Answer to Essential Question 14.5: To find the pressure we can apply the ideal gas law, in the form $P = nRT/V$. Because all the factors on the right-hand side are the same for the two containers the pressures must be equal. When applying the ideal gas law we do not have to worry about what the molecules consist of. We do have to account for this in determining which container has the larger internal energy, however. The internal energy for the monatomic gas is $E_{\text{int}} = (3/2)nRT$, while for the diatomic gas at room temperature it is $E_{\text{int}} = (5/2)nRT$. The monatomic ideal gas has 3/5 of the internal energy of the diatomic ideal gas.

14-6 The P-V Diagram

In Chapter 15, one of the tools we will use to analyze thermodynamic systems (systems involving energy in the form of heat and work) is the *P-V* diagram, which is a graph showing pressure on the *y*-axis and volume on the *x*-axis.

EXPLORATION 14.6 – Working with the *P-V* diagram

A cylinder of ideal gas is sealed by means of a cylindrical piston that can slide up and down in the cylinder without friction. The piston is above the gas. The entire cylinder is placed in a vacuum chamber, and air is removed from the vacuum chamber very slowly, slowly enough that the gas in the cylinder, and the air in the vacuum chamber, maintains a constant temperature (the temperature of the surroundings).

Step 1: *If you multiply pressure in units of kPa by volume in units of liters, what units do you get?*

$$1 \text{ kPa} \times 1 \text{ liter} = \left(1\times10^{3} \text{ Pa}\right)\times\left(1\times10^{-3} \text{ m}^{3}\right) = 1 \text{ Pa m}^{3} = 1 \text{ N m} = 1 \text{ J}.$$

Thus, the unit is the MKS unit the joule. This will be particularly relevant in the next chapter, when we deal with the area under the curve of the *P-V* diagram.

State	Pressure (kPa)	Volume (liters)
1	120	1.0
2	80	
3		2.0
4		3.0
5	30	
6		6.0

Table 14.2: A table giving the pressure and volume for a system of ideal gas with a constant temperature and a constant number of moles of gas.

Step 2: *Complete Table 14.2, giving the pressure and volume of the ideal gas in the cylinder at various instants as the air is gradually removed from the vacuum chamber in which the cylinder is placed.*

Using the ideal gas law, we can say that $PV = nRT$ = constant. In state 1, Table 14.2 tells us that the product of pressure and volume is 120 J. Thus the missing values in the table can be found from the equation PV = 120 J. In states 2 and 5, therefore, the gas occupies a volume of 1.5 liters and 4.0 liters, respectively. In states 3, 4, and 6, the pressure is 60 kPa, 40 kPa, and 20 kPa, respectively.

Step 3: *Plot these points on a P-V diagram similar to that in Figure 14.11, and connect the points with a smooth line. Note that such a line on a P-V diagram is known as an isotherm, which is a line of constant temperature.*

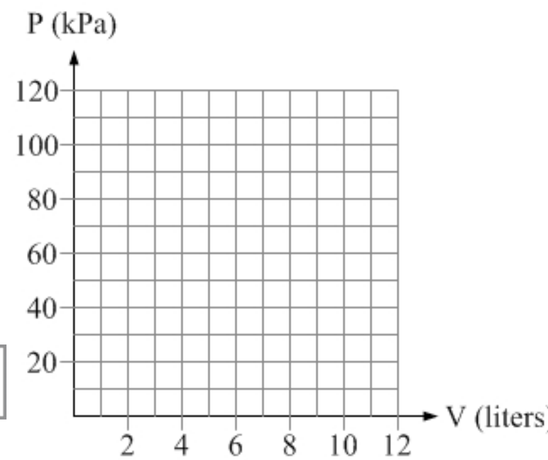

Figure 14.11: A blank *P-V* diagram.

The P-V diagram with the points plotted, and the smooth line drawn through the points representing the isotherm, is shown in Figure 14.12.

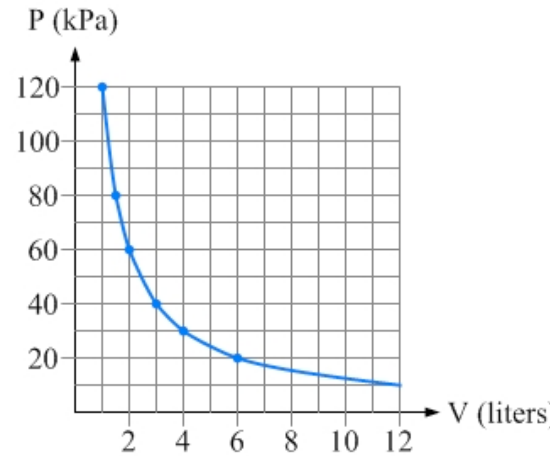

Figure 14.12: The P-V diagram corresponding to the points from Table 14.2. The smooth curve through the points is an isotherm, a line of constant temperature.

Step 4: *Repeat the process, but this time the absolute temperature of the gas is maintained at a value twice as large as that in the original process. Sketch that isotherm on the same P-V diagram.*

If the absolute temperature is doubled, the constant $P \times V$ must also double, from 120 J to 240 J. Starting with the original points we plotted, we can find points on the new isotherm by either doubling the pressure or doubling the volume. Several such points are shown on the modified P-V diagram in Figure 14.13, and we can see that this isotherm, at the higher temperature, is farther from the origin than the original isotherm. This is generally true, that the higher the temperature, the farther from the origin is the isotherm corresponding to that temperature.

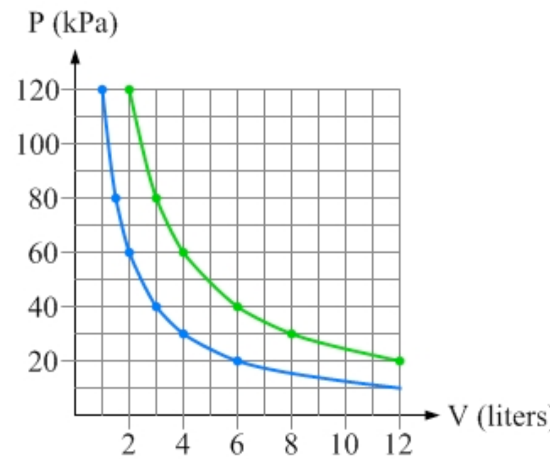

Figure 14.13: A P-V diagram showing two different isotherms. The isotherm that is farther from the origin has twice the absolute temperature as the isotherm closer to the origin.

Key Ideas about P-V diagrams: The P-V diagram (the graph of pressure as a function of volume) for a system can convey significant information about the state of the system, including the pressure, volume, and temperature of the system when it is in a particular state. It can be helpful to sketch isotherms on the P-V diagram to convey temperature information – an isotherm is a line of constant temperature.

Related End-of-Chapter Exercises: 11, 12, 49 – 52.

Essential Question 14.6: An isotherm on the P-V diagram has the shape it does because, from the ideal gas law, we are plotting pressure versus volume and the pressure is given by:

$$P = \frac{nRT}{V} .$$

For a particular isotherm, the value of nRT is constant, so an isotherm is a line with a shape similar to the plot of $1/V$ as a function of V. Let's say we now have two cylinders of ideal gas, sealed by pistons as in the previous Exploration. Cylinder A, however, has twice the number of moles of gas as cylinder B. We plot a P-V diagram for cylinder A, and plot the isotherm corresponding to a temperature of 300 K. We also draw a separate P-V diagram for cylinder B, and we find that the same points we connected to draw the 300 K isotherm on cylinder A's P-V diagram are connected to form an isotherm on cylinder B's P-V diagram. What is the temperature of that isotherm on cylinder B's P-V diagram?