Answer to Essential Question 16.5: The +*Q* charge exerts a force on the test charge that points directly away from the +*Q* charge. The unknown charge shifts the direction of the force on the test charge towards the unknown charge, so the unknown charge must be negative. If the unknown charge was –*Q*, by symmetry the test charge would experience a net force directed right, with no vertical component. Decreasing the magnitude of the unknown charge gives the net force an upward component; so the magnitude of the unknown charge is less than *Q*.

16-6 Electric Field: Special Cases

Let's consider two important special cases. The first involves a charged object in a uniform electric field, while the second involves the electric field from point charges.

EXPLORATION 16.6A – Motion of a charge in a uniform electric field

Step 1 – ***Sketch the free-body diagram for a proton placed in a uniform electric field of magnitude E = 250 N/C that is directed straight down. Then apply Newton's Second Law to find the acceleration of the proton. How important is gravity in this situation?*** The free-body diagram in Figure 16.12 shows the two forces applied to the proton, the downward force of gravity and the downward force applied by the electric field.
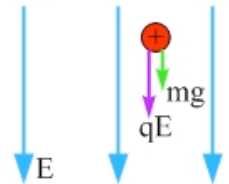

Figure 16.12: Free-body diagram for a proton in a uniform electric field directed down.

Taking down to be positive, applying $\sum \vec{F} = m\vec{a}$, gives: $+qE + mg = m\vec{a}$.

The values of the mass and charge of the proton are given in Table 16.1. Solving for the acceleration gives:

$$\vec{a} = +\frac{qE}{m} + g = +\frac{\left(1.60 \times 10^{-19} \text{ C}\right)\left(250 \text{ N/C}\right)}{\left(1.67 \times 10^{-27} \text{ kg}\right)} + 9.8 \text{ m/s}^2 = +2.40 \times 10^{10} \text{ m/s}^2.$$

Note that gravity is negligible in this case, because *g* is orders of magnitude less than *qE/m*.

Step 2 – ***How far has the proton traveled 25 µs after being released from rest in this field?*** The electric field is uniform, so the acceleration is constant. We can apply the constant acceleration equations from one-dimensional motion (Chapter 2) or projectile motion (Chapter 4).

From Eq. 2.11: $\vec{x} = \vec{x}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 = 0 + 0 + \frac{1}{2}\left(2.395 \times 10^{10} \text{ m/s}^2\right)\left(25 \times 10^{-6} \text{ s}\right)^2 = 0.75 \text{ m}.$

Key idea: The acceleration of a charged particle in a uniform electric field is constant, so the constant-acceleration equations from Chapters 2 and 4 apply. The scale of the accelerations and times are different but the physics is the same.   **Related End-of-Chapter Exercises: 27 and 28.**

EXAMPLE 16.6A – Where is the electric field equal to zero?
Two point charges, with charges of $q_1 = +2Q$ and $q_2 = -5Q$, are separated by a distance of 3.0 m, as shown in Figure 16.13. Determine all locations on the line passing through the two charges where their individual electric fields combine to give a net electric field of zero.
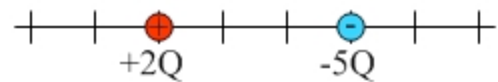

Figure 16.13: Two point charges separated by 3.0 m.

(a) For the net field to be zero, what condition(s) must the individual fields satisfy?

(b) Using qualitative arguments, can the electric field be zero at:
  I. a point on the line, to the left of the +2Q charge?
  II. a point on the line, in between the charges?
  III. a point on the line, to the right of the −5Q charge?
(c) Calculate the locations of all points near the charges where the net electric field is zero.

SOLUTION
  (a) The two individual fields add as vectors, so they must cancel one another for the net field to be zero. The two fields must be of equal magnitude and point in opposite directions.

  (b) Figure 16.14 shows three representative points on the line through the charges. In region I, to the left of the +2Q charge, the two fields are in opposite directions. There is only one point at which the fields cancel in that region because close to the +2Q charge the field from that charge dominates, and far from the charges the field from the −5Q charge dominates because that charge is larger. At some point in between the fields exactly balance. In region II, between the charges, there is no such point because both fields are directed right, and can not cancel.
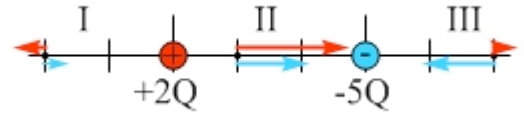

Figure 16.14: Choosing a point in each region allows us to do a qualitative analysis to determine where the net field is zero. At each point we draw two field vectors, one from each charge.

  We also see from Figure 16.14 that at a point to the right of the −5Q charge the two fields point in opposite directions. However, because such points are closer to the −5Q charge (the larger-magnitude charge), the field from the −5Q charge is always larger in magnitude than the field from the +2Q charge. Thus, there are no points in region III where the net field is zero.

  (c) Thus, we can conclude that there is only one point at which the net electric field is zero. Let's say this point is a distance x to the left of the +2Q charge. Equating the magnitude of the field from one charge at that point to the magnitude of the field from the second charge gives:

$$\frac{k|q_1|}{x^2} = \frac{k|q_2|}{(x+3.0 \text{ m})^2}.$$

Canceling factors of k leads to: $\quad \dfrac{2Q}{x^2} = \dfrac{5Q}{(x+3.0 \text{ m})^2}.$

Canceling factors of Q and re-arranging gives: $\quad \dfrac{(x+3.0 \text{ m})^2}{x^2} = \dfrac{5}{2}.$

Taking the square root of both sides gives: $\quad \dfrac{x+3.0 \text{ m}}{x} = \pm\sqrt{2.5}.$

Using the + sign gives, when we solve for x: $\quad x = \dfrac{3.0 \text{ m}}{\sqrt{2.5}-1} = 5.16 \text{ m}.$

  Thus, the electric field is zero at the point 5.16 m to the left of the +2Q charge.

Related End-of-Chapter Exercises: 57, 59, and 60.

Essential Question 16.6: The method in the previous example gives two solutions. What is the second solution in this case and what is its physical meaning?