14-1 The Ideal Gas Law

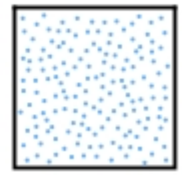

Figure 14.1: A container of ideal gas.

Let's say you have a certain number of moles of ideal gas that fills a container that has a known volume. Such a system is shown in Figure 14.1.

If you know the absolute temperature of the gas, what is the pressure? The answer can be found from the ideal gas law, which you may well have encountered before.

The ideal gas law connects the pressure P, the volume V, and the absolute temperature T, for an ideal gas of n moles:

$$PV = nRT \, , \qquad \text{(Equation 14.1: \textbf{The ideal gas law})}$$

where $R = 8.31 \text{ J/(mol K)}$ is the universal gas constant.

First of all, what is a mole? It is a not a cute, furry creature that you might find digging holes in your backyard. In this context, a mole represents an amount, and we use the term mole in the same way we use the word dozen. A dozen represents a particular number, 12. A mole also represents a particular number, 6.02×10^{23}, which we also refer to as Avogadro's number, N_A. Thus, a mole of something is Avogadro's number of those things. In this chapter, we generally want to know about the number of moles of a particular ideal gas. A toy balloon, for instance, has about 0.1 moles of air molecules inside it. Strangely enough, the number of stars in the observable universe can also be estimated at about 0.1 moles of stars.

In physics, we often find it convenient to state the ideal gas law not in terms of the number of moles but in terms of N, the number of atoms or molecules, where $N = n N_A$. Taking the ideal gas law and multiplying the right-hand side by N_A / N_A gives:

$$PV = nN_A \frac{R}{N_A} T = N \frac{R}{N_A} T \, .$$

The constant R / N_A has the value $k = 1.38 \times 10^{-23}$ J/K and is known as Boltzmann's constant. Using this in the equation above gives:

$$PV = NkT \, . \qquad \text{(Eq. 14.2: \textbf{Ideal gas law in terms of the number of molecules})}$$

Under what conditions is the ideal gas law valid? What is an ideal gas, anyway? For a system to represent an ideal gas it must satisfy the following conditions:

1. The system has a large number of atoms or molecules.
2. The total volume of the atoms or molecules should represent a very small fraction of the volume of the container.
3. The atoms or molecules obey Newton's Laws of motion; and they move about in random motion.
4. All collisions are elastic. The atoms or molecules experience forces only when they collide, and the collisions take a negligible amount of time.

The ideal gas law has a number of interesting implications, including –

Boyle's Law: at constant temperature, pressure and volume are inversely related;

Charles' Law: at constant pressure, volume and temperature are directly related;

Gay-Lussac's Law: at constant volume, pressure and temperature are directly related.

EXAMPLE 14.1 – Two containers of gas

The two sealed containers in Figure 14.2 contain the same type of ideal gas. Container 2 has twice the volume of container 1. Aside from that difference, the containers differ in only one of the following three parameters, pressure, number of moles, and temperature. (a) Could the containers differ only in pressure and volume? If so, explain how. (b) Could the containers differ only in the number of moles and volume? If so, explain how.

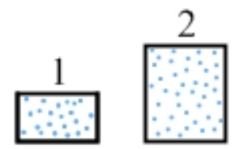

Figure 14.2: Two containers of ideal gas, one with twice the volume as the other.

SOLUTION

(a) Yes, if the number of moles of gas and the temperature are the same in each container, we must have the product of PV equal in the two containers, according to the ideal gas law. Thus, if container 2 has twice the volume as container 1, it must have half the pressure as container 1.

(b) Yes, if the pressure and the temperature are the same in each container, the number of moles of gas must be twice that in container 2 as it is in container 1. If we double the value of the volume, on the left side of Equation 14.1, we must double the value of *n*, the number of moles, on the right side of the equation, if everything else remains constant.

Prove to yourself that the containers could also differ only in volume and temperature.

An aside – Thinking about the rms average.

In Section 14.2, we will use the rms (root-mean square) average speed of a set of gas molecules. To gain some insight into the root-mean-square averaging process, let's work out the rms average of the set of numbers –1, 1, 3, and 5. The average of these numbers is 2. To work out the rms average, square the numbers to give 1, 1, 9, and 25. Then, find the average of these squared values, which is 9. Finally, take the square root of that average to find the rms average, 3.

Clearly, this is a funny way to do an average, because the average is 2 while the rms average is 3. There are two reasons why the rms average is larger than the average in this case. The first reason is that squaring the numbers makes everything positive – without this negative values cancel positive values when we add the numbers up. The second reason is that squaring the values weights the larger numbers more heavily (the 5 counts five times more than the 1 when doing the average, but 5^2 counts 25 times more than 1^2 when doing the rms average.) Note that we will discuss rms average values again later in the book when we talk about alternating current.

Related End-of-Chapter Exercises: 1, 2, 13, 17, 18.

Essential Question 14.1: A container of ideal gas is sealed so that it contains a particular number of moles of gas at a constant volume and an initial pressure of P_i. If the temperature of the system is then raised from 10°C to 30°C, by what factor does the pressure increase?