17-1 Electric Potential Energy

Whenever charged objects interact with one another, there is an energy associated with that interaction. In general, we have two special cases to consider. The first is the energy associated with a charged object in a uniform electric field, and the second is the energy associated with the interaction between point charges. These are analogous to the two situations we examined earlier for gravity.

> The potential energy associated with the interaction between one object with a charge *q* and a second object with a charge *Q* that is a distance *r* away is given by:
>
> $$U_E = \frac{kqQ}{r}$$ (Eq. 17.1: **Potential energy for the interaction between two charges**)
>
> where $k = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$ is a constant. If the charges are of opposite signs, then the potential energy is negative – this indicates an attraction. If the charges have the same sign, the potential energy is positive, indicating a repulsion.

Once again, we can see the parallel with gravity, for which the equivalent expression is:

$$U_G = -\frac{GmM}{r}$$ (Eq. 8.4: **Potential energy for the interaction between two masses**)

where $G = 6.67 \times 10^{-11} \text{N m}^2/\text{kg}^2$ is the universal gravitational constant. For the interaction between charges *k* takes the place of *G*, and the values of the charges *q* and *Q* take the place of the values of the masses, *m* and *M*.

Case 1 – A charged object in a uniform electric field

This situation is directly analogous to the situation of an object with mass in a uniform gravitational field. When we raise a ball of mass *m* through a height *h* in a uniform gravitational field *g* directed down, the change in potential energy is $\Delta U_G = mgh$. Note that we take *h* to be the component of the displacement parallel to, and opposite in direction to, the field. If the ball experiences a displacement $\Delta\vec{r}$ then an equivalent equation is $\Delta U_G = -m\vec{g} \bullet \Delta\vec{r} = -mg\Delta r\cos\theta$, where θ is the angle between the gravitational field and the displacement.

The equivalent expression for a charge object in a uniform electric field is:

$$\Delta U_E = -q\vec{E} \bullet \Delta\vec{r} = -qE\Delta r\cos\theta ,$$ (Eq. 17.2: **Change in potential energy in a uniform field**)

where θ is the angle between the electric field $\vec{E}$ and the displacement $\Delta\vec{r}$.

We are free to define the zero level of potential energy, but only in a uniform field.

Related End-of-Chapter Exercise: 6.

Case 2 – The electric potential energy of a set of point charges

In this situation, we look at the interaction between pairs of charges. We use equation 17.1 to calculate the energy of each pair of interacting objects, and then simply add up all these numbers because potential energy is a scalar. Note that we are not free to define the zero level. The zero is defined by equation 17.1, in fact, because the potential energy goes to zero as the distance between the charges approaches infinity.

Compare Exploration 17.1 to Exploration 8.4

EXPLORATION 17.1 – Calculate the total potential energy in a system

Three charged balls, of charge $-q$, $+2q$, and $-3q$, are placed in a line, as shown in Figure 17.1. What is the total electric potential energy of this system?

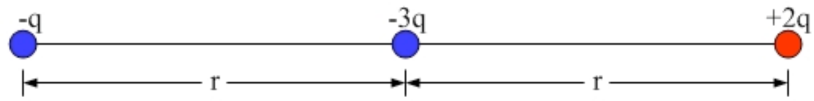

Figure 17.1: Three equally spaced balls placed in a line.

To determine the total potential energy of the system, consider the number of interacting pairs. In this case, there are three ways to pair up the objects, so there are three terms to add together to find the total potential energy. Because energy is a scalar, we do not have to worry about direction. Using a subscript of 1 for the ball of charge $-q$, 2 for the ball of charge $+2q$, and 3 for the ball of charge $-3q$, we get:

$$U_{Total} = U_{13} + U_{23} + U_{12} = \frac{k(-q)(-3q)}{r} + \frac{k(+2q)(-3q)}{r} + \frac{k(-q)(+2q)}{2r} = -\frac{4kq^2}{r}.$$

When a system has a negative total energy (including the total kinetic energy, of which there is none in this situation), that is indicative of a bound system. In general, there is a greater degree of attraction in the system than repulsion.

Key ideas for electric potential energy: Potential energy is a scalar. The total electric potential energy of a system of objects can be found by adding up the energy associated with each interacting pair of objects. **Related End-of-Chapter Exercises: 4, 42, 46.**

Work – an equivalent approach

Consider again the system shown in Figure 17.1. If the three charged balls start off infinitely far away from their final positions, and infinitely far from one another, how much work do we have to do to assemble the balls into the configuration shown in Figure 17.1? Assume that, aside from their interactions with us, the balls interact only with one another, electrostatically.

Pick one ball to bring into position first. Let's start with the ball with the $-3q$ charge. Because the other charged balls are still infinitely far away, it takes no work to bring the first ball into position. There are no other interactions to worry about.

Now, let's bring the ball with the $-q$ charge into position. The potential energy changes from 0, when the two balls are infinitely far away, to $+3kq^2/r$, when those two balls are in their final positions. This potential energy comes from work we do – we do $+3kq^2/r$ worth of work.

Finally, bring the ball with the $+2q$ charge into position. The potential energy associated with this ball changes from 0, when that ball is at infinity, to $-2kq^2/(2r) + (-6kq^2)/r = -7kq^2/r$, when the ball with the $+2q$ charge is in its final position.

Adding the two individual work values to find the total work to assemble the system gives $+3kq^2/r - 7kq^2/r = -4kq^2/r$, the same result we got for the potential energy of the system. The work done in assembling the system is equal to the system's potential energy.

Essential Question 17.1: Return to Exploration 17.1. If we replace the ball of charge $+2q$ by a ball of charge $-2q$, does the potential energy of the system increase, decrease, or stay the same?