

### PROBLEM 1 – 10 points

You have three identical metal spheres that have different initial net charges. Sphere A has a net charge of  $+10Q$ ; sphere B has a net charge of  $-6Q$ ; and sphere C has a net charge of  $+14Q$ . You first touch sphere B to sphere A, and then separate them; you then touch sphere A to sphere C, and then separate them; and finally you touch sphere C to sphere B, and then separate them.

[2 points] (a) Assuming no charge is transferred to you, what is the total combined charge on the three spheres at the end of the process?

**There are two main ideas here. One is the Law of Conservation of Charge – when two spheres touch the total charge on the two afterwards equals the total charge they had before. The other idea is the symmetry involved in the situation – because the spheres are identical each sphere should have an equal amount of charge after they are separated.**

**Let's set up a table to keep track of the charge on each sphere.**

	Charge on Sphere A	Charge on Sphere B	Charge on Sphere C
<b>Initially</b>	<b><math>+10Q</math></b>	<b><math>-6Q</math></b>	<b><math>+14Q</math></b>
<b>After B and A touch</b>	<b><math>+2Q</math></b>	<b><math>+2Q</math></b>	<b><math>+14Q</math></b>
<b>After A and C touch</b>	<b><math>+8Q</math></b>	<b><math>+2Q</math></b>	<b><math>+8Q</math></b>
<b>After C and B touch</b>	<b><math>+8Q</math></b>	<b><math>+5Q</math></b>	<b><math>+5Q</math></b>

**At every step there is a combined charge of  $+18Q$ , in accordance with charge conservation.**

[2 points] (b) What is the charge on sphere A at the end of the process?

**$+8Q$**

[2 points] (c) What is the charge on sphere B at the end of the process?

**$+5Q$**

[2 points] (d) What is the charge on sphere C at the end of the process?

**$+5Q$**

[2 points] (e) What if the spheres had been made from insulating material instead, with the same initial charges. Would any of the answers above definitely be the same after the same process of touching and separating described above?

[ ] all of the answers would still definitely be the same

[ ] none of the answers have to be the same

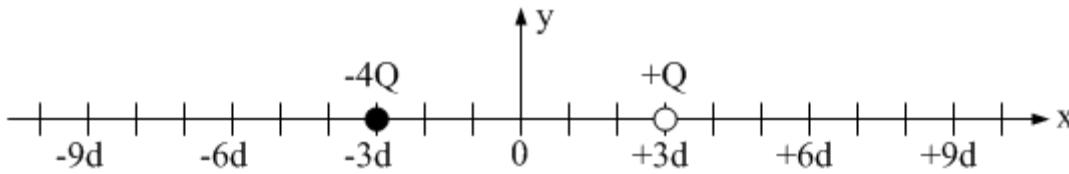
[ **X** ] the answer to part (a) would stay the same; the others could be different

[ ] the answer to part (a) could be different; the others have to stay the same

**Charge conservation still applies, but the charge does not need to be shared equally by the spheres.**

**PROBLEM 2 – 10 points**

Two charges are placed on the x-axis. The charge at  $x = -3d$  has a charge of  $-4Q$ , while the charge at  $+3d$  has a charge of  $+Q$ .



[2 points] (a) The net electric field due to the two charges is zero at at least one location on the x-axis near the two charges. In which region(s) is there such a point on the x-axis, where the net electric field is zero a finite distance from the charges? **Select all that apply.**

[ ] to the left of the  $-4Q$  charge

[ ] between the charges

[ **X** ] to the right of the  $+Q$  charge

**The fields point in the same direction between the charges, so they can not cancel. To the left of the  $-4Q$  charge the field from that charge dominates because, no matter what point you consider, the point is always closer to the larger-magnitude charge. To the right of the  $+Q$  charge there is one point where the net field is zero, where the smaller distance to the  $+Q$  charge can just make up for the fact that the  $-4Q$  charge has a larger magnitude.**

[5 points] (b) Determine the location of one such point on the x-axis near the charges where the net electric field is zero.

$$\vec{E}_1 + \vec{E}_2 = 0 \quad \text{Define right to be the positive direction.}$$

$$-\frac{k(4Q)}{(x+3d)^2} + \frac{k(Q)}{(x-3d)^2} = 0 \quad \text{Cancel the k and Q, and move one term to the other side.}$$

$$\frac{4}{(x+3d)^2} = \frac{1}{(x-3d)^2} \quad \text{Now cross-multiply.}$$

$$4 = \frac{(x+3d)^2}{(x-3d)^2} \quad \text{Now take the square root of both sides:}$$

$$2 = \frac{(x+3d)}{(x-3d)}$$

**Cross-multiply to get:**  $2x - 6d = x + 3d$ , which gives  $x = +9d$ .

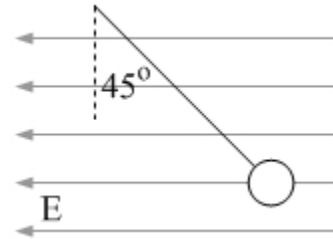
[3 points] (c) Are there any points near the charges, **but not on the x-axis**, where the net electric field due to the point charges is zero? Explain.

**No. To get a net field of zero, we need to have fields that are equal in magnitude and opposite in direction, but there is no way the two fields (one from each point charge) can be in exactly opposite directions for a point off the x-axis.**

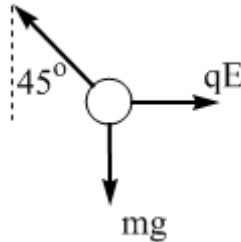
### PROBLEM 3 – 15 points

A small ball with a weight of 10 N is hung from a string that is made of insulating material.

The ball and string is first placed in a region of space in which there is a uniform electric field with  $E = 2.0 \times 10^4 \text{ N/C}$  directed horizontally to the left. The ball has an unknown net charge. When the ball is in equilibrium the string is at a  $45^\circ$  angle, as shown.



[3 points] (a) Sketch a free-body diagram for the ball in this situation.



[3 points] (b) Can we neglect gravity in this situation? Briefly justify your answer.

**Absolutely not. If gravity was negligible the string would be horizontal. In fact, because the string is at  $45^\circ$  the force associated with the electric field has exactly the same magnitude as the force of gravity acting on the ball.**

[4 points] (c) What is the sign and magnitude of the net charge on the ball?

**The ball is deflected to the right by the force applied by the field, but the field points to the left. This can only happen if the charge is negative.**

**Because the angle is  $45^\circ$  the force applied by the field,  $qE$ , has the same magnitude as the force of gravity, 10 N.**

**Solving for the magnitude of the charge gives:**  $q = \frac{10 \text{ N}}{E} = \frac{10 \text{ N}}{2.0 \times 10^4 \text{ N/C}} = 5.0 \times 10^{-4} \text{ C}.$

[5 points] (d) The string will break if its tension exceeds 25 N. The ball is now given a net charge of  $+1.0 \times 10^{-3} \text{ C}$  and placed in a region where you can introduce a uniform electric field in any direction you choose. The field is initially off, and the ball hangs vertically from the string, as shown. Assuming the magnitude of the electric field is changed gradually, what is the direction and magnitude of the **minimum** electric field necessary to cause the string to break?

**The optimal situation is when the force from the electric field is in the same direction as the force of gravity – everything works together to break the string, requiring the smallest electric field. The force of gravity is directed down, so the force from the electric field should also be down. Because the charge is positive the field should be directed down.**

**Together the force of gravity and the force from the field combine to break the string, so:**  
 $mg + qE = 25 \text{ N} \quad \rightarrow \quad qE = 25 \text{ N} - mg = 25 \text{ N} - 10 \text{ N} = 15 \text{ N}.$

**Solving for the magnitude of the field gives:**  $E = \frac{15 \text{ N}}{q} = \frac{15 \text{ N}}{1.0 \times 10^{-3} \text{ C}} = 1.5 \times 10^4 \text{ N/C}.$