Answer to Essential Question 16.2: No. Charge does not flow on an insulator, so touching charged rubber spheres together could transfer a small amount of charge between the spheres, but most of the excess charge would stay where it was. This is the big difference between conductors and insulators – charge flows easily in a conductor but does not flow through an insulator.

16-3 Coulomb's Law

To qualitatively measure the charge on an object, we can use an electroscope (see Figure 16.2). When the electroscope is charged (such as by rubbing it with a charged rod), the charge distributes itself over the entire electroscope because the electroscope is made from conducting material. Like charges repel, so the arm of the electroscope swings out, as in the electroscope on the right. The larger the charge, the more the arm swings out. To get a more quantitative measure of charge than we can get with an electroscope, we use Coulomb's law.

Figure 16.2: An uncharged electroscope, on the left, and a charged electroscope, on the right. Photo courtesy A. Duffy.

Point charges are charged objects that are so small that all the charge is effectively at one point. The force between point charges of charge q and Q, separated by a distance r, is given by:

$$\vec{F}_E = \frac{kqQ}{r^2}\hat{r} \qquad \text{(Eq. 16.2: \textbf{Coulomb's Law for the force between point charges})}$$

where $k = 8.99\times10^9 \text{ N m}^2/\text{C}^2$ is a constant. The unit vector $\hat{r}$ tells us the force is directed along the line joining the charges. If the charges have opposite signs, qQ is negative and the force is attractive, directed toward the object applying the force. If the charges have the same sign, qQ is positive and the force is repulsive, directed away from the object applying the force.

Comparing Coulomb's law to Newton's law of universal gravitation, which gives the force between two objects with mass, we see that they have the same form:

$$\vec{F}_G = -\frac{GmM}{r^2}\hat{r} \qquad \text{(Equation 8.1: \textbf{Newton's Law of Universal Gravitation})}$$

where $G = 6.67\times10^{-11} \text{N m}^2/\text{kg}^2$ is the universal gravitational constant. For Coulomb's law, k takes the place of G, and the charges q and Q take the place of the masses, m and M. Note that objects with mass always attract one another, while charged objects can attract or repel.

EXAMPLE 16.3 – Inside the hydrogen atom
A hydrogen atom contains an electron and a proton. In the Bohr model of the atom, the electron follows a circular orbit around the proton. Determine the ratio of the magnitudes of the electrostatic force to the gravitational force between the proton and electron.

SOLUTION
Note that we don't need to know the radius, because the radius cancels out in the ratio.

$$\frac{F_E}{F_G} = \frac{k|qQ|/r^2}{GmM/r^2} = \frac{k|qQ|}{GmM} = \frac{\left(9.0\times10^9 \text{ N m}^2/\text{C}^2\right)\times\left(1.6\times10^{-19} \text{ C}\right)\times\left(1.6\times10^{-19} \text{ C}\right)}{\left(6.67\times10^{-11} \text{ N m}^2/\text{kg}^2\right)\times\left(9.11\times10^{-31} \text{ kg}\right)\times\left(1.67\times10^{-27} \text{ kg}\right)} = 2.3\times10^{39}$$

The gravitational interaction is negligible, being 39 orders of magnitude smaller!

EXPLORATION 16.3 – The principle of superposition
　　　Three balls, with charges of $+q$, $-2q$, and $-3q$, are equally spaced along a line. The spacing between the balls is r. We can arrange the balls in three different ways, as shown in Figure 16.3. In each case, the balls are in an isolated region of space very far from anything else.
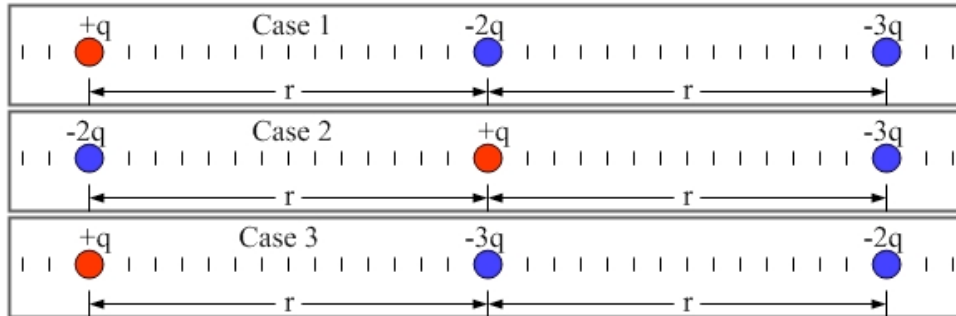

Figure 16.3: Three different arrangements of three balls of charge $+q$, $-2q$, and $-3q$ placed on a line with a distance r between neighboring balls. Each ball experiences two electrostatic forces, one from each of the other balls. We can neglect any other interactions.

Step 1 – *Consider Case 1. Is the electrostatic force that the ball of charge $+q$ exerts on the ball of charge $-3q$ affected by the fact that the ball of charge $-2q$ lies between the other two balls?*
No, we can apply the principle of superposition – the force from the interaction between any pair of charged objects is unaffected by the presence of any other charged object in the vicinity.

Step 2 – *In which case does the ball of charge $-2q$ experience the largest-magnitude net force? Argue qualitatively.* Let's attach arrows to the ball of charge $-2q$, as in Figure 16.4, to represent the two forces it experiences in each case. The length of each arrow is proportional to the force.
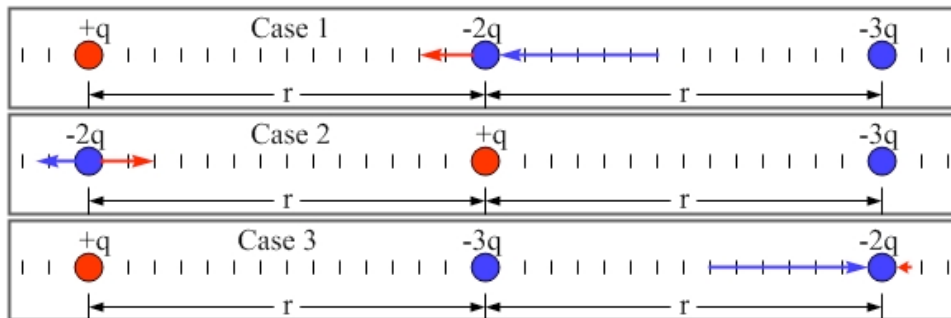

Figure 16.4: Attaching force vectors to the ball of charge $-2q$, which is attracted to the $+q$ ball and repelled by the $-3q$ ball. The length of each vector is drawn in units of kq^2/r^2.

　　　The net force is largest in case 1. In cases 2 and 3, the forces partly cancel, while only in case 1 are the directions of the two forces acting on the ball of charge $-2q$ the same, and the net force case 1 is clearly larger than it is in the other two cases.

Key ideas about adding electrostatic forces: The net force acting on an object can be found using the principle of superposition, remembering that each individual force is unaffected by the presence of other forces.　　　　**Related End of Chapter Exercises: 28, 29.**

Essential Question 16.3: An object with a charge of $+5Q$ is placed a distance r away from an object with a charge of $+2Q$. Which object exerts a larger electrostatic force on the other?