

Answer to Essential Question 14.2: The average velocity of the atoms is zero. This is because the motion of the atoms is random, and with a large number of atoms in the box there are as many, on average, going one way as the opposite way. Because velocity is a vector the individual vectors tend to cancel one another out. The average speed, however, is non-zero, and it makes sense that, the faster the atoms move, the more force they exert on the wall.

14-3 Temperature

Let's pick up where we left off at the end of the previous section. Because pressure is force divided by area, we can find the average pressure the atoms exert on the wall by dividing the average force by the wall area, L^2 . This gives:

$$P = \frac{\bar{F}_{\text{on wall}}}{A} = \frac{2N}{3L^3} K_{av} . \quad (\text{Equation 14.12: Pressure in the gas})$$

Now we have a factor of L^3 , which is V , the volume of the cube. We can thus write Equation 14.12 as:

$$PV = N \left(\frac{2}{3} K_{av} \right) . \quad (\text{Equation 14.13: The product } PV)$$

Compare Equation 14.13 to Equation 14.2, the ideal gas law in the form $PV = NkT$. These equations must agree with one another, so we must conclude that:

$$\frac{2}{3} K_{av} = kT ,$$

or, equivalently,

$$K_{av} = \frac{3}{2} kT . \quad (\text{Equation 14.14: Average kinetic energy is directly related to temperature})$$

This is an amazing result – it tells us what temperature is all about. Temperature is a direct measure of the average kinetic energy of the atoms in a material. It is further amazing that we obtained such a fundamental result by applying basic principles of physics (such as impulse, kinetic energy, and pressure) to an ideal gas. Consider now the following example.

EXAMPLE 14.3 – Two containers of ideal gas

Container A holds N atoms of ideal gas, while container B holds $5N$ atoms of the same ideal gas. The two containers are at the same temperature, T .

- In which container is the pressure highest?
- In which container do the atoms have the largest average kinetic energy? What is that average kinetic energy in terms of the variables specified above?
- In which container do the atoms have the largest total kinetic energy? What is that total kinetic energy in terms of the variables specified above?

SOLUTION

(a) We don't know anything about the volumes of the two containers, so there is not enough information to say how the pressures compare. All we can say is that the product of the pressure multiplied by the volume is five times larger in container B than in container A , because PV is proportional to the product of the number of atoms multiplied by the absolute temperature.

(b) The fact that the temperatures are equal tells us that the average kinetic energy of the atoms is the same in the two containers. Applying Equation 14.14, we get in each case:

$$K_{av} = \frac{3}{2}kT .$$

(c) The total kinetic energy is the average energy multiplied by the number of atoms, so container *B* has the larger total kinetic energy. Container *B* has a total kinetic energy of:

$$K_B = 5NK_{av} = 5N \frac{3}{2}kT = \frac{15}{2}NkT .$$

Related End-of-Chapter Exercises: 8, 10, 37.

Absolute zero

Another interesting concept contained in the ideal gas law is the idea of absolute zero. Let's say we seal a sample of ideal gas in a container that has a constant volume. The container has a pressure gauge connected to it that allows us to read the pressure inside. We then measure the pressure as a function of temperature, placing the container into boiling water (100 °C), ice water (0 °C), and liquid nitrogen (−196 °C). The pressures at these temperatures are 129 kPa, 93.9 kPa, and 26.6 kPa, respectively. Plotting pressure as a function of temperature results in the graph shown in Figure 14.4. We find that our three points, and other points we care to measure, fall on a straight line. Extrapolating this line to zero pressure tells us that the pressure equals zero at a temperature of −273 °C (also known as 0 K).

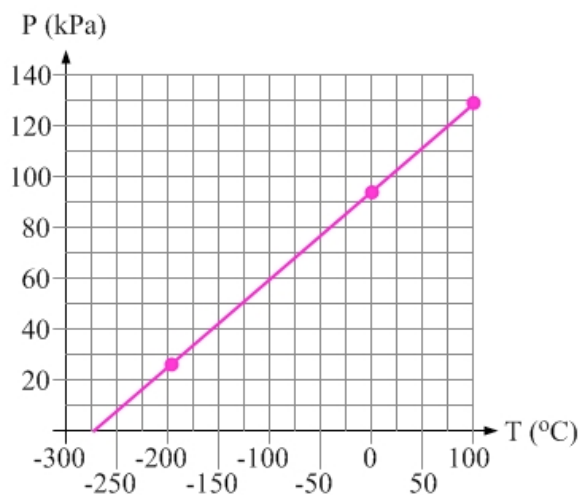


Figure 14.4: A graph of pressure as a function of temperature for a constant-volume situation. Extrapolating the graph to zero pressure shows that absolute zero corresponds to a temperature of −273 °C.

Based on the previous section, we would conclude that the pressure drops to zero at absolute zero because the atoms or molecules have no kinetic energy. This is not quite true, although applying ideas of quantum mechanics is necessary to understand why not. If the atoms and molecules stopped completely, we would be able to determine precisely where they are. Heisenberg's uncertainty principle, an idea from quantum mechanics, tells us that this is not possible, that the more accurately we know an object's position the more uncertainty there is in its momentum. The bottom line is that even at absolute zero there is motion, known as zero-point motion. Absolute zero can thus be defined as the temperature that results in the smallest possible average kinetic energy.

Essential Question 14.3: At a particular instant compare the kinetic energy of one particular atom in container *A* to that of one particular atom in container *B*. Which atom has the larger kinetic energy? The two containers are at the same temperature, and there are five times more atoms in container *B* than in container *A*.