine how the capacitance changes, and then use the various equations to determine what happens to other parameters.

EXPLORATION 17.7B – The battery is disconnected from the capacitor before the changes

Let's return our parallel-plate capacitor to the same initial state as in the previous Exploration. The wires connecting the capacitor to the battery are then removed. After this the spacing between the plates is doubled, and then a dielectric with a dielectric constant 5 times that of air is inserted into the capacitor, completely filling the space between the plates.

Step 1 – *What stays constant during this process?* The charge is constant. With the capacitor disconnected from the battery, the charge is stranded on the plates.

Step 2 – *Complete Table 17.2 to show the values of the various parameters after each step.* The completed table is Table 17.4. The charge is constant, so we start with that column. Applying Equation 17.7, we can then determine what happens to the capacitance. Doubling the distance between the plates reduces the capacitance by a factor of 2; increasing the dielectric constant by a factor of 5 then increases *C* by a factor of 5, for an overall increase by a factor of 5/2 = 2.5.

Knowing *Q* and *C*, we can use Equation 17.6, $Q = C\Delta V$, to find the potential difference across the capacitor. Note that ΔV is inversely proportional to the capacitance. Equation 17.8, $E = |\Delta V| / d$, tells us what happens to the electric field, and the form of equation 17.11 that says the energy is proportional to $Q\,\Delta V$ tells us what happens to the energy. In this case, inserting the dielectric decreases the field, as we would expect from our previous discussion of dielectrics.

Situation	Potential difference	Capacitance	Charge	Electric field	Energy
Initially	V_0	C_0	Q_0	E_0	U_0
Spacing is doubled	$2\,V_0$	$0.5\,C_0$	Q_0	E_0	$2\,U_0$
Dielectric inserted	$0.4\,V_0$	$2.5\,C_0$	Q_0	$0.2\,E_0$	$0.4\,U_0$

Table 17.4: Keeping track of the various parameters as changes are made to the capacitor.

Key idea: When a capacitor is not connected to anything the charge on the capacitor remains constant. If changes are made we first determine how the capacitance changes, and then use the various equations to determine what happens to other parameters.

Related End-of-Chapter Exercises: 25 – 30.

Essential Question 17.7: In Exploration 17.7B, the energy stored by the capacitor doubles when the spacing between the plates doubles. Where does this extra energy come from?