

Answer to Essential Question 17.7: The capacitor plates, being oppositely charged, attract one another. Positive work is required to pull the plates farther apart. The energy associated with that process is the extra energy stored by the capacitor.

Chapter Summary

Essential Idea: Electric Potential Energy and Electric Potential.

In this chapter we continued looking at parallels between how charged particles interact and how objects with mass interact. As with gravitational situations, conservation of energy can be applied to many situations involving charged particles. We also went beyond what we had done with gravity, defining electric potential. The analogy still holds – we can define a gravitational potential for objects with mass that is much like electric potential for charged objects.

Electric Potential Energy

The potential energy associated with the interaction between one object with a charge q and a second object with a charge Q that is a distance r away is given by:

$$U_E = \frac{kqQ}{r} \quad (\text{Eq. 17.1: Potential energy for the interaction between two charges})$$

where $k = 8.99 \times 10^9 \text{ N m}^2 / \text{C}^2$ is a constant. If the charges are of opposite signs then the potential energy is negative – this indicates an attraction. If the charges have the same sign the potential energy is positive, indicating a repulsion.

Potential energy is a scalar. The total electric potential energy of a system of charged objects can be found by adding up the energy associated with each interacting pair of objects. For a charged particle in a uniform electric field we use the change in potential energy:

$$\Delta U_E = -q\vec{E} \cdot \Delta\vec{r} = -qE\Delta r \cos\theta, \quad (\text{Eq. 17.2: Change in potential energy in a uniform field})$$

where θ is the angle between the electric field \vec{E} and the displacement $\Delta\vec{r}$.

In a uniform field, we are free to define the zero level of potential energy. With equation 17.1, however, the potential energy is zero when the charges are separated by an infinite distance.

Electric Potential

Electric potential helps us understand how a charged object, or a set of charged objects, affects the region around it. The electric potential, V , at a particular point can be defined in terms of the electric potential energy, U , associated with an object of charge q placed at that point:

$$V = \frac{U}{q}, \quad \text{or} \quad U = qV. \quad (\text{Eq. 17.3: Connecting electric potential and potential energy})$$

The unit for electric potential is the volt (V). $1\text{V} = 1 \text{ J/C}$.

A special case is the electric potential from a point charge with a charge Q :

$$V = \frac{kQ}{r}, \quad (\text{Equation 17.4: Electric potential from a point charge})$$

where r is the distance from the charge to the point in space where we are finding the potential. Electric potential is a scalar, so it has a sign but not a direction.

Equipotentials connect points of the same potential. Equipotentials are perpendicular to field lines, because field points in the direction of decreasing potential. In a uniform field, or for a small displacement in a non-uniform field, we can connect the potential difference to the field:

$$\Delta V = -E \Delta r \cos \theta, \quad (\text{Equation 17.5: Potential difference in a uniform field})$$

where θ is the angle between the electric field \vec{E} and the displacement $\Delta \vec{r}$.

Working with Force, Field, Potential Energy, and Potential

Keep two points in mind when working with force, field, potential energy, and potential:

- Force and field are vectors, while potential energy and potential are scalars.
- Forces and potential energies arise from interactions between charges, requiring at least two charges. Field and potential exist with a single charged object.

Capacitors and Dielectrics

The charge Q stored in a capacitor is proportional to the capacitor's potential difference, ΔV :

$$Q = C \Delta V, \quad (\text{Eq. 17.6: Charge on a capacitor is proportional to its voltage})$$

where C is the capacitance of the capacitor, a measure of how much charge is stored for a particular voltage. The capacitance depends on the geometry of the capacitor, as well as on what, if anything, is between the plates of the capacitor. For a parallel-plate capacitor made of two plates of equal area A , separated by a distance d , the capacitance is given by:

$$C = \frac{\kappa \epsilon_0 A}{d}, \quad (\text{Eq. 17.7: Capacitance of a parallel-plate capacitor})$$

where κ is the dielectric constant of the material between the capacitor plates, and

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N m}^2) \text{ is the permittivity of free space.}$$

The MKS unit for capacitance is the farad (F). $1 \text{ F} = 1 \text{ C/V}$.

The dielectric constant of an insulator is the ratio of the external field to the net field inside:

$$\kappa = \frac{\vec{E}_0}{\vec{E}_{net}} \geq 1. \quad (\text{Eq. 17.9: The dielectric constant})$$

$$E = \frac{|\Delta V|}{d}. \quad (\text{Eq. 17.8: Magnitude of the electric field in a parallel-plate capacitor})$$

$$U = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} Q \Delta V = \frac{Q^2}{2C}. \quad (\text{Equation 17.11: Energy stored in a capacitor})$$

The energy is stored in the electric field. The energy density (the energy per unit volume) of an electric field is:

$$\text{Energy density} = \frac{1}{2} \kappa \epsilon_0 E^2. \quad (\text{Eq. 17.10: Energy per unit volume in an electric field})$$

When a capacitor is connected to a battery the capacitor voltage (i.e., potential difference) is equal to the battery voltage. When a capacitor is charged and then all connections to the capacitor are removed, however, the charge on the capacitor is constant.