Answer to Essential Question 16.3: Newton's third law tells us that the electrostatic forces the two objects exert on one another are equal in magnitude (and opposite in direction). This follows from Coulomb's law, because whether we look at the force exerted by the first object or the second object the factors going into the equation are the same in both cases.

16-4 Applying the Principle of Superposition

EXPLORATION 16.4 – Three objects in a line

Let's return again to the situation of three different arrangements of three balls that we looked at in Exploration 16.3. The balls, with charges of $+q$, $-2q$, and $-3q$, are equally spaced along a line. The spacing between the balls is r. In each case, the balls are in an isolated region of space very far from anything else.

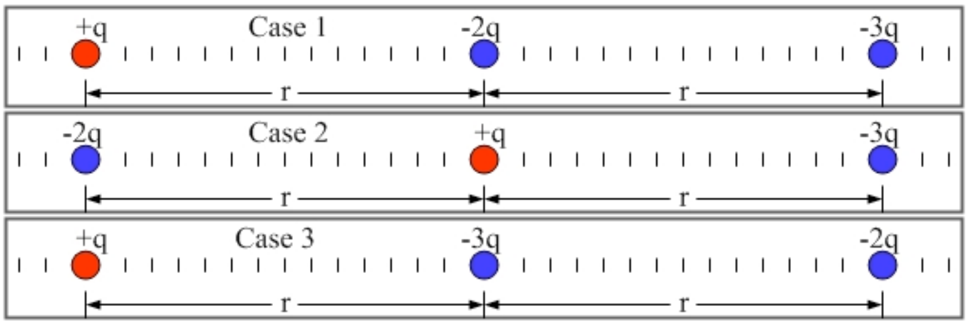

Figure 16.5: Three different arrangements of three balls of charge $+q$, $-2q$, and $-3q$ placed on a line with a distance r between neighboring balls.

Step 1 – ***Calculate the force experienced by the ball of charge –2q in each case.***

To do this, we will make extensive use of Coulomb's law. Let's define right to be the positive direction, and use the notation $\vec{F}_{21}$ for the force that the ball of charge $-2q$ experiences from the ball of charge $+q$. The + and – signs in the equation come from the direction of the force, not the signs on the charges. In each case:

$$\vec{F}_{2,net} = \vec{F}_{21} + \vec{F}_{23}$$

Case 1: $$\vec{F}_{2,net} = \vec{F}_{21} + \vec{F}_{23} = -\frac{kq(2q)}{r^2} - \frac{k(2q)(3q)}{r^2} = -\frac{2kq^2}{r^2} - \frac{6kq^2}{r^2} = -\frac{8kq^2}{r^2}$$

Case 2: $$\vec{F}_{2,net} = \vec{F}_{21} + \vec{F}_{23} = +\frac{kq(2q)}{r^2} - \frac{k(2q)(3q)}{(2r)^2} = +\frac{2kq^2}{r^2} - \frac{3kq^2}{2r^2} = +\frac{kq^2}{2r^2}$$

Case 3: $$\vec{F}_{2,net} = \vec{F}_{21} + \vec{F}_{23} = -\frac{kq(2q)}{(2r)^2} + \frac{k(2q)(3q)}{r^2} = -\frac{kq^2}{2r^2} + \frac{6kq^2}{r^2} = +\frac{11kq^2}{2r^2}$$

The ball of charge $-2q$ does experience the largest-magnitude net force in case 1.

Key ideas about adding electrostatic forces: Again, we see that the net force acting on an object can be found using the principle of superposition, remembering that each individual force is unaffected by the presence of other forces. In addition, + and – signs should be based on the direction of the force, rather than the signs of the charges.
Related End of Chapter Exercises: 30, 31.

EXAMPLE 16.4 – A two-dimensional situation

Compare this example to Example 8.1. Three balls, with charges of $+q$, $-2q$, and $+3q$, are placed at the corners of a square measuring L on each side, as shown in Figure 16.6. Assume this set of three balls is not interacting with anything else in the universe, and assume that gravitational interactions are negligible. What is the magnitude and direction of the net electrostatic force on the ball of charge $+q$?

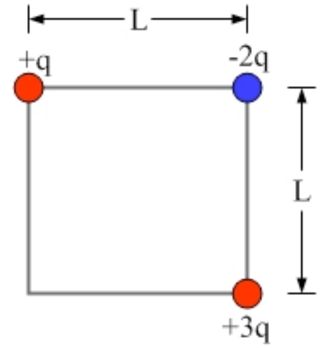

Figure 16.6: Three charged balls placed at the corners of a square.

SOLUTION

Let's attach force vectors (see Figure 16.7) to the ball of charge $+q$, which is attracted to the $-2q$ ball and repelled by the $+3q$ ball. The length of each vector is proportional to the magnitude of the force it represents.

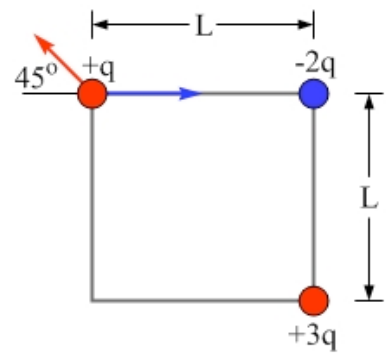

Figure 16.7: Attaching force vectors to the ball of charge $+q$.

We can find the two individual forces acting on the ball of charge $+q$ using Coulomb's law. Let's define +x to the right and +y up.

From the ball of charge $-2q$: $\vec{F}_{21} = \dfrac{kq(2q)}{L^2}$ to the right.

From the ball of charge $+3q$: $\vec{F}_{31} = \dfrac{kq(3q)}{L^2 + L^2}$ at 45° above the $-x$-axis.

Finding the net force is a vector addition problem.

In the x-direction, we get:

$$\vec{F}_{1x} = \vec{F}_{21x} + \vec{F}_{31x} = +\frac{2kq^2}{L^2} - \frac{3kq^2}{2L^2}\cos 45^\circ = \left(2 - \frac{3}{2\sqrt{2}}\right)\frac{kq^2}{L^2}.$$

Note that the signs on each term come not from the signs on the charges, but from comparing the direction of the forces to the directions we chose to be positive above.

In the y-direction, we get: $\vec{F}_{1y} = \vec{F}_{21y} + \vec{F}_{31y} = 0 + \dfrac{3kq^2}{2L^2}\sin 45^\circ = \left(+\dfrac{3}{2\sqrt{2}}\right)\dfrac{kq^2}{L^2}$.

The Pythagorean theorem gives the magnitude of the net force on the ball of charge $+q$:

$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2} = \sqrt{\left(4 - \frac{6}{\sqrt{2}} + \frac{9}{8} + \frac{9}{8}\right)}\frac{kq^2}{L^2} = 1.42\frac{kq^2}{L^2}.$$

The angle is given by: $\tan\theta = \dfrac{F_{1y}}{F_{1x}} = \dfrac{\dfrac{3}{2\sqrt{2}}}{\dfrac{4\sqrt{2}-3}{2\sqrt{2}}} = \dfrac{3}{4\sqrt{2}-3}$.

So, the angle is 48.5° above the $+x$-axis.

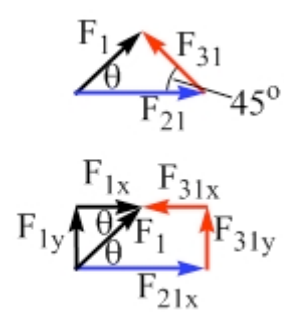

Figure 16.8: The triangle representing the vector addition problem above.

Related End-of-Chapter Exercises: 4, 5, 16, 17, 41, 49, 50, 53.

Essential Question 16.4: In Exploration 16.4, on the previous page, which ball experiences the largest-magnitude net force in (i) Case 1, (ii) Case 2, and (iii) Case 3?