Answer to Essential Question 14.1: It is tempting to say that the pressure increases by a factor of 3, but that is incorrect. Because the ideal gas law involves *T*, not ΔT, we must use temperatures in Kelvin rather than Celsius. In Kelvin, the temperature rises from 283K to 303K. Finding the ratio of the final pressure to the initial pressure shows that pressure increases by a factor of 1.07:

$$\frac{P_f}{P_i} = \frac{nRT_f / V}{nRT_i / V} = \frac{T_f}{T_i} = \frac{303K}{283K} = 1.07 .$$

14-2 Kinetic Theory

We will now apply some principles of physics we learned earlier in the book to help us to come to a fundamental understanding of temperature. Consider a cubical box, measuring *L* on each side. The box contains *N* identical atoms of a monatomic ideal gas, each of mass *m*.

We will assume that all collisions are elastic. This applies to collisions of atoms with one another, and to collisions involving the atoms and the walls of the box. The collisions between the atoms and the walls of the box give rise to the pressure the walls of the box experience because the gas is enclosed within the box, so let's focus on those collisions.

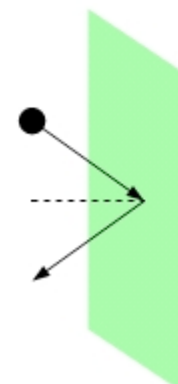

Figure 14.3: An atom inside the box bouncing off the right-hand wall of the box.

Let's find the pressure associated with one atom because of its collisions with one wall of the box. As shown in Figure 14.3 we will focus on the right-hand wall of the box. Because the atom collides elastically, it has the same speed after hitting the wall that it had before hitting the wall. The direction of its velocity is different, however. The plane of the wall we're interested in is perpendicular to the *x*-axis, so collisions with that wall reverse the ball's *x*-component of velocity, while having no effect on the ball's *y* or *z* components of velocity. This is like the situation of the hockey puck bouncing off the boards that we looked at in Chapter 6.

The collision with the wall changes the *x*-component of the ball's velocity from $+v_x$ to $-v_x$, so the ball's change in velocity is $-2v_x$ and its change in momentum is $\Delta\vec{p} = -2mv_x$, where the negative sign tells us that the change in the atom's momentum is in the negative *x*-direction.

In Chapter 6, we learned that the change in momentum is equal to the impulse (the product of the force $\vec{F}$ and the time interval Δt over which the force is applied). Thus:

$$\vec{F}_{\text{wall on molecule}} = \frac{-2mv_x}{\Delta t} .$$ (Equation 14.3: **The force the wall exerts on an atom**)

The atom feels an equal-magnitude force in the opposite direction (Newton's third law):

$$\vec{F}_{\text{molecule on wall}} = \frac{+2mv_x}{\Delta t} .$$ (Equation 14.4: **The force the atom exerts on the wall**)

What is this time interval, Δt? The atom exerts a force on the wall only during the small intervals it is in contact with the wall while it is changing direction. It spends most of the time not in contact with the wall, not exerting any force on it. We can find the time-averaged force the atom exerts on the wall by setting Δt equal to the time between collisions of the atom with that wall. Because the atom travels a distance *L* across the box in the *x*-direction at a speed of v_x, it takes a time of L / v_x to travel from the right wall of the box to the left wall, and the same amount of time to come back again. Thus:

$$\Delta t = \frac{2L}{v_x}. \qquad \text{(Equation 14.5: } \textbf{Time between collisions with the right wall}\text{)}$$

Substituting this into the force equation, Equation 14.4, tells us that the magnitude of the average force this one atom exerts on the right-hand wall of the box is:

$$\bar{F}_{\text{molecule on wall}} = \frac{2mv_x}{\Delta t} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}. \qquad \text{(Eq. 14.6: } \textbf{Average force exerted by one atom}\text{)}$$

To find the total force exerted on the wall we sum the contributions from all the atoms:

$$\bar{F}_{\text{on wall}} = \sum \frac{mv_x^2}{L} = \frac{m}{L}\sum v_x^2. \qquad \text{(Equation 14.7: } \textbf{Average force from all atoms}\text{)}$$

The Greek letter $\sum$ (sigma) indicates a sum. Here the sum is over all the atoms in the box.

If we have N atoms in the box then we can write this as:

$$\bar{F}_{\text{on wall}} = \frac{Nm}{L}\left(\frac{\sum v_x^2}{N}\right). \qquad \text{(Equation 14.8: } \textbf{Average force from all atoms}\text{)}$$

The term in brackets represents the average of the square of the magnitude of the x-component of the velocity of each atom. For a given atom if we apply the Pythagorean theorem in three dimensions we have $v_x^2 + v_y^2 + v_z^2 = v^2$. Doing this for all the atoms gives:

$$\sum v_x^2 + \sum v_y^2 + \sum v_z^2 = \sum v^2,$$

and there is no reason why the sum over the x-components would be any different from the sum over the y or z-components – there is no preferred direction in the box. We can thus say that $3\sum v_x^2 = \sum v^2$ or, equivalently, $\sum v_x^2 = \frac{1}{3}\sum v^2$.

Substituting this into the force equation, Equation 14.8, above gives:

$$\bar{F}_{\text{on wall}} = \frac{Nm}{3L}\left(\frac{\sum v^2}{N}\right). \qquad \text{(Equation 14.9: } \textbf{Average force on a wall}\text{)}$$

The term in brackets represents the square of the rms average speed. Thus:

$$\bar{F}_{\text{on wall}} = \frac{Nm}{3L}v_{rms}^2. \qquad \text{(Equation 14.10: } \textbf{Average force on a wall}\text{)}$$

By multiplying by 2 and dividing by 2, we can transform Equation 14.10 to:

$$\bar{F}_{\text{on wall}} = \frac{2N}{3L}\left(\frac{1}{2}mv_{rms}^2\right) = \frac{2N}{3L}K_{av}, \qquad \text{(Eq. 14.11: } \textbf{Force connected to kinetic energy}\text{)}$$

The term in brackets is a measure of the average kinetic energy, K_{av}, of the atoms.

Related End-of-Chapter Exercise: 36.

Essential Question 14.2: Why is the rms average speed, and not the average velocity, involved in the equations above? What is the average velocity of the atoms of ideal gas in the box?