

Answer to Essential Question 15.5: The ideal gas law tells us that temperature is proportional to PV . $P_2V_2 = 480 \text{ J}$ for state 2 in both processes we are considering, so the temperature in state 2 is the same in both cases. $\Delta E_{\text{int}} = nC_V \Delta T$, and all three factors on the right-hand side are the same for the two processes, so the change in internal energy is the same (+360 J, in fact). Because the gas does no work in the isochoric process, and a positive amount of work in the isobaric process, the First Law tells us that more heat is required for the isobaric process (+600 J versus +360 J).

15-6 Thermodynamic Cycles

Many devices, such as car engines and refrigerators, involve taking a thermodynamic system through a series of processes before returning the system to its initial state. Such a cycle allows the system to do work (e.g., to move a car) or to have work done on it so the system can do something useful (e.g., removing heat from a fridge). Let's investigate this idea.

EXPLORATION 15.6 – Investigate a thermodynamic cycle

One cycle of a monatomic ideal gas system is represented by the series of four processes in Figure 15.15. The process taking the system from state 4 to state 1 is an isothermal compression at a temperature of 400 K. Complete Table 15.1 to find Q , W , and ΔE_{int} for each process, and for the entire cycle.

Process	Special process?	Q (J)	W (J)	ΔE_{int} (J)
1 \rightarrow 2	No	+1360		
2 \rightarrow 3	Isobaric			
3 \rightarrow 4	Isochoric		0	
4 \rightarrow 1	Isothermal			0
Entire Cycle	No			0

Table 15.1: Table to be filled in to analyze the cycle. See Step 1 for a justification of the 0's in the table.

Step 1 – Fill in any zeroes in the table. There is no work done in the 3 \rightarrow 4 process, and the change in internal energy is zero in the 4 \rightarrow 1 process. However, ΔE_{int} for the entire cycle is also zero (this is always true), because the system returns to its original state, and therefore its original temperature.

Step 2 – Analyze the 1 \rightarrow 2 process. The work done by the gas is

$$W_{1 \rightarrow 2} = P_{\text{av}} \Delta V = 100 \text{ kPa} \times (8.0 \text{ L} - 4.0 \text{ L}) = 400 \text{ J}.$$

From the ideal gas law, PV is proportional to temperature. Because $P_1 V_1 = 320 \text{ J}$ corresponds to $T_1 = 400 \text{ K}$ (so $nR = 0.80 \text{ J/K}$ in this case), then $P_2 V_2 = 960 \text{ J} = 3(P_1 V_1)$ corresponds to a tripling of the temperature to $T_2 = 1200 \text{ K}$. We know that $C_V = 3R/2$, so:

$$\Delta E_{\text{int},1 \rightarrow 2} = \frac{3}{2} nR \Delta T = \frac{3}{2} nR (T_2 - T_1) = \frac{3}{2} (0.80 \text{ J/K})(1200 \text{ K} - 400 \text{ K}) = +960 \text{ J}.$$

Using the first law: $Q_{1 \rightarrow 2} = \Delta E_{\text{int},1 \rightarrow 2} + W_{1 \rightarrow 2} = +960 \text{ J} + 400 \text{ J} = +1360 \text{ J}$.

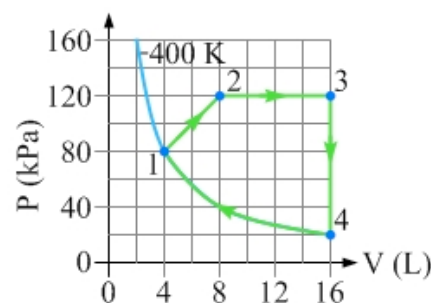


Figure 15.15: The series of four processes making up the cycle.

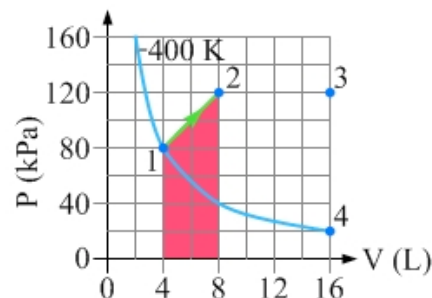


Figure 15.16: The work done by the gas in the 1 \rightarrow 2 process equals the area under the curve, shown shaded.

Step 3 – Analyze the 2 → 3 process. The work done by the gas in the 2 → 3 process is the area of the shaded region in Figure 15.17:

$$W_{2 \rightarrow 3} = P\Delta V = 120 \text{ kPa} \times (16.0 \text{ L} - 8.0 \text{ L}) = 960 \text{ J}.$$

Doubling the volume while keeping the pressure constant doubles the absolute temperature. Thus $T_3 = 2T_2 = 2400 \text{ K}$, and:

$$\Delta E_{\text{int},2 \rightarrow 3} = nC_V\Delta T = \frac{3}{2}nR(T_3 - T_2) = \frac{3}{2}(0.80 \text{ J/K})(2400 \text{ K} - 1200 \text{ K}) = +1440 \text{ J}.$$

Applying the first law tells us that

$$Q_{2 \rightarrow 3} = \Delta E_{\text{int},2 \rightarrow 3} + W_{2 \rightarrow 3} = +1440 \text{ J} + 960 \text{ J} = +2400 \text{ J}.$$

Step 4 – Analyze the 3 → 4 process. With no work done in this process:

$$Q_{3 \rightarrow 4} = \Delta E_{\text{int},3 \rightarrow 4} = \frac{3}{2}nR(T_4 - T_3) = \frac{3}{2}(0.80 \text{ J/K})(400 \text{ K} - 2400 \text{ K}) = -2400 \text{ J}.$$

Step 5 – Analyze the 4 → 1 process. $\Delta E_{\text{int}} = 0$ for an isothermal process.

The work done by the gas is the area of the shaded region in Figure 15.18, in blue to remind us that it is negative. Using Equation 15.8:

$$Q_{4 \rightarrow 1} = W_{4 \rightarrow 1} = nRT \ln(V_f/V_i) = (0.80 \text{ J/K})(400 \text{ K}) \ln(4.0 \text{ L}/16 \text{ L}) = -444 \text{ J}.$$

Step 6 – Complete the table. Each process, and the entire cycle, satisfies the first law, and values in each column sum to the value for the cycle.

Process	Special process?	Q (J)	W (J)	ΔE_{int} (J)
1 → 2	No	+1360	+400	+960
2 → 3	Isobaric	+2400	+960	+1440
3 → 4	Isochoric	-2400	0	-2400
4 → 1	Isothermal	-444	-444	0
Entire Cycle	No	+916	+916	0

Table 15.2: The completed table. Check that each row satisfies the first law and each column adds up.

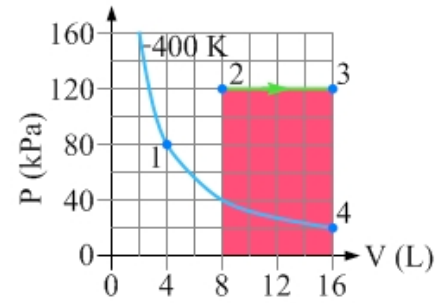


Figure 15.17: The work done by the gas in the 2 → 3 process equals the shaded area under the curve.

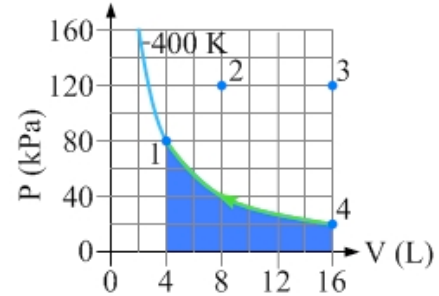


Figure 15.18: The work done by the gas in the 4 → 1 is negative, and is the shaded area under the curve.

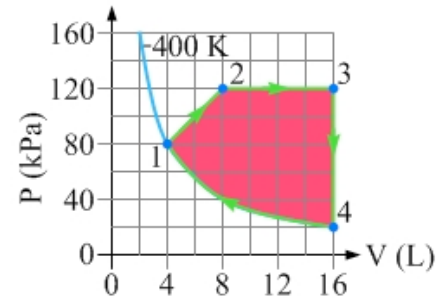


Figure 15.19: The enclosed area is the net work done in the cycle.

Key Ideas for a Thermodynamic Cycle: The cycle satisfies the first law of thermodynamics, as does each of its processes. The change in internal energy for any cycle is always zero because the system returns to its original state, and the area of the enclosed region on the P-V diagram is the net work done by the gas in the cycle. **Related End-of-Chapter Exercises: 32 – 38.**

Essential Question 15.6: Complete Table 15.3, for a cycle consisting of three processes. The process taking the system from state 1 to state 2 is adiabatic.

Process	Q (J)	W (J)	ΔE_{int} (J)
1 → 2			-300
2 → 3	+800	0	
3 → 1			
Entire Cycle		-200	

Table 15.3: Fill in the missing values.