Answer to Essential Question 17.1: The potential energy of the system would increase and, in fact, would become $+10kq^2/r$. This is because the balls would all have charge of the same sign, and, overall, the particles of the system would repel one another.

17-2 Example Problems Involving Potential Energy

EXAMPLE 17.2A – Stopping an electron

An electron with a speed of 1.2×10^5 m/s enters a region where there is a uniform electric field with E = 500 N/C, as shown in Figure 17.2.

(a) Show that gravitational influences can be neglected in this situation.

(b) If, after traveling for some distance d in the field, the electron comes to rest for an instant, in which direction is the field?
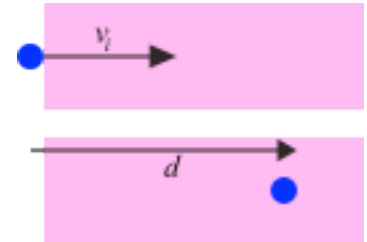
(c) Calculate d.

Figure 17.2: The top diagram shows the electron entering the electric field. The bottom diagram shows the point where the electron comes instantaneously to rest.

SOLUTION

(a) Let's assume the electron is at the surface of the Earth, where the acceleration due to gravity is the familiar g = 9.8 m/s². What is the acceleration associated with the electric field? The magnitude of the force applied to the electron by the field is $F = qE$. Dividing by the mass gives the magnitude of the acceleration of the electron:

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(500 \text{ N/C})}{9.11 \times 10^{-31} \text{ kg}} = 8.78 \times 10^{13} \text{ N/kg} .$$

This is 13 orders of magnitude larger than g, which justifies neglecting gravity.

(b) To stop the electron, the force on the electron from the field must be directed opposite to the electron's initial velocity. Because the electron has a negative charge, the field and force are in opposite directions. Thus, the electric field must be in the same direction as the electron's initial velocity.

(c) Let's apply energy conservation here, using the usual energy equation: $U_i + K_i + W_{nc} = U_f + K_f$.

In this case, there are no non-conservative forces acting, and the final state is the point at which the electron comes instantaneously to rest. Thus, the final kinetic energy is zero. In addition, we can define the initial electric potential energy to be zero. This gives $K_i = U_f$.

The final potential energy is given by: $U_f = U_i + \Delta U = 0 + (-(-e)Ed) = +eEd$.

Putting everything together gives: $\frac{1}{2}mv_i^2 = +eEd$.

Solving for the distance gives: $d = \frac{mv_i^2}{2eE} = \frac{(9.11 \times 10^{-31} \text{ kg})(1.2 \times 10^5 \text{ m/s})^2}{2(1.60 \times 10^{-19} \text{ C})(500 \text{ N/C})} = 8.2 \times 10^{-5}$ m.

Compare the following Example to Example 8.4.

EXAMPLE 17.2B – Applying conservation ideas

Two identical balls, of mass 1.0 kg and charge +2.0 µC, are initially separated by 2.0 m in a region of space in which they interact only with one another. When they are released from rest, they accelerate away from one another. When they are 4.0 m apart, how fast are they going?

SOLUTION

Figure 17.3 shows the balls at the beginning and when they are separated by 4.0 m. Analyzing forces, we find that the force on each ball decreases as the distance between the balls increases. This makes it difficult to apply a force analysis. Energy conservation is a simpler approach. Our energy equation is:

$U_i + K_i + W_{nc} = U_f + K_f$.

Figure 17.3: The balls are initially at rest. Repulsion from their like charges causes them to accelerate away from one another.

In this case, no non-conservative forces act and, in the initial state, the kinetic energy is zero because both objects are at rest. This gives $U_i = U_f + K_f$. The final kinetic energy represents the kinetic energy of the system, the sum of the kinetic energies of the two objects.

Do we need to account for gravity here? In this situation we do not, which we can tell by calculating that the gravitational potential energy is much less than the electric potential energy. The change in potential energy is really what is important, but that is of the same order of magnitude as the potential energy.

$$U_{iG} = -\frac{Gmm}{r} = -\frac{\left(6.67\times10^{-11}\ \text{N m}^2/\text{kg}^2\right)(1.0\ \text{kg})^2}{r} \qquad U_{iE} = \frac{kqq}{r} = -\frac{\left(8.99\times10^9\ \text{N m}^2/\text{C}^2\right)(2.0\times10^{-6}\ \text{C})^2}{r}$$

Let's solve this generally, using a mass of m and a final speed of v for each ball.

The energy equation becomes: $\dfrac{kqq}{2.0\ \text{m}} = \dfrac{kqq}{4.0\ \text{m}} + \dfrac{1}{2}mv^2 + \dfrac{1}{2}mv^2$.

Simplifying: $\dfrac{kq^2}{4.0\ \text{m}} = mv^2$.

Solving for v gives:

$$v = \sqrt{\frac{kq^2}{m(4.0\ \text{m})}} = \sqrt{\frac{\left(8.99\times10^9\ \text{N m}^2/\text{C}^2\right)\left(2.0\times10^{-6}\ \text{C}^2\right)}{(1.0\ \text{kg})(4.0\ \text{m})}} = 9.5\times10^{-2}\ \text{m/s} .$$

Related End-of-Chapter Exercises: Problems 1, 2, 3, 32, 33.

Essential Question 17.2: How would we solve Example 17.2B if the balls had different masses? To be specific, let's say the ball on the left has a mass of 1.0 kg while the ball on the right has a mass of 3.0 kg. What ar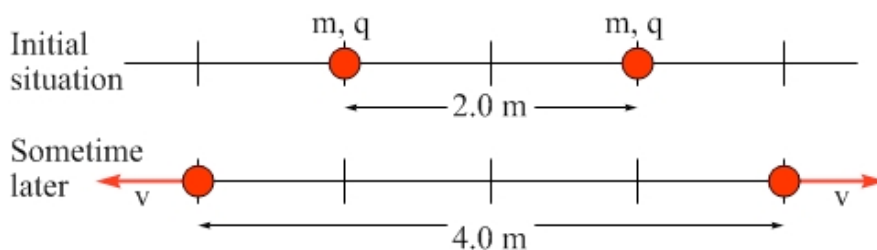e the speeds of the two balls in that case?