Answer to Essential Question 17.4: A fundamental difference between field and potential is that field is a vector while potential is a scalar. To find the net field we need to account for direction, adding the various fields as vectors. Finding potential simply involves adding numbers.

17-5 Working with Force, Field, Potential Energy, and Potential

Let's consider two important special cases. The first involves a charged object in a uniform electric field, while the second involves the electric field from point charges.

EXAMPLE 17.5A – Charged particles at the corners of a square

Consider the four situations shown in Figure 17.10, involving four charged particles of charge $+q$ or $-q$ placed so that there is one charge at each corner of the square.

(a) In which case does the charge at the top right corner of the square experience the net force of the largest magnitude?
(b) Is the net electric field at the center of the square zero in any of the configurations? If so, which?
(c) Is the net electric potential at the center of the square zero in any of the configurations? If so, which?
(d) Is the electric potential energy of the system equal to zero in any of the configurations? If so, in which configuration(s)?

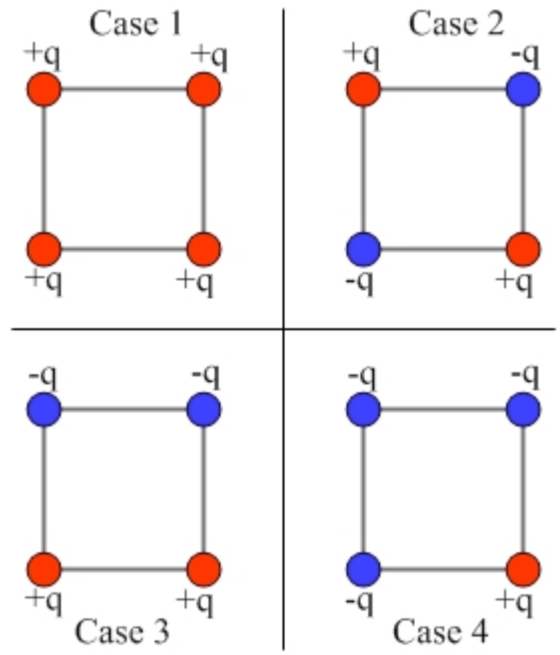

Figure 17.10: Four different arrangements of equal-magnitude point charges placed so that there is one charged particle at each corner of a square.

SOLUTION

(a) The charged particle at the top right corner experiences the largest magnitude net force in case 1, where all the forces have a component directed away from the center. In the other cases the particle at the top right experiences a mix of attractive and repulsive forces, which in this case causes some partial cancellation of the forces.

(b) The net electric field is zero at the center of the square in cases 1 and 2. If we pair up the charges that are diagonally across from one another, in cases 1 and 2 the fields within each pair cancel, giving no net field at the center.

(c) Each positive charge makes a positive contribution to the potential, while each negative charge makes a negative contribution. To produce a zero net potential at the center in this situation we need two positive charges and two negative charges, which is true in case 2 and case 3. The potential at the center of the square is positive in case 1 and negative in case 4.

(d) To find the total potential energy, we can work out the energy associated with each interacting pair of charges (with four charges there are six such interactions) by applying Equation 17.1, and add up these six terms to find the total potential energy. When we do this in case 4, we find that the total potential energy is zero. The total potential energy is non-zero in all other cases. If the square measures $L \times L$, the addition of the six terms gives:

$$U_{net,case4} = \frac{k(-q)(+q)}{L} + \frac{k(-q)(-q)}{L} + \frac{k(+q)(-q)}{L} + \frac{k(-q)(-q)}{L} + \frac{k(-q)(-q)}{\sqrt{2}\,L} + \frac{k(-q)(+q)}{\sqrt{2}\,L} = 0$$

Related End-of-Chapter Exercises for Example 17.5A: 19 - 23.

EXAMPLE 17.5B – Asking questions

Two charged particles, one with a charge of $+q$ and the other with a charge of $-3q$, are placed on the x-axis at $x = 0$ and $x = +4a$, respectively. This is shown in Figure 17.11. Give an example of a question pertaining to this situation that involves:

(a) Force; (b) Electric field; (c) Electric field, with a follow-up question on force; (d) Electric potential energy; (e) Electric potential; (f) Electric potential, with a follow-up question on potential energy.

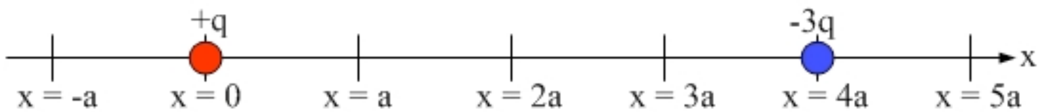

Figure 17.11: Two charged particles are separated by a distance of $4a$ on the x-axis.

SOLUTION

(a) Because force involves an interaction between objects, and we only have two objects in the system, there are a limited number of questions we could ask involving force unless we add another charge to the system. One is: "What is the magnitude and direction of the force exerted on the $+q$ charge by the $-3q$ charge?"

(b) "What is the magnitude and direction of the net electric field at $x = +5a$?" Because the two charged particles create an electric field at all points around them, we can ask an infinite number of questions involving field – we have an infinite number of points to choose from.

(c) "Consider the point $x = +2a$, $y = +2a$. (i) What is the magnitude and direction of the net electric field at that point? (ii) If a third charged particle with a charge of $+2q$ is placed at that point, what is the magnitude and direction of the force it experiences because of the two charges?"

(d) The pattern for (a) – (c) can be repeated for potential energy and potential. "What is the electric potential energy associated with this system of two charges?"

(e) "What is the net electric potential at x = +5a due to the two charges?" As with field, we can ask about the potential at any point.

(f) "Consider the point $x = +2a$, $y = +2a$. (i) What is the net electric potential at that point? (ii) If a third charged particle with a charge of $+2q$ is placed at that point, what is the change in potential energy for the system?"

Related End-of-Chapter Exercises: 16 - 18.

Essential Question 17.5: Return to the situation described in Example 17.5A. In which configurations, could you bring a fifth charged particle from infinitely far away and place it at the center of the square without doing any net work? In which configurations would you do negative net work in bringing a fifth charged particle from infinitely far away to the center of the square?