Answer to Essential Question 15.4: The work done by the gas in the expansion is shown by the area shaded in red in Figure 15.11. We can find a numerical value for the work done by applying the First Law of Thermodynamics, $Q = \Delta E_{\text{int}} + W$. Because Q = 0, we have:

$$W = -\Delta E_{\text{int}} = -nC_V \Delta T = -\frac{3}{2} nR \left(T_f - T_i\right) = -\frac{3}{2} nRT_f + \frac{3}{2} nRT_i .$$

We could solve for the number of moles of gas, but let's instead apply the ideal gas law:

$$W = -\frac{3}{2} P_f V_f + \frac{3}{2} P_i V_i = \frac{3}{2}\left(P_i V_i - P_f V_f\right) = \frac{3}{2}\left(160 \text{ kPa} \times 4.0 \text{ L} - 28.3 \text{ kPa} \times 11.31 \text{ L}\right) = 480 \text{ J}$$

15-5 A Summary of Thermodynamic Processes

There is no single step-by-step strategy that can be applied to solve every problem involving a thermodynamic process. Instead let's summarize the tools we have to work with. These tools can be applied in whatever order is appropriate to solve a particular problem.

Tools for Solving Thermodynamics Problems

- The P-V diagram can help us to visualize what is going on. In addition, the work done by a gas in a process is the area under the curve defining that process on the P-V diagram.
- The ideal gas law, $PV = nRT$.
- The first law of thermodynamics, $Q = \Delta E_{\text{int}} + W$.
- The general expression for the change in internal energy, $\Delta E_{\text{int}} = n C_V \, \Delta T$.
- In specific special cases (see the summary in Figure 15.14), there are additional relationships that can be used to relate the different parameters.

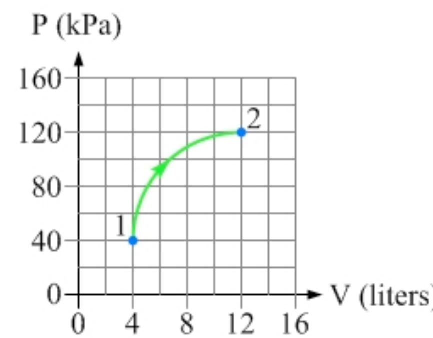

Figure 15.12: The process that moves the system from state 1 to state 2 follows a circular arc on the P-V diagram that covers ¼ of a circle.

EXAMPLE 15.5 – Applying the tools

A system of monatomic ideal gas is taken through the process shown in Figure 15.12. For this process find (a) the work done by the gas, (b) the change in internal energy, and (c) the heat added to the gas.

SOLUTION

(a) The area under the curve has been split into two parts in Figure 15.13, a ¼-circle and a rectangle. Each box on the P-V diagram measures 20 kPa × 2.0 L, representing an area of 40 J. The rectangular area covers 8 boxes, for a total of 320 J of work. The radius of the quarter-circle is four boxes, so the area of that quarter circle is given by:

$$W_{1/4} = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi \left(4 \text{ units}\right)^2 = 4\pi \text{ boxes} = 4\pi \text{ boxes} \times 40 \text{ J/box} = 500 \text{ J} .$$

Thus, the total work done by the gas is 320 J + 500 J = 820 J.

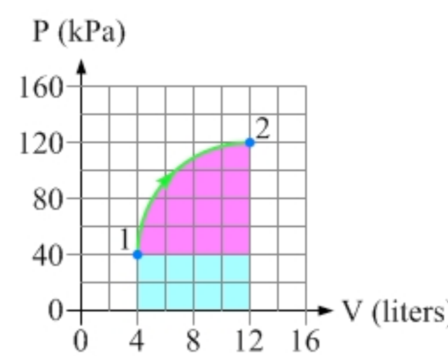

Figure 15.13: To find the work done by the gas here, it is simplest to split the area under the curve into two pieces, a ¼-circle and a rectangle.

(b) The gas is monatomic, so $C_V = 3R/2$.

Combining Eq. 15.4 with the ideal gas law:

$$\Delta E_{int} = \frac{3}{2} nR\Delta T = \frac{3}{2}\left(nRT_f - nRT_i\right) = \frac{3}{2}\left(P_f V_f - P_i V_i\right) = \frac{3}{2}(120 \text{ kPa} \times 12 \text{ L} - 4.0 \text{ kPa} \times 4.0 \text{ L}) = 1920 \text{ J}$$

(c) Using the first law of thermodynamics: $Q = \Delta E_{int} + W = 1920 \text{ J} + 820 \text{ J} = 2740 \text{ J}$.

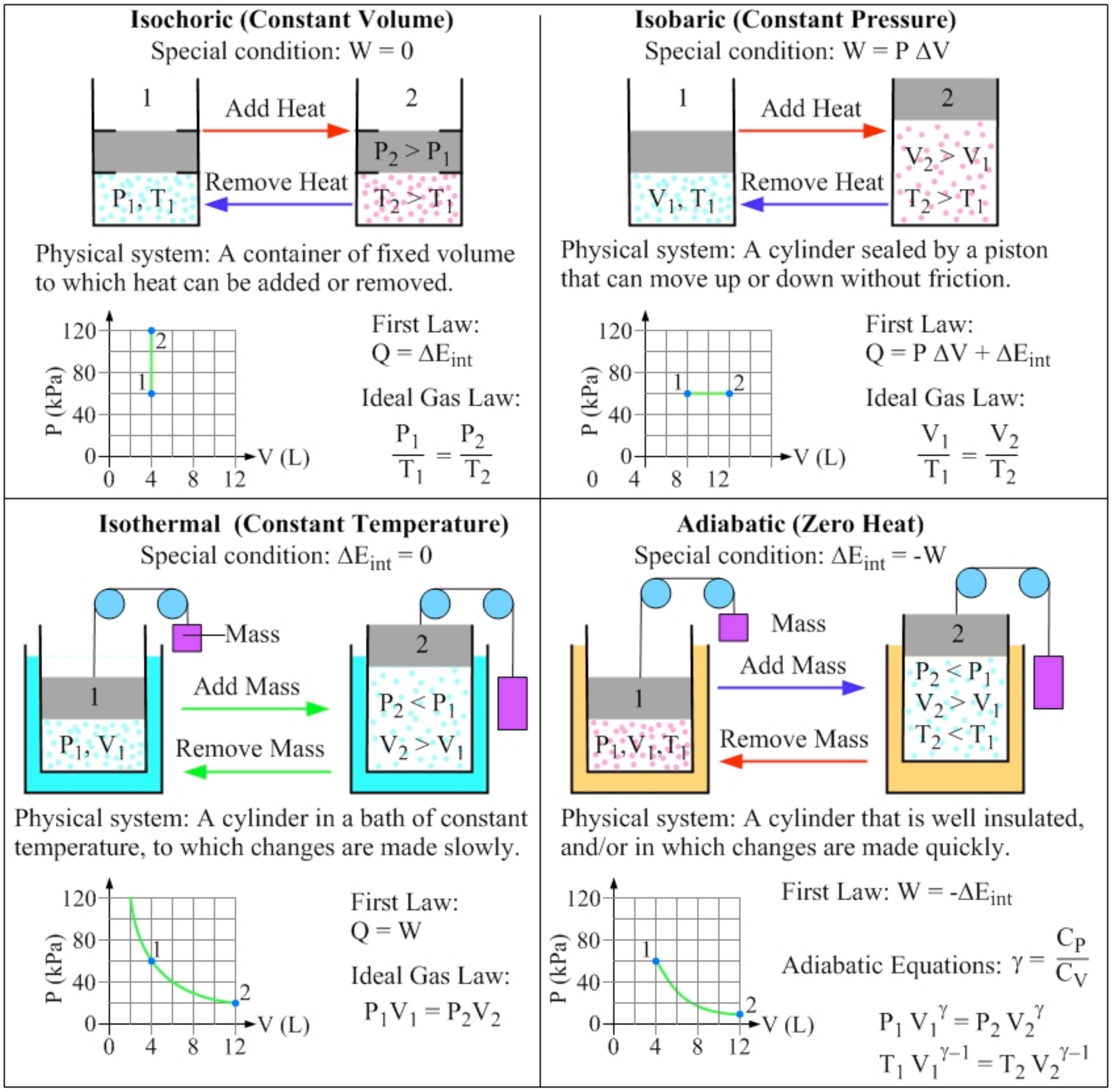

Figure 15.14: A summary of four special-case thermodynamic processes. For each, we see the special condition associated with that process; a pictorial representation and description in words of a corresponding physical system; a P-V diagram for the process; and equations we can apply to solve problems associated with the process.

Related End-of-Chapter Exercises for this section: 6, 7, 28 – 31.

Essential Question 15.5: Compare the P-V diagrams for the isochoric and isobaric processes in Figure 15.14. Assuming these pertain to the same system, and state 1 is the same in both cases, in which case is the change in internal energy larger? In which case is more heat involved?