Answer to Essential Question 14.3: There is no way we can answer this question. The ideal gas law, and kinetic theory, tells us about what the atoms are doing on average, but they tell us nothing about what a particular atom is doing at a particular instant in time. Atoms are continually colliding with one another and these collisions generally change both the magnitude and direction of the atom's velocity, and thus change the atom's kinetic energy. We can find the probability that an atom has a speed larger or smaller than some value, but that's about it.

14-4 Example Problems

EXPLORATION 14.4 – Finding pressure in a cylinder that has a movable piston

A cylinder filled with ideal gas is sealed by means of a piston. The piston is a disk, with a weight of 20.0 N, that can slide up or down in the cylinder without friction but which is currently at its equilibrium position. The inner radius of the cylinder, and the radius of the piston, is 10.0 cm. The top of the piston is exposed to the atmosphere, and the atmospheric pressure is 101.3 kPa. Our goals for this problem are to determine the pressure inside the cylinder, and then to determine what changes if the temperature is raised from 20°C to 80°C.

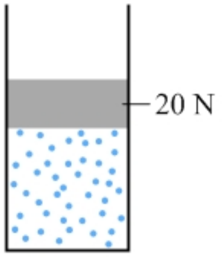

Figure 14.5: A diagram of the ideal gas sealed inside a cylinder by a piston that is free to move up and down without friction.

Step 1: ***Picture the scene***. A diagram of the situation is shown in Figure 14.5.

Step 2: ***Organize the data***. The best way to organize what we know in this case is to draw a free-body diagram of the piston, as in Figure 14.6. Three forces act on the piston: the force of gravity; a downward force associated with the top of the piston being exposed to atmospheric pressure; and an upward force from the bottom of the piston being exposed to the pressure in the cylinder.

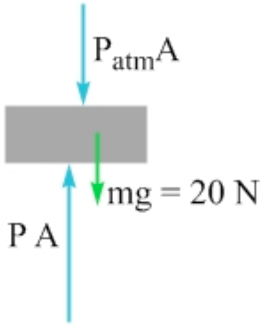

Figure 14.6: The free-body diagram of the piston, showing the forces acting on it.

Step 3: ***Solve the problem***. The piston is in equilibrium, so let's apply Newton's second law, $\sum \vec{F} = m\vec{a} = 0$, to the piston. Choosing up to be positive gives:

$+PA - mg - P_{atm}A = 0$, where A is the cross-sectional area of the piston.

Solving for P, the pressure inside the cylinder, gives:

$$P = \frac{mg + P_{atm}A}{A} = \frac{mg}{\pi r^2} + P_{atm} = \frac{20.0 \text{ N}}{\pi (0.100 \text{ m})^2} + 101300 \text{ Pa} = 101900 \text{ Pa} .$$

The pressure inside the cylinder is not much larger than atmospheric pressure.

Step 4: ***The temperature of the gas inside the piston is gradually raised from 20°C to 80°C, bringing the piston to a new equilibrium position. What happens to the pressure of the gas, and what happens to the volume occupied by the gas? Be as quantitative as possible.***

To answer the question about pressure we can once again draw a free-body diagram of the piston. However, the fact that the piston has changed position to a new equilibrium position in the cylinder changes nothing on the free-body diagram. Thus, the pressure in the cylinder is the same as it was before. The fact that the temperature increases, however, means the volume increases by the same factor. Because the pressure is constant, we can re-arrange the ideal gas law to:

$$\frac{P}{nR} = \frac{T}{V} = \text{constant}\,.$$

This tells us that $\frac{T_i}{V_i} = \frac{T_f}{V_f}$.

Re-arranging to find the ratio of the volumes, and using absolute temperatures, gives:

$$\frac{V_f}{V_i} = \frac{T_f}{T_i} = \frac{(273+80)\text{K}}{(273+20)\text{K}} = 1.20\,.$$

The volume expands by 20%, increasing by the same factor as the absolute temperature.

Key Idea for a cylinder sealed by a movable piston: When ideal gas is sealed inside a cylinder by a piston that is free to move without friction, the pressure of the gas is generally determined by balancing the forces on the piston's free-body diagram rather than from the volume or temperature of the gas. **Related End-of-Chapter Exercises: 5, 26, 27, 30, 35.**

EXAMPLE 14.4 – Comparing two pistons

The two cylinders in Figure 14.7 contain an identical number of moles of the same type of ideal gas, and they are sealed at the top by identical pistons that are free to slide up and down without friction. The top of each piston is exposed to the atmosphere. One piston is higher than the other. (a) In which cylinder is the volume of the gas larger? (b) In which piston is the pressure higher? (c) In which piston is the temperature higher?

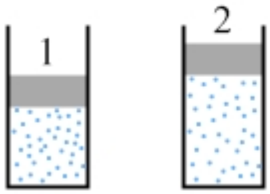

Figure 14.7: The cylinders contain the same number of moles of ideal gas, but the piston in cylinder 2 is at a higher level. The pistons are identical, are free to slide up and down without friction, and the top of each piston is exposed to the atmosphere.

SOLUTION

(a) Cylinder 2 has a larger volume. Note that the volume in question is not the volume of the molecules themselves, but the volume of the space the molecules are confined to. In other words, it is the volume inside the cylinder itself, below the piston.

(b) Despite the fact that the piston in cylinder 2 is at a higher level than the piston in cylinder 1, the pressure is the same in both cylinders is the same. This is because the free-body diagrams in Figure 14.8 applies to both pistons. The pressure in both cylinders exceeds atmospheric pressure by an amount that is just enough to balance the pressure associated with the downward force of gravity acting on the piston. The pressure is equal in both cases because the pistons are identical.

(c) Applying the ideal gas law tells us that the temperature is larger in cylinder 2, because $T = PV/nR$ and the only factor that is different on the right-hand side of that equation is the volume. In this case the absolute temperature is proportional to the volume.

Related End-of-Chapter Exercises: 6, 7, 20 – 25, 28, 29.

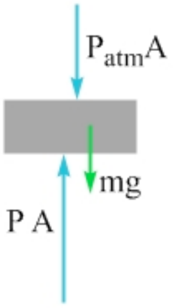

Figure 14.8: The free-body diagram applies equally well to both pistons.

Essential Question 14.4: Piston 2, in Figure 14.7, could be the same piston as piston 1, but just at a later time. What could you do to move the system from the piston 1 state to the piston 2 state?